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## Management Strategies and Dynamic Financial Analysis

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## Outline

1. Motivation
2. Model Framework
3. Management Strategies
4. Performance Measurement
5. Simulation Study
6. Role of Non-linear Dependencies
7. Conclusion and Outlook

## 1. Motivation: Three pillars of Solvency II

### Solvency II

#### First pillar:

Quantitative regulations for capital requirements

- Technical provisions, minimum capital, target capital
- Use of standard models and internal models (Dynamic Financial Analysis)

#### Second pillar:

Qualitative elements of supervision

- Appropriate processes and decisions in the context of a risk management system
- Principles for internal risk management and control

#### Third pillar:

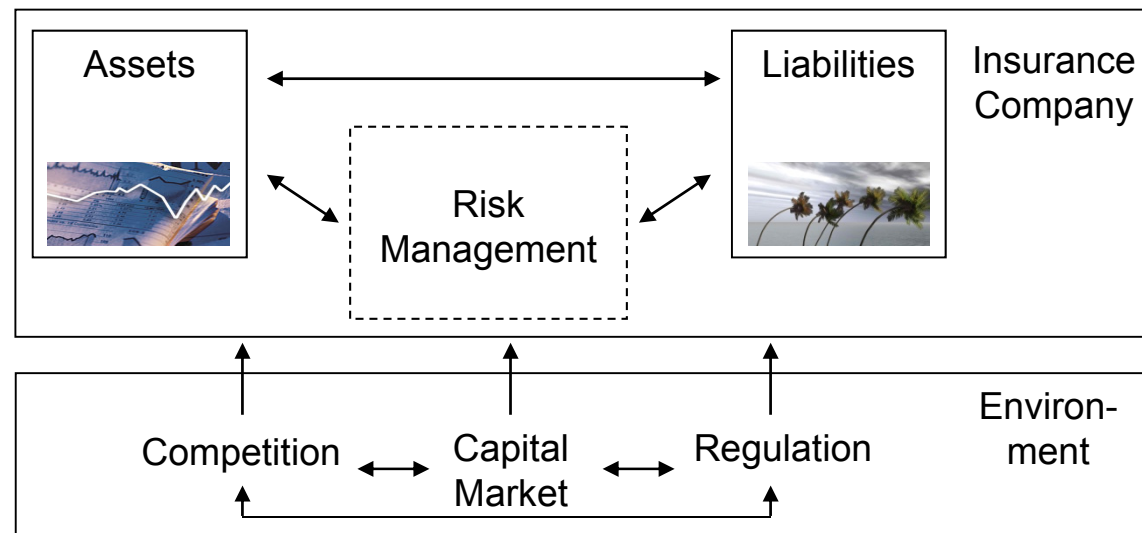
Market transparency and disclosure requirements

- A transparent process will require less regulation as market participants themselves force appropriate insurer behavior
- Harmonization with IFRS

## 1. Motivation: Dynamic Financial Analysis (DFA)

- Projects results under a variety of possible scenarios, showing how outcomes might be affected by changing internal and external conditions

- Used in practice for cash flow projection and decision support



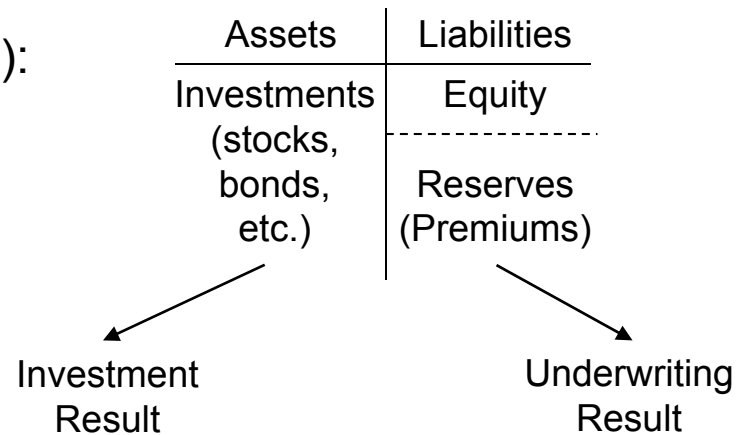
- Aim of this paper:

1. Implement management strategies in a DFA framework
2. Study the effects on the insurer's risk and return position
3. Give helpful insights for the development of DFA tools

## 2. Model Framework

- Simplified model of a property-liability insurer

- Balance sheet (t=0):



- Statement of Income (t=1):
  - Premiums
  - Claims
  - Costs (Upfront, Claim Settlement)
  - = Underwriting Result
  - + Investment Result
  - = Earnings

