Optimal Layers for Catastrophe Reinsurance

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Agenda

- Introduction
- Optimal reinsurance: academics
- Optimal reinsurance: RAROC
- Optimal reinsurance: our method
- A case study
- Conclusions
- Q&A

1. Introduction

- Bad property loss ratios of insurance industry, especially homeowners line
- Increasing property losses from wind-hail perils
- Insurers buy cat reinsurance to hedge against catastrophe risks
1. Introduction

Reinsurance decision is a balance between cost and benefit
- Cost: reinsurance premium – loss recovered
- Benefit: risk reduction
  - Stable income stream over time
  - Protection again extreme events
  - Reduce likelihood of being downgraded

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1. Introduction

How to measure risk reduction
- Variance and standard deviation
- Not downside risk measures
- Desirable swings are also treated as risk
- VaR (Value-at-Risk), TVaR, XTVaR
  - VaR: predetermined percentile point
  - TVaR: expected value when loss>VAR
  - XTVaR: TVaR-mean

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1. Introduction

How to measure risk reduction
- Lower partial moment and downside variance
  \[ LPM(L \mid T, k) = \frac{1}{T} \int_{-\infty}^{0} (L - T)^k dF(L) \]
  - T is the maximum acceptable losses, benchmark for "downside"
  - k is the risk perception parameter to large losses, the higher the k, the stronger risk aversion to large losses
  - When k=1 and T is the 99th percentile of loss, LPM is equal to 0.01*VaR
  - When k=2 and T is the mean, LPM is semi-variance
  - When K=2 and T is the target, LPM is downside variance
1. Introduction

How to measure risk reduction
- EPD expected policyholder deficit
  \[ \text{EPD} = \text{probability of default} \times \text{average loss from default} \]
- Cost of default option
  - An insurer will not pay claims once the capital is exhausted
  - A put option that transfers default risk to policyholders
- PML (probable maximum loss per event) and AAL (average annual Loss)

2. Optimal reinsurance: academics


- Cat reinsurance has zero correlation with market index, and therefore zero beta in CAPM.
- Because of zero beta, reinsurance premium should be a dollar-to-dollar.
- Reinsurance reduces risk at zero cost. Therefore optimizing profit-risk tradeoff implies minimizing risk
  - buy largest possible protection without budget constraints
  - buy highest possible retention with budget constraints
2. Optimal reinsurance: academics

Academic Assumption

Those studies do not help practitioners
- Reinsurance is costly.
- Reinsurers need to hold a large amount of capital and require a market return on such a capital.
- Reinsurance premium/Loss recovered can be over 10 in reality
- No reinsurers can fully diversify away cat risk
- Only consider the risk side of equation and ignore cost side.

3. Optimal reinsurance: RAROC

RAROC (Risk-adjusted return on capital) approach is popular in practice
- Economic capital (EC) covers extreme loss scenarios
- Reinsurance cost = reinsurance premium – expected recovery
- Capital Saving = EC w/o reinsurance – EC w reinsurance
- Cost of Risk Capital (CORC) = Reinsurance cost / Capital Saving
- CORC balances profit (numerator) and risk (denominator)
3. Optimal reinsurance: RAROC

- There is no universal definition of economic capital
- Use VaR or TVaR to measure risk
  - Only consider extreme scenarios. Insurance companies also dislike small losses
  - Linear risk perception. 100 million loss is 10 times worse than 10 million loss by VaR. In reality, risk perception is exponentially increasing with the size of loss.

