Actionable predictive learning for insurance profit maximization

Leo Guelman$^{1,2}$ and Montserrat Guillén$^2$

$^1$Royal Bank of Canada - RBC Insurance  
$^2$Riskcenter, University of Barcelona

CAS RPM Seminar  
Washington, D.C.  
March, 2014
The Casualty Actuarial Society is committed to adhering strictly to the letter and spirit of the antitrust laws. Seminars conducted under the auspices of the CAS are designed solely to provide a forum for the expression of various points of view on topics described in the programs or agendas for such meetings.

Under no circumstances shall CAS seminars be used as a means for competing companies or firms to reach any understanding expressed or implied that restricts competition or in any way impairs the ability of members to exercise independent business judgment regarding matters affecting competition.

It is the responsibility of all seminar participants to be aware of antitrust regulations, to prevent any written or verbal discussions that appear to violate these laws, and to adhere in every respect to the CAS antitrust compliance policy.
Motivation

- **Predictive Modeling** is a core strategic capability of many top insurers (widely applied in marketing, underwriting, pricing, claims management, fraud detection, etc.)

- **Common goal of models**: to predict a response variable using a collection of **observable attributes** (e.g., Age, Yrs. Licensed, Gender, Territory, Claims and Conviction History, etc.)

  Tons of literature on the above, but less attention has been paid to:

  - In many important settings, the values of certain attributes can be proactively chosen at the discretion of a decision maker – called **actionable attributes or “treatments”**. For instance, we can choose:
    - Which policyholders should be contacted to prevent them from switching to an alternative insurer?
    - Which Auto insurance clients should be offered a Life policy?
    - By how much should we change the rates at policy renewal?
The values chosen for the actionable attributes have important implications for the ultimate profitability of the insurance company.

There is no “global” better action ⇒ Relevant in the context of treatment heterogeneity effects.

The objective is NOT to predict a response variable with high accuracy (as in predictive modeling), but to select the optimal action or treatment for each client.

Optimal personalized treatment ⇒ the one that maximizes the probability of a desirable outcome (e.g., Profits).

Not addressed by traditional predictive modeling techniques (GLMs, CART, SVM, Neural Nets, etc.).
A toy example: The red/blue envelope problem

- Consider a Client Retention Program aimed to increase the overall retention rate of an insurance portfolio
- Treatment consists in a promotion sent either in a red or blue envelope

<table>
<thead>
<tr>
<th>Client Type</th>
<th>Red envelope</th>
<th>Blue envelope</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>NOT renew</td>
<td>NOT renew</td>
</tr>
<tr>
<td>B</td>
<td>Renew</td>
<td>Renew</td>
</tr>
<tr>
<td>C</td>
<td>NOT renew</td>
<td>Renew</td>
</tr>
<tr>
<td>D</td>
<td>Renew</td>
<td>NOT Renew</td>
</tr>
</tbody>
</table>

Clients ‘A’ and ‘B’ are indifferent to the color of the envelope

The optimal personalized treatment is to send a Blue envelope to ‘C’ clients and a Red envelope to ‘D’ clients
Literature is relatively scarce and mostly published recently

**Personalized Medicine**: (Qian and Murphy, 2011; Zhao et al., 2012; Su et al., 2009)

**Marketing**: (Jaskowski and Jaroszewicz, 2012; Radcliffe and Surry, 2011; Lo, 2002)

**Economics**: Imai and Ratkovic (2013)

**Insurance**: Personalized treatments in the context of Pricing, Client Retention and Cross-Selling


The problem of selecting the optimal treatment is non-trivial...

- The **outcome of interest** – i.e., the optimal treatment – is **unknown** on a given training data set.

- Each client can only be exposed to one treatment condition ⇒ we can only observe the response under the exposed condition. The counterfactual response is never observed ⇒ the “true” optimal treatment is not observed (Holland, 1986).