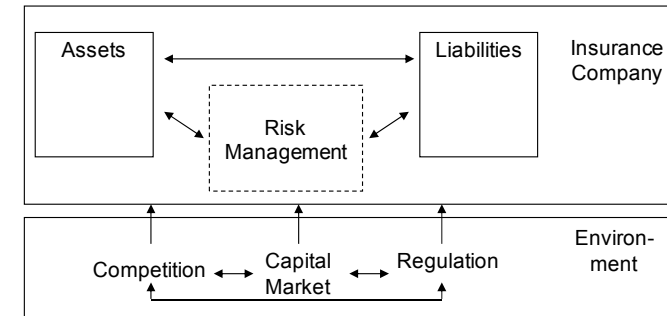


## 2. Model Framework: Earnings



$$(1) EC_t = EC_{t-1} + E_t$$

$$(2) E_t = I_t + U_t - \max(tr \cdot (I_t + U_t), 0)$$



$EC_t$ : Equity Capital at the end of period t

$E_t$  : Earnings

$I_t$  : Investment Result

$U_t$  : Underwriting Result

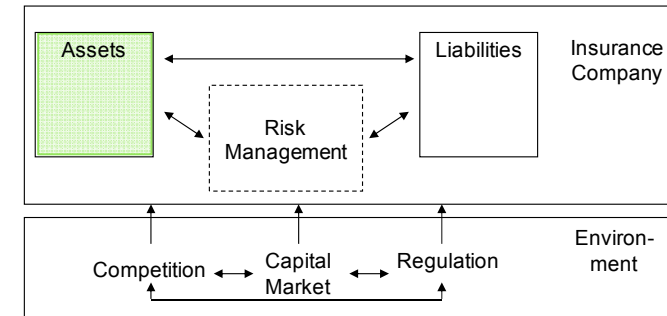
$tr$  : Tax rate

## 2. Model Framework: Investment result



$$(3) I_t = r_{pt} \cdot (EC_{t-1} + P_{t-1} - Ex_{t-1}^P)$$

$$(4) r_{pt} = \alpha_{t-1} \cdot r_{1t} + (1 - \alpha_{t-1}) \cdot r_{2t}$$



$r_{pt}$  : Return of investment portfolio

$P_{t-1}$  : Premiums

$Ex_{t-1}^P$  : Upfront costs (depending on premiums)

$\alpha_{t-1}$  : Portion invested in high-risk investments

$r_{1t}$  : Return of high-risk investment (e.g., stocks)

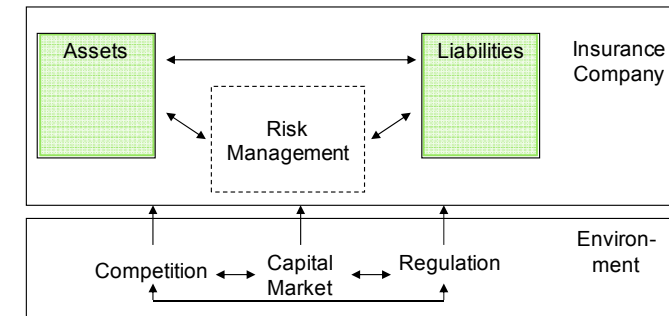
$r_{2t}$  : Return of low-risk investment (e.g., bonds)

## 2. Model Framework: Underwriting result



$$(5) U_t = P_{t-1} - C_t - Ex_{t-1}^P - Ex_t^C$$

$$(6) P_{t-1} = cr_{t-1}^{EC_{t-1}} \cdot \pi_{t-1} \cdot \beta_{t-1} \cdot MV$$



- Consumer response ( $cr$ ) to changes in solvency

$$cr = 1, \text{ if } EC_t > MCR$$

$$cr < 1, \text{ if } EC_t < MCR$$

- Underwriting cycle ( $\pi$ ): Markov chain with different states

- Claims:  $C_t = C_{cat_t} + C_{ncat_t}$

$Ex_t^C$  : Claim settlement costs

$cr$  : Consumer response

$\Pi_t$  : Underwriting cycle

$MCR$  : Minimum capital required (Solvency I)

$\beta_{t-1}$  : Portion in the underwriting market



## 2. Model: Implementation in R (simplified one period example)

```

E=0
EC=15
MV=200
β=0.2
P=MV*β
ExP<-0.05*P
tr=0.25
α=0.2

for (i in 1:10000) {
# Assets
rp<-α*rnorm(1,0.1,0.2)+
(1-α)*rnorm(1,0.05,0.05)
l<-rp*(EC+P-ExP)

# Liabilities
mu=log(0.85)-0.5*log(1+0.085^2/0.85^2)
sigma= (log(1+0.085^2/0.85^2))^(1/2)
C<-rlnorm(1,mu,sigma)*P
ExC<-0.05*C
U<-P-C-ExP-ExC

# Aggregation
E[i]<-l+U-max(tr*(l+U),0)
} #end for i

hist (E)

mean(E)
sd(E)
summary(E)

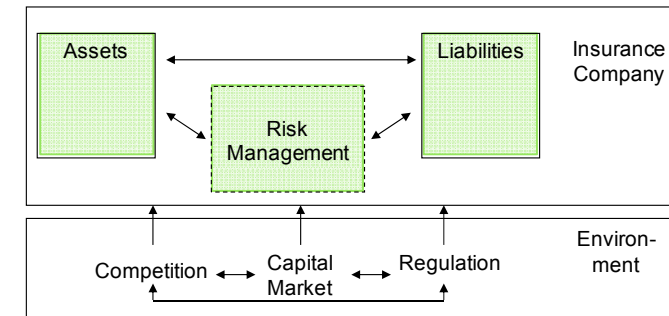
```



### 3. Management Strategies



- At the beginning of each period management can change:
  - Portion of the risky investment ( $\alpha$ )
  - Share in the underwriting business ( $\beta$ )
- Three Strategies under consideration:



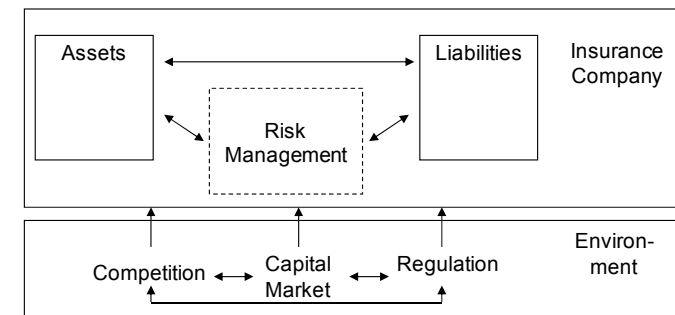
Strategy	Solvency	High Risk	Growth	
Target	Risk Reduction	Risk Taking	Risk Reduction and Risk Taking	
Trigger	$EC_t < MCR_t \cdot 1.5$	$EC_t < MCR_t \cdot 1.5$	$EC_t < MCR_t \cdot 1.5$	$EC_t > MCR_t \cdot 1.5$
Rule	$\alpha$ and $\beta$ 0.05 ↓	$\alpha$ and $\beta$ 0.05 ↑	$\alpha$ and $\beta$ 0.05 ↓	$\beta$ 0.05 ↑