4. Optimal Reinsurance: DRAP Approach

Downside Risk-adjusted Profit (DRAP)

\[
DRAP = \text{Mean}(r) - \theta \times LPM(r | T, k)
\]

\[
LPM(r | T, k) = \int_{-\infty}^{\infty} (T - r)^k dF(r)
\]

- \( r \) is underwriting profit rate
- \( \theta \) is the risk aversion coefficient
- \( T \) is the bench mark for downside
- \( K \) measures the increasing risk perception toward large losses
4. Optimal Reinsurance: DRAP Approach

**Loss Recovery**

\[ G(x, R, L) = \begin{cases} 0 & \text{if } x_i \leq R \\ (x_i - R) \Phi & \text{if } R < x_i \leq R + L \\ L \Phi & \text{if } x_i > R + L \end{cases} \]

- R is retention
- L is the limit
- \( \Phi \) is the coverage percentage
- \( x_i \) is cat loss from the \( i \)th event

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4. Optimal Reinsurance: DRAP Approach

**Underwriting profit**

\[ r = 1 - \frac{\text{EXP} + \text{RP}(R, L) + \sum_{i=1}^{N} G(x, R, L) + \text{RI}(x, R, L)}{\text{EP}} \]

- EP: gross earned premium
- EXP: expense
- Y: non-cat losses
- RP(R, L): reinsurance premium
- RI(x, R, L): reinstatement premium
- N: number of cat event

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4. Optimal Reinsurance: DRAP Approach

- AB is efficient frontier
- U1, U2, U3 are utility curves
- C is the optimal reinsurance that maximizes DRAP

\[ \text{Max} \left( \text{Mean}(r) - \theta^{*} \text{LPM}(r \mid T, k) \right) \]
4. Optimal Reinsurance: DRAP Approach

Advantages to conventional mean-variance studies in academics
- An ERM approach.
- Considers both catastrophe and non-catastrophe losses simultaneously
- Overall profitability impacts the layer selection. High profitability enhances an insurer’s ability to more cat risk.
- Use a downside risk measure (LPM) other than two-side risk measure (variance)

Parameter estimations
- Theta may not be constant by the size of loss
  - For loss that causes a bad quarter, theta is low
  - For loss that causes a bad year and no annual bonus, theta will be high
  - For loss that cause a financial downgrade or replacement of management, theta will be even higher
- Theta is time variant
- Theta varies by individual institution

Parameter estimations
- Theta is difficult to measure.
- How much management is willing to pay to be risk free?
- How much investors require to take the risk?
  - index risk premium = index return – risk free rate
  - Insurance risk premium= insurance return-risk free rate
  - cat risk premium= cat bond yield-risk free rate
4. Optimal Reinsurance: DRAP Approach

Parameter estimations
- k may not be constant by the size of loss
  - For smaller loss, loss perception is close to 1, k=1;
  - For severe loss, k>1
- Academic tradition: k=2
- Recent literature: increasing evidences that risks measured by moments >2 were priced

4. Optimal Reinsurance: DRAP Approach

Parameter estimations
- T is the benchmark for “downside”
  - Target profit: below target is risk
  - Zero: underwriting loss is risk
  - Zero ROE: underwriting loss larger than investment income is risk
  - Large negative: severe loss is treated as risk

5. Case Study

A hypothetical company
- Gross earned premium from all lines: 10 billion
- Expense ratio: 33%
- Lognormal non-cat loss from actual data
  - mean=5.91 billion; std=402 million
- Lognormal cat loss estimated from AIR data
  - mean # of event=39.7; std=4.45
  - mean loss from an event=10.02 million; std=50.77 million
  - total annual cat loss mean=398 million; std=323 million
5. Case Study

- K=2
- T=0%
- Theta is tested at 16.71, 22.28, and 27.85, which represents that primary insurer would like to pay 30%, 40%, and 50% of gross profit to be risk free, respectively.
- UW profit without Insurance is 3.92%
- Variance 0.263%
- Downside variance is 0.07% (T=0%)
- Probability of underwriting loss is 18.41%
- Probability of severe loss (<-15%) is 0.48%

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5. Case Study

Reinsurance quotes (million)