- A key distinction for building personalized treatment learning models is between **randomized experiments** and **observational data**.
Let’s formalize the problem

- For now assume a controlled randomized experiment – i.e., clients are randomly assigned to two treatments, denoted by \( A \in \{0, 1\} \)

- Let \( Y(a) \in \{0, 1\} \) denote a **binary potential response** of a client if assigned to treatment \( A = a, \ a = \{0, 1\} \)

- The **observed response** is \( Y = AY(1) + (1 - A)Y(0) \)

- Clients are characterized by a \( p \)-dimensional vector of baseline **predictors** \( X = (X_1, \ldots, X_p)\top \)

- Data consists of \( L \) i.i.d. realizations of \((Y, A, X), \ \{(Y_\ell, A_\ell, X_\ell), \ \ell = 1, \ldots, L\}\).
Let’s formalize the problem

- At the most granular level, the personalized treatment effect is a comparison between $Y(1)$ and $Y(0)$ on the same client. Usually,

$$Y_\ell(1) - Y_\ell(0) \forall \ell = \{1, \ldots, L\}$$

- But as discussed above, this is an unobserved quantity

- In practice, the best we can do is to estimate the personalized treatment effect by conditioning on clients with profile $X = x$

- Thus, we define the *personalized treatment effect* (PTE) by

$$\tau(x) = E[Y_\ell(1) - Y_\ell(0)|X_\ell = x] = E[Y_\ell|X_\ell = x, A_\ell = 1] - E[Y_\ell|X_\ell = x, A_\ell = 0].$$
The two-model approach to PTE estimation

1. Estimate $E[Y|X, A = 1]$ using the treated clients only
2. Estimate $E[Y|X, A = 0]$ using the control clients only
3. An estimate of the PTE for a client with predictors $X_\ell = x$ is

$$\hat{\tau}(x) = (\hat{Y}_\ell|X = x, A_\ell = 1) - (\hat{Y}_\ell|X = x, A_\ell = 0).$$

Pros:
- Any conventional statistical or algorithmic binary classification method may serve to fit the models.

Cons:
- Models developed to predict the wrong target!
  - The method emphasize the prediction accuracy on the response, not the accuracy in estimating the change in the response caused by the treatment
  - Relevant predictors for $Y$ are usually different from relevant PTE predictors
Y(Treat) = Attrition rate on treated clients
Y(Ctrl) = Attrition rate on control clients
PTE = Personalized treatment effect: Y(Ctrl) - Y(Treat)
L = Number of clients
Algorithm 1 Causal conditional inference tree

1: for each terminal node do
2: Test the global null hypothesis \( H_0 \) of no interaction effect between the treatment \( A \) and any of the \( p \) predictors at a level of significance \( \alpha \) based on a permutation test (Strasser and Weber, 1999)
3: if the null hypothesis \( H_0 \) cannot be rejected then
4: Stop
5: else
6: Select the \( j^* \)th predictor \( X_{j^*} \) with the strongest interaction effect (i.e., the one with the smallest adjusted \( P \) value)
7: Choose a partition \( \Omega^* \) of the covariate \( X_{j^*} \) in two disjoint sets \( M \subset X_{j^*} \) and \( X_{j^*} \setminus M \) based on the \( G^2(\Omega) \) split criterion
8: end if
9: end for

\[
G^2(\Omega) = \frac{(L - 4)\left\{ (\bar{Y}_{nL}(1) - \bar{Y}_{nL}(0)) - (\bar{Y}_{nR}(1) - \bar{Y}_{nR}(0)) \right\}^2}{\hat{\sigma}^2 \left\{ \frac{1}{LnL(1)} + \frac{1}{LnL(0)} + \frac{1}{LnR(1)} + \frac{1}{LnR(0)} \right\}}
\]
R implementation: The uplift package in CRAN

The highlights:

- Implements various functions for training personalized treatment learning models (a.k.a., uplift)
- Currently 5 estimation methods are implemented
  - Causal conditional inference forests (ccif)
  - Uplift random forests (upliftRF)
  - Modified covariate method (tian_transf)
  - Modified outcome method (rvtu)
  - Uplift k-nearest neighbor (upliftKNN)
- Exploratory Data Analysis (EDA) tools designed for PTE models
- Functions for evaluating performance of PTE models
- Profiling results of PTE models
- PTE Monte Carlo simulations
- Package in continuous development
A cross-sell example: Auto ⇒ Property Insurance

- A randomized experiment with a cross-sell binary “treatment”