## 4. Performance Measurement

	Symbol	Measure	Interpretation
Return	$E(G)$	Expected gain per annum	Absolute return
	ROI	Expected return on investment per annum	Relative return
Risk	$\sigma(G)$	Standard deviation of gain per annum	Total risk
	RP	Ruin probability	Downside risk
	EPD	Expected policyholder deficit	Downside risk
Performance	$SR_{\sigma}$	Sharpe ratio	Return/total risk
	$SR_{RP}$	Modified Sharpe ratio (RP)	Return/downside risk
	$SR_{EPD}$	Modified Sharpe ratio (EPD)	Return/downside risk

## 5. Simulation Study: Model Specifications

- Time horizon:  $T = 5$  years, equity capital in  $t = 0$ : €15 million
- Trigger for the management strategies: Solvency I  $MCR \cdot 1.5$
- Investments ( $\alpha$ ): High-risk  $N(0.1, 0.2)$ , low-risk  $N(0.05, 0.05)$
- Underwriting business ( $\beta$ ): Market volume €200 million
  - Log-normally distributed claims  $LN(0.85, 0.085)$
  - Underwriting cycle with three different states (1.05, 1, 0.95) and the transition probabilities  $p_{sj} = \begin{pmatrix} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.5 & 0.4 \end{pmatrix}$
  - Consumer response: 0.95 if  $EC < MCR \cdot 1.5$
- Tax rate: 25%



## 5. Simulation Study: Results



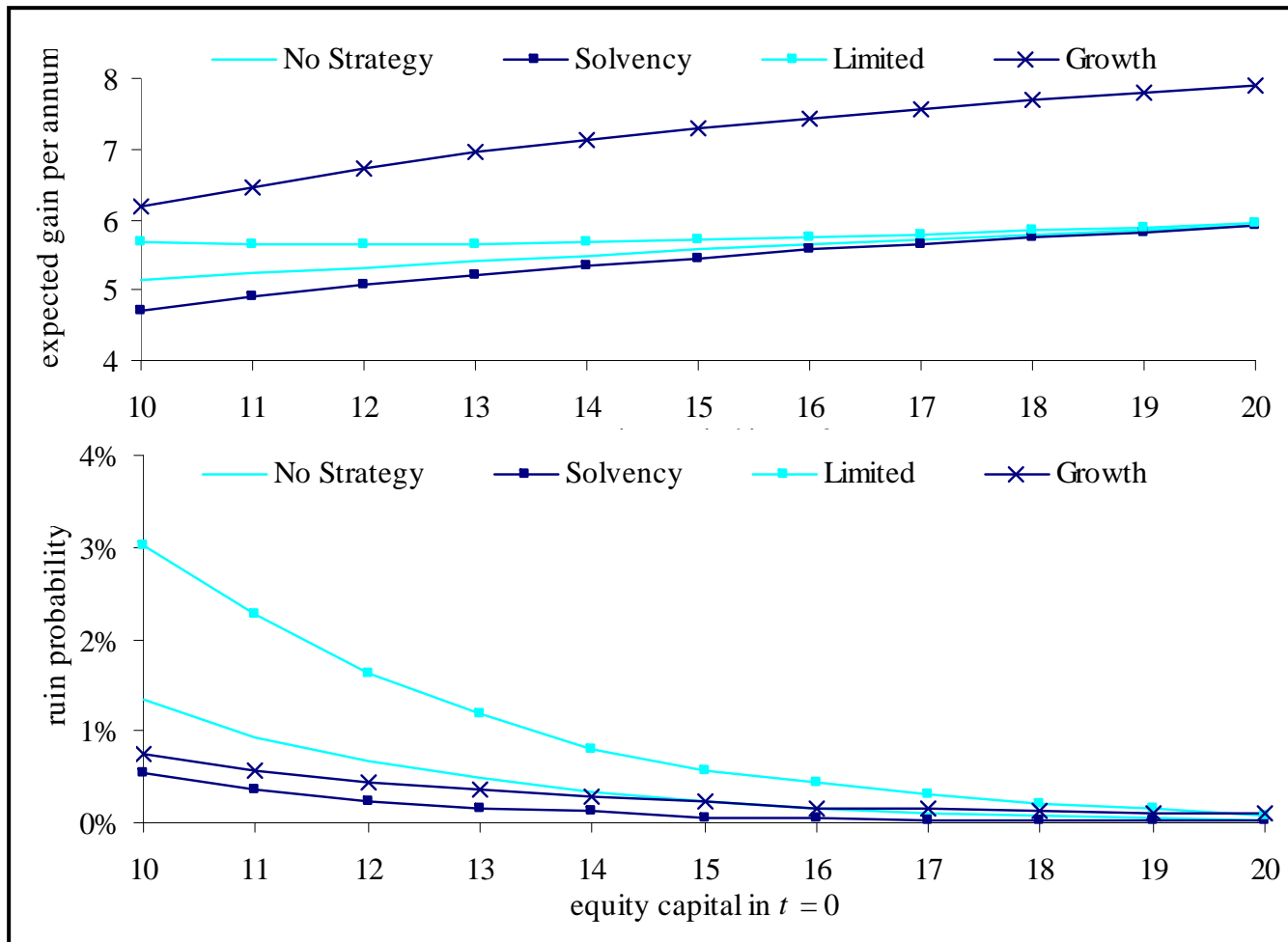
Strategy		No Strategy	Solvency	High Risk	Growth
Return	E(G) in million €	5.57	5.46	5.70	7.30
	ROI in %	23.35	23.05	23.73	27.99
Risk	$\sigma(G)$ in million €	2.88	2.95	2.89	4.19
	RP in %	0.22	0.06	0.63	0.20
	EPD in million €	0.0045	0.0006	0.0225	0.0035
Performance	$SR_{\sigma}$	1.93	1.85	1.97	1.74
	$SR_{RP}$	12.42	48.75	4.50	18.52
	$SR_{EPD}$	6.18	43.48	1.26	10.49