<table>
<thead>
<tr>
<th>Retention (million)</th>
<th>Upper Bound of Level (million)</th>
<th>Reinsurance Limits (million)</th>
<th>Reinsurance Price</th>
<th>Rate on line</th>
</tr>
</thead>
<tbody>
<tr>
<td>305</td>
<td>420</td>
<td>315</td>
<td>26.8</td>
<td>10.00%</td>
</tr>
<tr>
<td>420</td>
<td>610</td>
<td>515</td>
<td>21.7</td>
<td>13.42%</td>
</tr>
<tr>
<td>610</td>
<td>610</td>
<td>385</td>
<td>18.8</td>
<td>6.59%</td>
</tr>
<tr>
<td>610</td>
<td>1,030</td>
<td>420</td>
<td>25.2</td>
<td>5.99%</td>
</tr>
<tr>
<td>1,030</td>
<td>1,030</td>
<td>750</td>
<td>28.7</td>
<td>3.72%</td>
</tr>
<tr>
<td>1,030</td>
<td>1,030</td>
<td>1,250</td>
<td>38.1</td>
<td>3.19%</td>
</tr>
</tbody>
</table>

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5. Case Study

Recoveries and penetrations by layers

<table>
<thead>
<tr>
<th>Retention (million)</th>
<th>Upper Limit (million)</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Recovery/reinsurance Premium</th>
<th>Penetration Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>305</td>
<td>420</td>
<td>8,859,074</td>
<td>20,492,219</td>
<td>42.59%</td>
<td>10.18%</td>
</tr>
<tr>
<td>420</td>
<td>610</td>
<td>8,435,968</td>
<td>35,997,450</td>
<td>77.10%</td>
<td>6.06%</td>
</tr>
<tr>
<td>515</td>
<td>610</td>
<td>6,454,494</td>
<td>41,089,396</td>
<td>32.61%</td>
<td>3.15%</td>
</tr>
<tr>
<td>610</td>
<td>1,030</td>
<td>7,430,052</td>
<td>53,899,264</td>
<td>31.44%</td>
<td>3.15%</td>
</tr>
<tr>
<td>1,030</td>
<td>1,030</td>
<td>4,556,545</td>
<td>55,432,115</td>
<td>16.03%</td>
<td>1.1%</td>
</tr>
<tr>
<td>1,030</td>
<td>1,030</td>
<td>3,373,573</td>
<td>40,827,831</td>
<td>6.56%</td>
<td>0.46%</td>
</tr>
</tbody>
</table>
5. Case Study

Reinsurance Price Curves Fitting
- $(x_1, x_2)$ represents reinsurance layer
- $p(x_1, x_2)$ represents rate-on-line

\[ p(x_1, x_2) = \int f(x)dx \]

- Add quadratic term, logarithm, and inverse term to reflect nonlinear relations

\[ f(x) = \beta_1 x + \beta_2 x^2 + \beta_3 \log(x) + \beta_4 x^{-1} \]

\[ p(x_1, x_2) = \beta_1 (x_1 - x_2) + \frac{1}{2} \beta_2 (x_2^2 - x_1^2) + \frac{1}{3} \beta_3 (\log(x_2) - \log(x_1)) + \beta_4 (\log(x_2) - \log(x_1)) \]