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchased Home policy = N</td>
<td>30,184</td>
<td>3,322</td>
</tr>
<tr>
<td>Purchased Home policy = Y</td>
<td>789</td>
<td>75</td>
</tr>
<tr>
<td>Cross-sell rate</td>
<td>2.55%</td>
<td>2.21%</td>
</tr>
</tbody>
</table>

The average treatment effect is **0.34%** (2.55% - 2.21%), which is not statistically significant (*P* value = 0.23)

Can we identify a subgroup of clients for which the treatment was effective? If so, target those clients in the future.
A cross-sell example: Auto ⇒ Property
Consider the existing portfolio of an insurer where the premium $P_{\ell t}$ charged to policyholder $\ell = \{1, \ldots, L\}$ in year $t$ is given by

$$P_{\ell t} = \hat{L}C_{\ell t} + E_{\ell t} + A_{\ell t}$$

where

$\hat{L}C_{\ell t} = \text{Expected loss cost}$

$E_{\ell t} = \text{Expenses}$

$A_{\ell t} = \text{Profit loading}$

**Loss cost** estimation has seen an enormous advance with predictive modeling

**Profits** have remained obscure and rather forgotten.
We can think of $A_\ell$ as an actionable attribute or “treatment” which can take values on a continuous scale.

The problem is to select the optimal personalized treatment: the one that maximizes the overall profitability of the insurance portfolio

$$\sum_{\ell=1}^{L} P_\ell t - \hat{C}_\ell t - E_\ell t$$

Assuming $\hat{C}_\ell$ and $E_\ell$ are exogenous, then selecting the optimal $A_\ell \Rightarrow$ selecting the optimal $P_\ell$

The impact of a change in $P_\ell$ on the overall profitability of the portfolio is a-priori uncertain as a big enough $P_\ell$ will make a policyholder more likely to switch to an alternative insurer.

This requires understanding the precise impact of a change in $P_\ell$ on the probability of renewal for each policyholder $\ell$ – i.e., the price elasticity.
Price Elasticity as a missing data problem

- *Price elasticity involves a comparison of the potential renewal outcomes for alternative rate changes (the “treatments”) defined on the same policyholder.*

- Due to the **fundamental problem of personalized treatment learning models** ⇒ each policyholder can only be exposed to one rate change value, so only one of the potential renewal outcomes is an observed outcome. *The counterfactual outcomes are never observed.*

- One way to think about the **counterfactual outcomes** is that their values are “missing” and therefore they should be multiply imputed to represent their uncertainty.
Price Elasticity as a missing data problem

- To simplify, let's bin the rate change into five ordered values $A = \{1 < \ldots < 5\}$ and assume a 1-year horizon.

- The entries $r_{\ell a}$ below denote the observed renewal outcome $\in \{0, 1\}$ of policyholder $\ell = \{1, \ldots, L\}$ when exposed to rate change level $A = a$; $a = \{1 < \ldots < 5\}$

- Dots indicate counterfactual outcomes, which are missing.

- The price elasticity estimation problem $\equiv$ the problem of filling in the missing values in the client-by-rate change table with reliable estimates.

**Table: Client-by-Rate change table**

<table>
<thead>
<tr>
<th>Client</th>
<th>Rate Change Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$r_{31}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>
A key additional complexity

• **Reliable estimates of effects attributable to treatments require experimental data** (i.e., coming from randomized experiments)

• This means that for reliable price elasticity estimation, data must come from a randomized assignment of policyholders to rate change levels

• **This condition rarely holds in practice**: rate changes are mostly derived from a pricing modeling exercise ⇒ rate change is a deterministic function of the policyholder’s observed risk characteristics

• Thus, we end up with **observational data** – i.e., not derived from experimentation

• **Policyholders exposed to different rate change levels are not directly comparable.**
But...what is the problem?