## 5. Simulation Study: Robustness Checks / Sensitivity Analysis

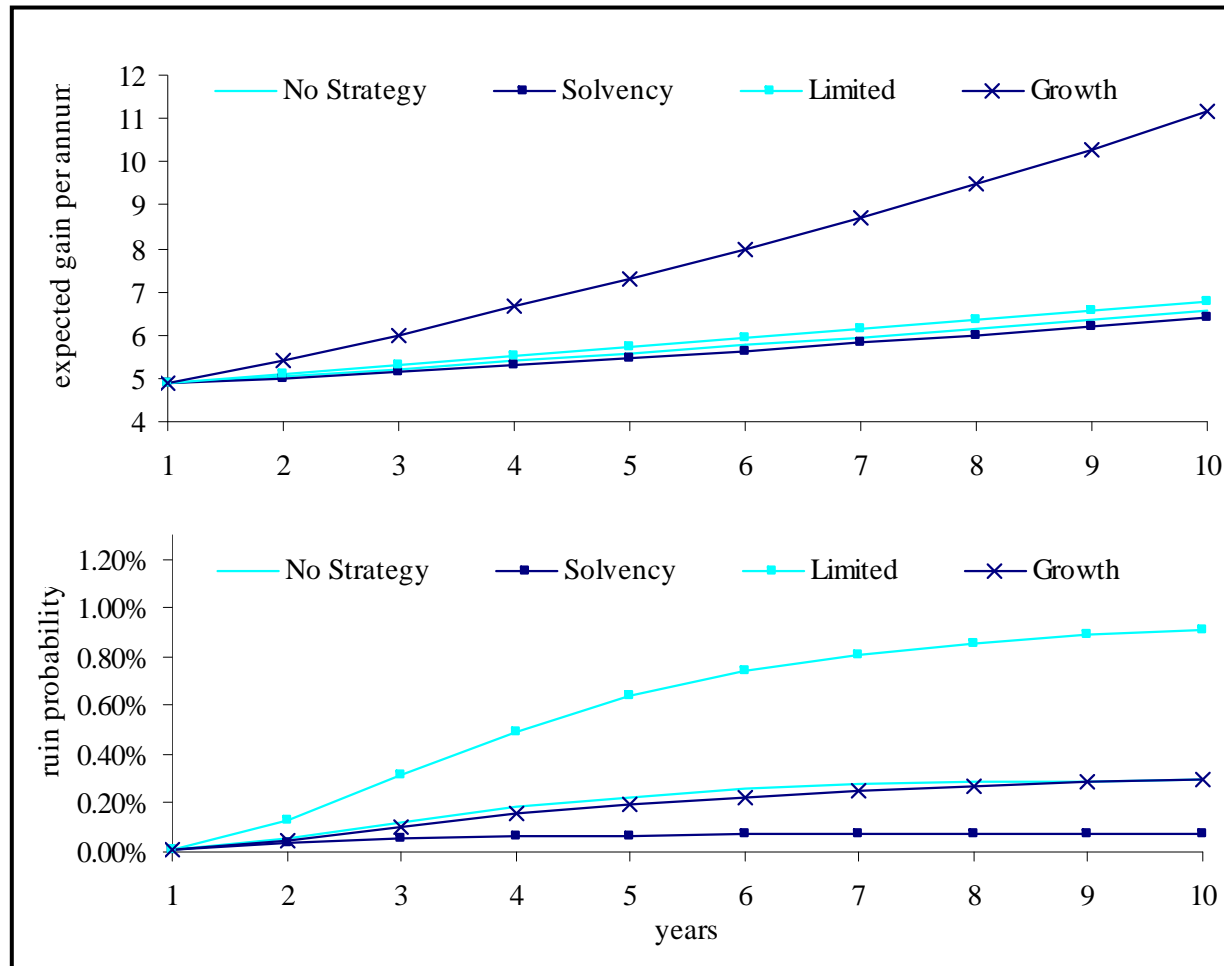
- Variation of the equity capital in  $t=0$  (from €10 to €20 million)
- Variation of the time horizon (from 1 to 10 years)
- Variation of starting values (application of different  $\alpha$  and  $\beta$  in  $t=0$ )
- Variation of the step length (for changes induced by the management, different step lengths for  $\alpha$  and  $\beta$  are assumed)
- Variation of consumer response function



## 5. Simulation Study: Variation of the equity capital in $t=0$



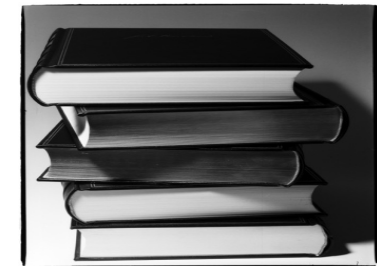
## 5. Simulation Study: Variation of the time horizon





