Casualty Loss Reserve Seminar
Testing Loss Reserve Models by Simulation

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 Charge to Subcommittee

• Given “data” that was simulated from a model, can one distinguish it from real data?

  Rephrasing the question

• Given a simulation model and some real data, is it likely that the data could have come from a simulation of that model?

• This presentation:
  – Shows how to answer that question.
  – Provides an answer to examples with real data.
Given an (say 10 x 10) array of cells \{AY, Lag\} indexed by Accident Year and Settlement Lag

- Observed data \{x_{AY,Lag}\} for \(AY + Lag < 11\)
- Random variables \{X_{AY,Lag}\} for \(AY + Lag > 10\)

- A stochastic model specifies the distribution of sums involving \{X_{AY,Lag}\}
- “Deterministic” models only specify the mean
- e.g. BF -- \(E[X_{AY,Lag}] = \{\text{Premium}_{AY} \times ELR \times Dev_{Lag}\}\)
The Test

- Given an observation $x_{AY, Lag}$ and $n$ simulated data points for each $\{X_i, AY, Lag\}$, define:

  $$p(x_{AY, Lag}) = \frac{\text{Count}(X_i, AY, Lag) \leq x_{AY, Lag}}{n}$$

- The set of points $\{p(x_{AY, Lag})\}$ should be uniformly distributed.

- Test uniformity
  - PP Plot with Kolmogorov – Smirnov critical values
  - Histogram
Diagnostics on Two Independent Uniform Samples

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**PP Plot**

- Observed $P$
  - 0.0, 0.4, 0.8
- Predicted $P$
  - 0.0, 0.2, 0.4, 0.6, 0.8, 1.0

**Histogram**

- Frequency
  - 0, 2, 4, 6, 8
- pctloss
  - 0.0, 0.2, 0.4, 0.6, 0.8, 1.0
A Generic Loss Reserve Simulation Model

• Most stochastic loss reserve models provide a distribution of parameter estimates.
• LSMWP models provide simulations of outcomes.
• Observe outcomes, and so we test outcomes.
• Simulation Algorithm
  – Select random \( \{ELR_{AY}\} \) and \( \{Dev_{Lag}\} \) parameters.
  – For each \( AY,Lag \) of interest, Simulate \( Loss_{AY,Lag} \)
  – Sum the \( \{Loss_{AY,Lag}\} \) over all \( (AY,Lag) \) cells of interest
Results on Real Data

- Commercial Auto – 1997 Schedule P
- Fit current favorite model
- Tested individual \((AY, Lag)\) combinations
- Training data from 1997 statement
- Test data from 1998-2006 statements for the same accident years
Insurer 388 Train Data

PP Plot for Insurer 388

AY vs Cell Percentiles

Predicted P
Train Data

AY
Train Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Lag
Train Data

CY
Train Data
Insurer 388 Test Data

PP Plot for Insurer 388

AY vs Cell Percentiles

Predicted P
Test Data

AY
Test Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Observed P

Observed P

Observed P

Observed P

Lag
Test Data

CY
Test Data
Insurer 914 Train Data

PP Plot for Insurer 914

AY vs Cell Percentiles

Predicted P
Train Data

AY
Train Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Observed P

0.0 0.4 0.8

0.0 0.2 0.4 0.6 0.8 1.0

Lag
Train Data

Observed P

0.0 0.4 0.8

0.0 2 4 6 8 10

CY
Train Data
Insurer 914 Test Data

PP Plot for Insurer 914

Predicted P
Test Data

AY vs Cell Percentiles

Observed P

AY
Test Data

Lag vs Cell Percentiles

Observed P

Lag
Test Data

CY vs Cell Percentiles

Observed P

CY
Test Data
Insurer 1767 Train Data

PP Plot for Insurer 1767

AY vs Cell Percentiles

Predicted P
Train Data

AY
Train Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Lag
Train Data

CY
Train Data
Insurer 1767 Test Data

PP Plot for Insurer 1767

Observed P
Predicted P
Test Data

AY vs Cell Percentiles

Observed P
AY
Test Data

Lag vs Cell Percentiles

Observed P
Lag
Test Data

CY vs Cell Percentiles

Observed P
CY
Test Data
Insurer 86 Train Data

PP Plot for Insurer 86

AY vs Cell Percentiles

Predicted P
Train Data

AY
Train Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Observed P

Observed P

Observed P

Observed P
Insurer 86 Test Data

PP Plot for Insurer 86

AY vs Cell Percentiles

Predicted P
Test Data

AY
Test Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Observed P

Observed P

Lag
Test Data

CY
Test Data
Insurer 3360 Train Data

PP Plot for Insurer 3360

AY vs Cell Percentiles

Predicted P
Train Data

Observed P

AY
Train Data

Lag vs Cell Percentiles

CY vs Cell Percentiles

Observed P

Lag
Train Data

Observed P

CY
Train Data
Testing the Total Reserve

Percentiles of Distribution

\[
\sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}
\]

• One observation per insurer.
• Test to see if observation is in the normal range e.g. (0.025, 0.975)
• Can test the methodology by looking at percentiles for 50 insurers.
  – They should be uniformly distributed.
Retrospective Test of Outcomes for Total Reserve

Outcome Percentiles

Percentiles of Distribution

\[ \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag} \]
Aggregate Results – Actual/Expected

Total for All 50 Insurers

Black line fit with training data

Red line fit with all data
Aggregate Results – Actual/Expected

Total for All 50 Insurers

Black line fit with training data

Red line fit with all data
Questions Raised

• Stochastic model fits well for some insurers, and poorly for others
• How can we tell which is which without retrospective tests?
  – Caption to cartoon in last weeks *New Yorker*
    • “To be right, you have to live in the past.”
• Can we find an approach that works well for all insurers?
• If we can’t, how do we manage insurer risk?