- **The standard approach:** model the policyholder’s lapse outcome as a function of the rate change and the policyholder’s covariates

- **The key assumption:** the inclusion of those covariates adjust for the exposure correlations between price elasticity and other explanatory variables

- **Problem:** non-overlapping supports of $X$ between policyholders exposed to different rate change levels

- **As an extreme example:** Assume policyholder’s Age is associated with the lapse outcome

<table>
<thead>
<tr>
<th>Age</th>
<th>Rate Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
</tr>
<tr>
<td>&lt; 25 yrs.</td>
<td>✓</td>
</tr>
<tr>
<td>≥ 25 yrs.</td>
<td>✓</td>
</tr>
</tbody>
</table>

- “✓” indicates whether historical data is available
- Clients $\geq 25$ yrs. exposed to a 5% rate change don’t have a good comparison in the 10% rate change group
But...what is the problem?

- Regression analysis masks this fact and assumes that the estimated price elasticity model is good for all policyholders (even for those never observed under a specific rate change).

- In real data sets, extreme examples such as the above are rare, but **non-overlap situations are common**.

- **Non-overlap** refers to the extent to which the distribution of the key renewal/lapse predictors differ across policyholders historically exposed to different rate change levels.

- The problem is even worse with a **large number of predictors**, as groups may differ in a multivariate direction and so non-overlap problems are more difficult to detect.
Propensity scores and Matching algorithms

Some good news...

- Under certain data conditions (Rosenbaum and Rubin, 1983):
  - We can construct a randomized-type of experiment from observational data ⇒ helpful for determining price elasticity at the portfolio level
  - It's possible to infer the “missing” counterfactual renewal outcomes (and thus fill-in the missing values in the client-by-rate change table) ⇒ helpful for determining price elasticity at the individual policyholder level

- The key concepts are propensity scores (Rosenbaum and Rubin, 1983) and matching algorithms (Gu and Rosenbaum, 1993)
Let’s say in the training data we have 2 policyholders which are very similar in terms of their relevant lapse predictors $X$ – i.e., about the same age, driving record, living in the same neighbourhood, etc.

But, they have been exposed to different rate change levels – e.g., 5% and 10% (enough historical data may allow us to find such pair)

The observed outcomes are used to fill-in the counterfactual outcomes
Matching algorithms have many variants. There are 3 key choices:

1. The **definition of distance** between two policyholders in terms of their characteristics

2. The choice of the **algorithm** used to form the matched pairs and make the distance small (greedy vs. optimal matching)

3. The **structure of the match** (i.e., the number of treated and control subjects that should be included in each match set)

In Guelman and Guillén (2014), we used **optimal pair matching**
⇒ equivalent to finding a flow of minimum cost in a certain network (a standard combinatorial optimization problem)
Even with a moderate number of predictors, exact matches on $X$ are not feasible $\Rightarrow$ propensity scores come into play.

Given a binary treatment $A \in \{0, 1\}$, the **propensity score** is the conditional probability of assignment to treatment 1 given $X$,

$$\pi(X_\ell) = P(A_\ell = 1 | X_\ell)$$

In a randomized experiment, $\pi(X_\ell) = 1/2 \ \forall \ X_\ell$

In an observational study, the propensity score can be estimated (e.g., logistic regression).

With more than two treatments, we could (i) consider all possible treatment dichotomies or (ii) build a multinomial response model.
An important property of the propensity score allows us to match only on the propensity score.

The **Balancing Property**: Treatment $A$ and the observed covariates $X$ are conditionally independent given the propensity score $\pi(X)$,

$$A \perp X | \pi(X)$$

i.e., conditional on the propensity score $\pi(X)$, the distribution of $X$ is similar for $A=1$ and $A=0$. 
Propensity score for 20% vs. 5% rate change dichotomy

Matched clients

Propensity Score density rate_change +5% +20%

Common Support

Matched clients
Replace the actual renewal outcomes with probability estimates

<table>
<thead>
<tr>
<th>Client</th>
<th>Rate Change Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>. . ( \hat{r}_{12} ) . . .</td>
</tr>
<tr>
<td>2</td>
<td>. ( \hat{r}<em>{31} ) . ( \hat{r}</em>{23} ) . .</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{r}_{31} ) . . . .</td>
</tr>
<tr>
<td>...</td>
<td>... ... ... ... ... .</td>
</tr>
<tr>
<td>L</td>
<td>. . . . ( \hat{r}_{L5} )</td>
</tr>
</tbody>
</table>

Infer the counterfactual renewal outcomes from the matched pairs (as far as the overlap situation permits)