## 6. Role of Non-linear Dependencies (Eling/Toplek, 2009)

- Mapping of nonlinear dependencies in a DFA context: focus on linear correlation, heavy-tailed and skewed risks frequent in insurance
- Literature:
  - DFA: Lowe/Stanard (1997), Kaufmann/Gadmer/Klett (2001), Blum et al. (2001), D'Arcy/Gorvett (2004)
  - Copulas: Wang (1998), Frees/Valdez (1998), Tibiletti (1995), Wang (1996), Klugman/Parsa (1999), Dias (2004)
- Contribution of Eling/Toplek (2009) :
  - Integrating different copulas in a DFA context
  - Studying their effects on the insurer's risk and return position
  - Giving helpful insights for the development of DFA tools, for regulators, and risk managers



## 6. Role of Non-linear Dependencies

- Relevance of nonlinear dependencies:



Last Updated: Monday, 19 November 2007, 08:23 GMT

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### Swiss Re hit by sub-prime losses

Swiss Re expects to lose 1.2bn Swiss francs (\$1.07bn; £525m) on insurance cover a client took out against any fall in the value of its mortgage debt.

Such investments have fallen sharply in value since the summer, due to the crisis in the US mortgage industry, centred on the sub-prime sector.



Swiss Re's UK office is based in the distinctive building of the same name



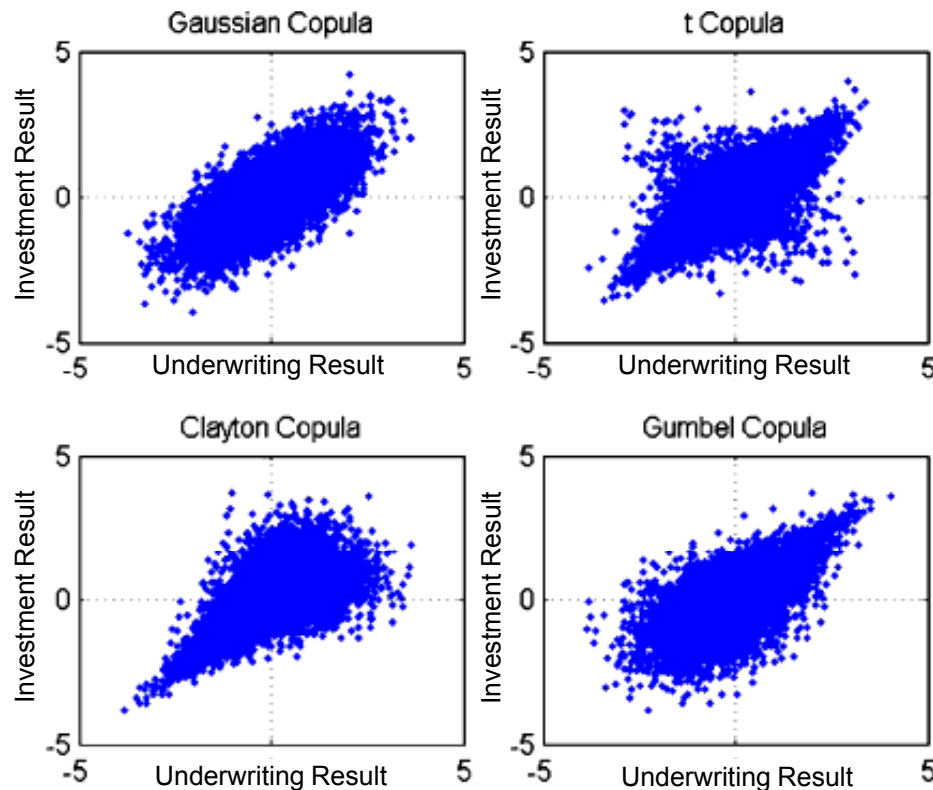
Further examples:

1. Assets: Stocks vs. hedge funds, bonds vs. hedge funds (LTCM)
2. Liabilities: Cat vs. non-cat losses, homeowners vs. householders
3. Assets vs. liabilities: September 11, 2001...

=> Such nonlinear dependencies can be modeled using copulas

## 6. Role of Non-linear Dependencies

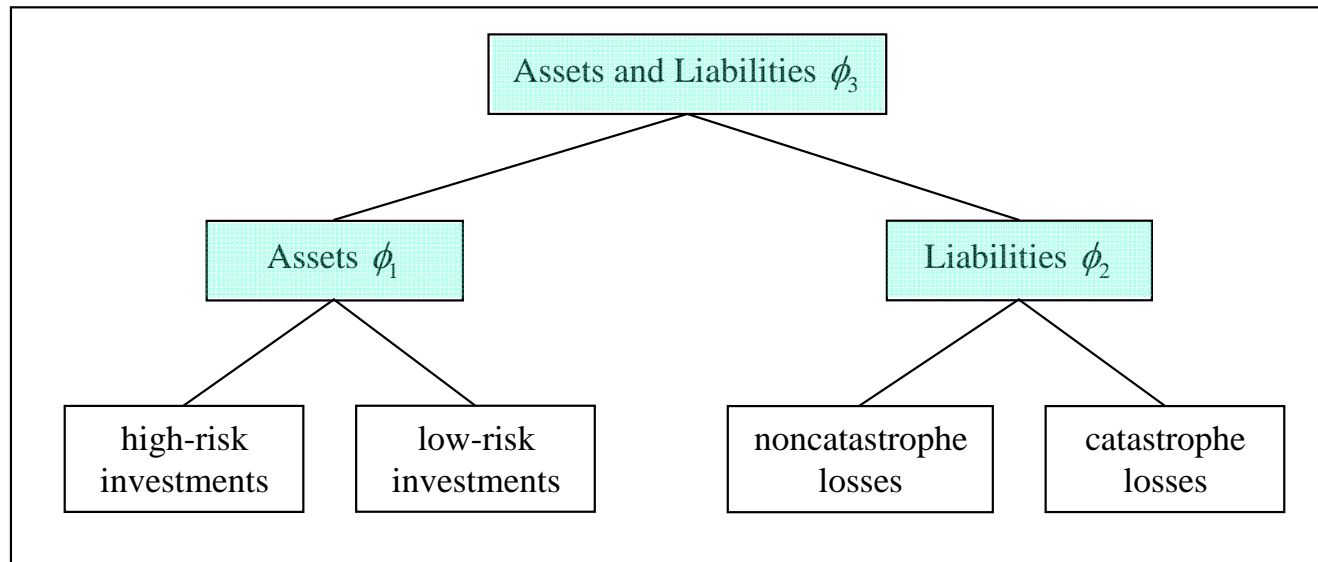
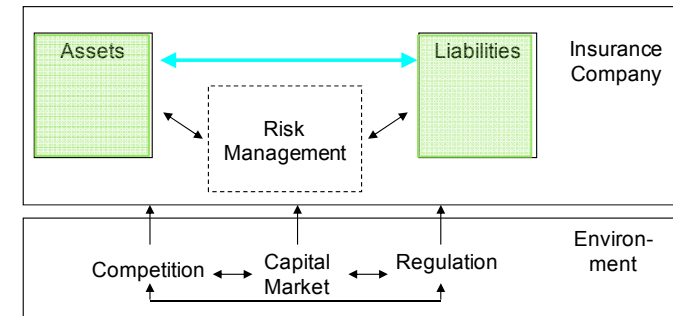
- Different structure of dependence for 10,000 standard normal random variables with Kendall's tau=0.7 (Natale, 2008):



Copula	Tail Dependence
Gaussian	No
Clayton	Lower
t	Upper and Lower
Gumbel	Upper

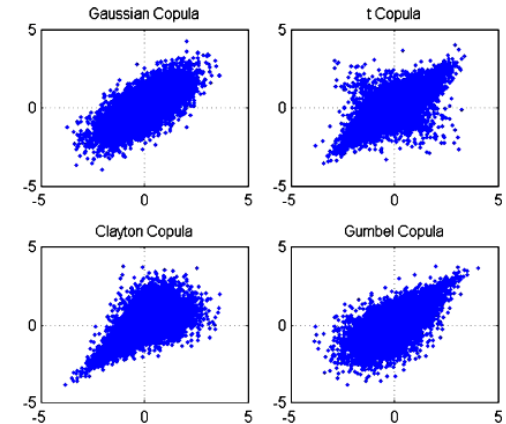
## 6. Role of Non-linear Dependencies

- Correlated model elements:  
 separate correlations for investments, losses,  
 and between assets and liabilities



## 6. Role of Non-linear Dependencies

- Copulas integrated:
  - Gauss-Copula
  - $t$ -Copula
  - Three Archimedean Copulas



Copula	Tail Dependence	Generator $\phi(t)$	Kendall's tau $\rho_\tau$
$C_\theta^{Gumbel}$	upper	$(-\ln t)^\theta$	$1 - 1/\theta$
$C_\theta^{Clayton}$	lower	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta/(\theta + 2)$
$C_\theta^{Frank}$	none	$-\ln\left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right)$	$1 - 4\theta^{-1}\left(1 - \theta^{-1} \int_0^\theta t/(\exp(t) - 1) dt\right)$

## 6. Role of Non-linear Dependencies

- Results (nearly the same model but with different calibration):

Dependence structure	No corr.	Gauss	$t$	Gumbel	Clayton	Frank
Tail dependence	none	none	upper and lower	upper	lower	none
$E(G)$ in million €	203.39	201.21	200.93	201.77	199.33	201.72
$\sigma(G)$ in million €	75.18	92.04	92.40	93.57	101.69	89.91
RP	0.07%	0.34%	0.57%	0.20%	1.00%	0.18%
EPD in million €	0.07	0.36	0.71	3.16	7.38	0.55
$SR_{\sigma}$	2.53	2.04	2.03	2.02	1.83	2.10
$SR_{RP}$	1408.19	278.09	164.56	473.32	92.99	529.51
$SR_{EPD}$	13.39	2.64	1.33	0.30	0.13	1.70