<table>
<thead>
<tr>
<th>Client</th>
<th>Rate Change Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \hat{r}<em>{11} ) ( \hat{r}</em>{12} ) ( \hat{r}<em>{13} ) ( \hat{r}</em>{14} ) ( \hat{r}_{15} )</td>
</tr>
<tr>
<td>2</td>
<td>( \hat{r}<em>{21} ) ( \hat{r}</em>{22} ) ( \hat{r}<em>{23} ) ( \hat{r}</em>{24} ) .</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{r}_{31} ) . . . .</td>
</tr>
<tr>
<td>...</td>
<td>... ... ... ... ... .</td>
</tr>
<tr>
<td>L</td>
<td>( \hat{r}<em>{L1} ) ( \hat{r}</em>{L2} ) ( \hat{r}<em>{L3} ) ( \hat{r}</em>{L4} ) ( \hat{r}_{L5} )</td>
</tr>
</tbody>
</table>
Develop a “global model” of the response.

- Develop a global model \( \hat{r}_{lt}(x_{\ell}) \), obtained by fitting the estimates \( \hat{r}_{lt} \) of the observed responses, plus the estimates of a subset of the counterfactual responses on the vector of observed characteristics \( x_{\ell} \) and rate change level \( a = \{1 < \ldots < 5\} \).
- This model allows us to predict the renewal outcome for each rate change \( A = a \) and value of \( X \).

Table: Client-by-Rate change table filled with “global” renewal probability estimates

<table>
<thead>
<tr>
<th>Client</th>
<th>Rate Change Level</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 3</td>
<td>Level 4</td>
<td>Level 5</td>
</tr>
<tr>
<td>1</td>
<td>( \hat{r}_{11} )</td>
<td>( \hat{r}_{12} )</td>
<td>( \hat{r}_{13} )</td>
<td>( \hat{r}_{14} )</td>
<td>( \hat{r}_{15} )</td>
</tr>
<tr>
<td>2</td>
<td>( \hat{r}_{21} )</td>
<td>( \hat{r}_{22} )</td>
<td>( \hat{r}_{23} )</td>
<td>( \hat{r}_{24} )</td>
<td>( \hat{r}_{25} )</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{r}_{31} )</td>
<td>( \hat{r}_{32} )</td>
<td>( \hat{r}_{33} )</td>
<td>( \hat{r}_{34} )</td>
<td>( \hat{r}_{35} )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>L</td>
<td>( \hat{r}_{L1} )</td>
<td>( \hat{r}_{L2} )</td>
<td>( \hat{r}_{L3} )</td>
<td>( \hat{r}_{L4} )</td>
<td>( \hat{r}_{L5} )</td>
</tr>
</tbody>
</table>
The proposed framework to fill-in the counterfactual renewal outcomes with probability estimates allows us to more efficiently solve the Economic Price Optimization problem.

**The problem:** which rate change should we expose each policyholder to maximize the overall expected profit of the portfolio subject to a fixed overall retention rate?

Recall that: An **Optimal personalized treatment** is the one that maximizes the probability of a desirable outcome (treatment \(\equiv\) rate change and the outcome \(\equiv\) profits)
The optimization problem: An integer program

Maximize an expected profit function

\[ \max_{\ell, a} \sum_{\ell} \sum_{a} Z_{\ell a} \left[ P_{\ell} (1 + RC_a) (1 - \hat{R}_{\ell a}) (1 - \hat{r}_{\ell a}) \right] \]

subject to a retention constraint

\[ \sum_{a} Z_{\ell a} = 1 \quad \forall \ell \]
\[ Z_{\ell a} \in \{0, 1\} \]
\[ \sum_{\ell} \sum_{a} Z_{\ell a} \hat{r}_{\ell a} / L \leq \alpha. \]
• We introduced the concept of predictive learning with actionable attributes (in the context of marketing and pricing intervention activities)

• The values chosen for these attributes have important implications for the ultimate profitability of the insurer

• Off-the-shelf predictive modeling algorithms can generally not be used to tackle learning with actionable attributes

• The nature of the data is key: experimental vs. observational (experimental data is more common in marketing than in pricing interventions)

• Discussed methods and tools useful for each data context.
Your turn...