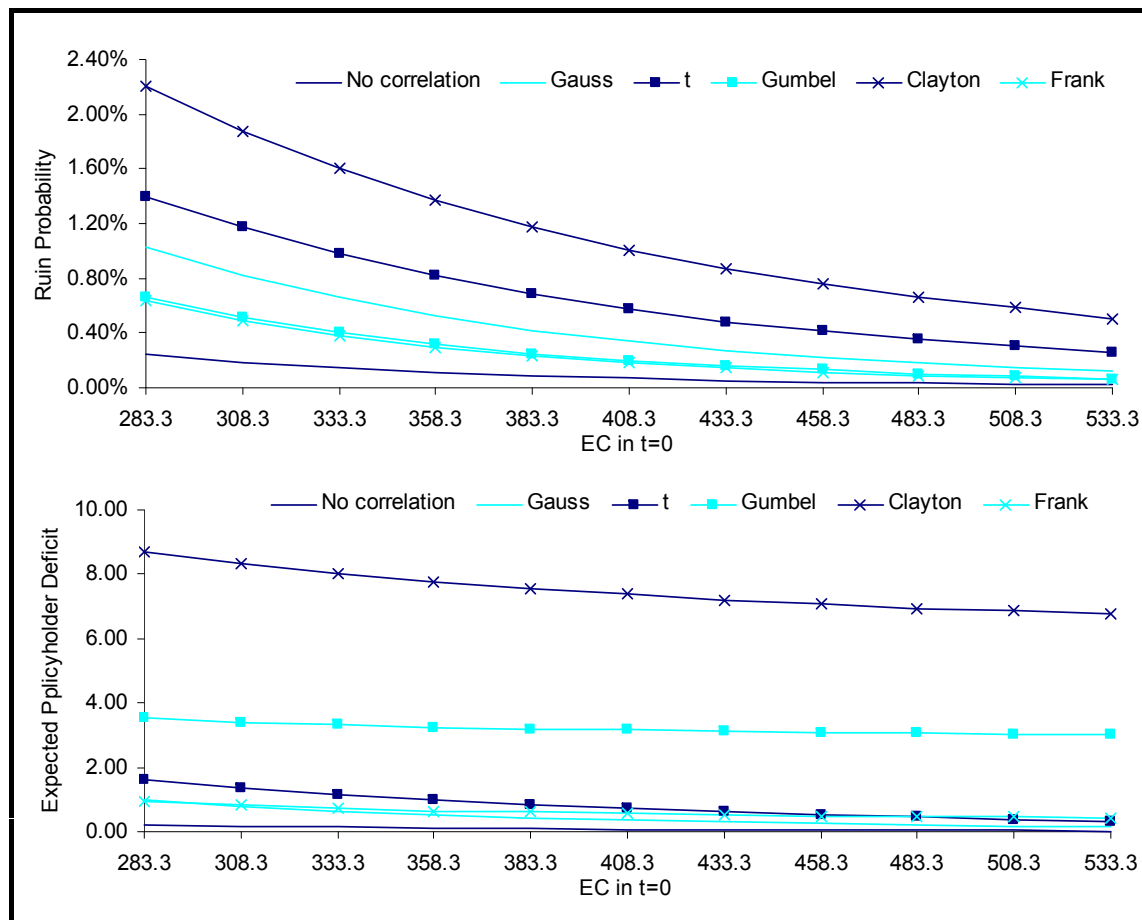
## 6. Role of Non-linear Dependencies

### Sensitivity Analysis

- Variation of the equity capital in  $t=0$   
(from €283.3 to €533.3 million)
- Variation of correlation settings
  - Correlation of assets between 0.1 and 0.5
  - Correlation of liabilities between 0.1 and 0.5
- Other robustness tests (not presented here)
  - Variation of the time horizon (from 1 to 10 years)
  - Variation of starting values (application of different  $\alpha$  and  $\beta$  in  $t=0$ )
  - Variation of the parameter changes (for changes induced by the management, different step lengths for  $\alpha$  and  $\beta$  are assumed)
  - Variation of consumer response function

## 6. Role of Non-linear Dependencies

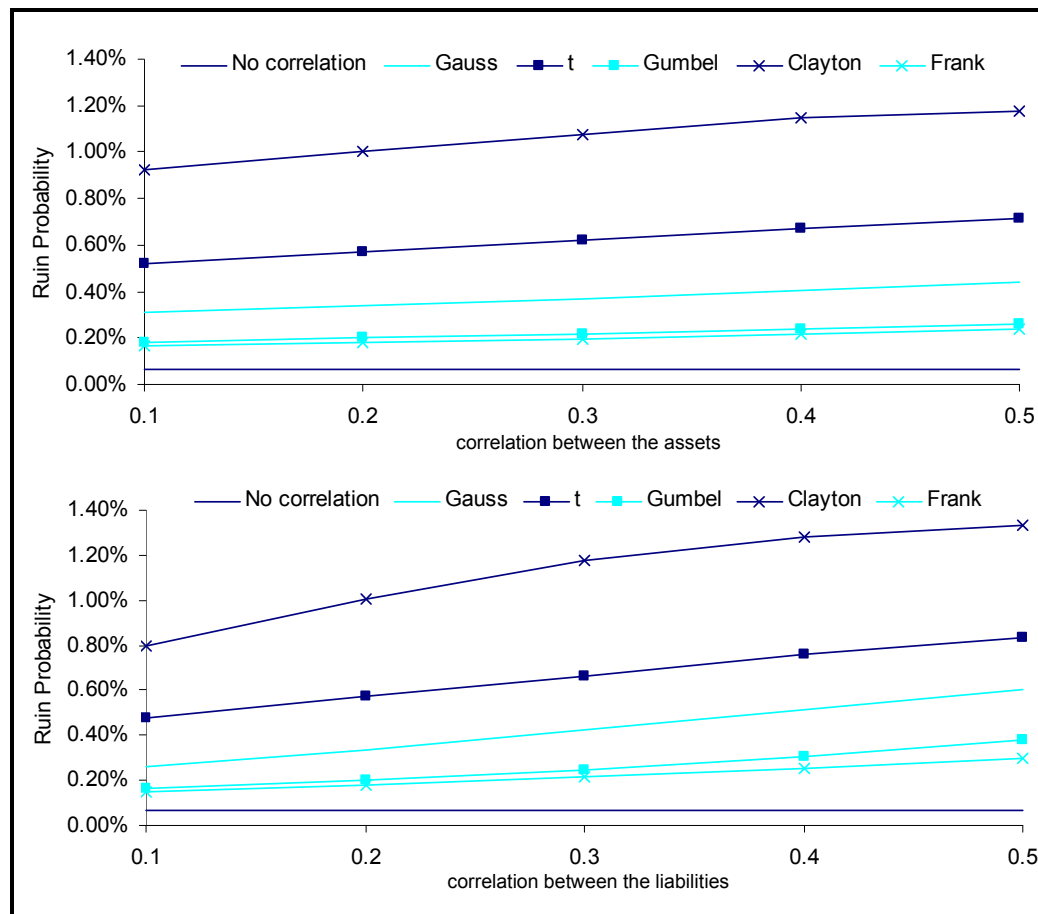
- Variation of the equity capital in  $t=0$  (from €283.3 to €533.3 million):





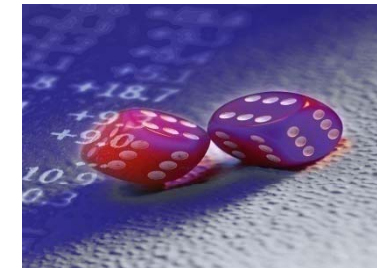
## 6. Role of Non-linear Dependencies

- Variation of correlation settings:



## 6. Role of Non-linear Dependencies

- Implemented three risk management strategies:



Strategy	Solvency	Reinsurance (Stop Loss)	Growth	
Target	Risk Reduction	Risk Reduction	Risk Reduction and Risk Taking	
Trigger	$EC_t < MCR_t \cdot 1.5$	Losses > €1000 million	$EC_t < MCR_t \cdot 1.5$	$EC_t > MCR_t \cdot 1.5$
Rule	$\alpha$ and $\beta$ 0.05 ↓	Indemnity = $\min(\max(C_t - 1000, 0), 200)$	$\alpha$ and $\beta$ 0.05 ↓	$\beta$ 0.05 ↑

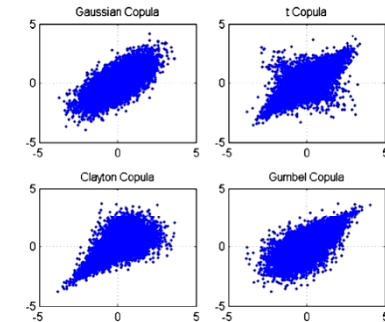
## 6. Role of Non-linear Dependencies

- Results:

Dependence structure	No corr.	Gauss	t	Gumbel	Clayton	Frank
Tail dependence	no	no	upper and lower	upper	lower	no
Solvency strategy						
E(G) in million €	203.06	200.38	200.10	201.09	198.30	201.02
RP	0.06%	0.32%	0.55%	0.19%	0.96%	0.17%
EPD in million €	0.07	0.33	0.67	3.13	7.24	0.54
Growth strategy						
E(G) in million €	252.16	248.07	247.70	249.20	245.04	249.09
RP	0.12%	0.56%	0.91%	0.33%	1.50%	0.30%
EPD in million €	0.14	0.70	1.35	4.21	10.17	0.86
Reinsurance strategy						
E(G) in million €	195.48	194.00	193.91	194.29	192.97	194.19
RP	0.02%	0.16%	0.31%	0.08%	0.57%	0.08%
EPD in million €	0.02	0.13	0.27	3.01	6.56	0.43

## 6. Role of Non-linear Dependencies

- Analyzed the influence of nonlinear dependencies and the resulting effects on a non-life insurer's risk and return
- Three main conclusions:
  1. Large differences in risk assessment for different copulas
    - return not affected, ruin probability and expected policyholder deficit extremely affected
    - lower tail dependent copulas induce highest risk in our model
  2. Increase of equity capital reduces ruin probability, but not necessarily the expected policyholder deficit
  3. Reinsurance contracts are useful in reducing ruin probability, but not as good in reducing the expected policyholder deficit

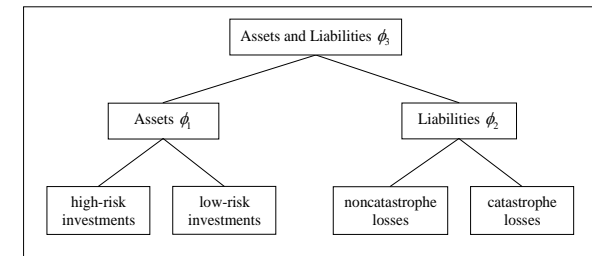


## 7. Conclusion and Outlook



- Implementation of management strategies in a DFA framework
- Effects on the insurer's risk and return position:
  - Solvency strategy: Reasonable for managers desiring to protect the company from insolvency
  - Growth strategy: An alternative for managers pursuing a higher return and willing to take higher risks
- Outlook:
  - Search for optimal management strategies in our model framework
  - Comparison of optimization results with the results of the heuristic management strategies
  - Consideration of Bernstein Copulas: Diers/Eling/Marek (WRIEC 2010)
  - Empirical Considerations (non-linear dependencies, next slide)

## 7. Conclusion and Outlook



### Dependence structure between assets and liabilities

- NAIC data 2001 to 2006; 3000 non-life insurers
- Investment result vs. underwriting result
- Goodness of fit test for various copulas (Akaike's information criterion)

Panel A: Ranking of Copulas according to AIC

	2001	2002	2003	2004	2005	2006
Gaussian	4	5	5	5	5	3
t	1	1	1	1	1	2
Gumbel	5	4	3	3	3	5
Clayton	3	3	4	2	2	1
Frank	2	2	2	4	4	4

Kendall's Tau

= -0.09

=> Gaussian is among the worst in all years

=> t Copula is the best in 2001, 2002, 2003, 2004, and 2005

=> Clayton Copula is best in 2006