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5. Case Study

Reinsurance Price Fitting

<table>
<thead>
<tr>
<th>Retention Level</th>
<th>Upper Bound of Layer</th>
<th>Limit</th>
<th>Rate-on-line</th>
<th>Fitted Rate</th>
<th>Fitted Rate-on-line</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>340</td>
<td>199</td>
<td>21.7</td>
<td>21.69</td>
<td>21.64</td>
</tr>
<tr>
<td>340</td>
<td>420</td>
<td>269</td>
<td>6.58%</td>
<td>6.57%</td>
<td></td>
</tr>
<tr>
<td>420</td>
<td>500</td>
<td>333</td>
<td>5.58%</td>
<td>5.57%</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>610</td>
<td>396</td>
<td>5.27%</td>
<td>5.26%</td>
<td></td>
</tr>
<tr>
<td>610</td>
<td>915</td>
<td>530</td>
<td>3.59%</td>
<td>3.59%</td>
<td></td>
</tr>
<tr>
<td>915</td>
<td>1,390</td>
<td>710</td>
<td>2.77%</td>
<td>2.77%</td>
<td></td>
</tr>
<tr>
<td>1,390</td>
<td>2,380</td>
<td>1,050</td>
<td>2.18%</td>
<td>2.18%</td>
<td></td>
</tr>
<tr>
<td>2,380</td>
<td>3,050</td>
<td>1,700</td>
<td>1.73%</td>
<td>1.73%</td>
<td></td>
</tr>
<tr>
<td>3,050</td>
<td>4,200</td>
<td>2,690</td>
<td>1.55%</td>
<td>1.55%</td>
<td></td>
</tr>
<tr>
<td>4,200</td>
<td>6,100</td>
<td>4,190</td>
<td>1.47%</td>
<td>1.47%</td>
<td></td>
</tr>
<tr>
<td>6,100</td>
<td>9,150</td>
<td>6,090</td>
<td>1.38%</td>
<td>1.38%</td>
<td></td>
</tr>
<tr>
<td>9,150</td>
<td>12,200</td>
<td>8,990</td>
<td>1.31%</td>
<td>1.31%</td>
<td></td>
</tr>
<tr>
<td>12,200</td>
<td>15,300</td>
<td>11,890</td>
<td>1.25%</td>
<td>1.25%</td>
<td></td>
</tr>
</tbody>
</table>

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5. Case Study

Performance of Reinsurance Layers $\theta=22.28$

<table>
<thead>
<tr>
<th>Retention Level</th>
<th>Upper Limit of Layer</th>
<th>Rate-on-line</th>
<th>Risk-adjusted Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>340</td>
<td>19.43%</td>
<td>0.21%</td>
</tr>
<tr>
<td>340</td>
<td>420</td>
<td>18.14%</td>
<td>0.07%</td>
</tr>
<tr>
<td>420</td>
<td>500</td>
<td>17.88%</td>
<td>0.07%</td>
</tr>
<tr>
<td>500</td>
<td>610</td>
<td>17.62%</td>
<td>0.07%</td>
</tr>
<tr>
<td>610</td>
<td>915</td>
<td>17.37%</td>
<td>0.07%</td>
</tr>
<tr>
<td>915</td>
<td>1,390</td>
<td>17.16%</td>
<td>0.07%</td>
</tr>
<tr>
<td>1,390</td>
<td>2,380</td>
<td>16.97%</td>
<td>0.07%</td>
</tr>
<tr>
<td>2,380</td>
<td>3,050</td>
<td>16.80%</td>
<td>0.07%</td>
</tr>
<tr>
<td>3,050</td>
<td>4,200</td>
<td>16.64%</td>
<td>0.07%</td>
</tr>
<tr>
<td>4,200</td>
<td>6,100</td>
<td>16.49%</td>
<td>0.07%</td>
</tr>
<tr>
<td>6,100</td>
<td>9,150</td>
<td>16.35%</td>
<td>0.07%</td>
</tr>
<tr>
<td>9,150</td>
<td>12,200</td>
<td>16.24%</td>
<td>0.07%</td>
</tr>
<tr>
<td>12,200</td>
<td>15,300</td>
<td>16.15%</td>
<td>0.07%</td>
</tr>
</tbody>
</table>

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5. Case Study

Efficient Frontier

Figure 3: Reinsurance Efficient Frontier

5. Case Study

- Optimal Reinsurance Layers \( \theta = 16.71, 22.28, 27.85 \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Retention (mill)</th>
<th>Upper Limit (mill)</th>
<th>Mean</th>
<th>Downside Variance</th>
<th>Risk-Adjusted Profit Rate 3.771</th>
<th>Risk-Adjusted Profit Rate 3.667</th>
<th>Risk-Adjusted Profit Rate 3.610</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.71</td>
<td>705</td>
<td>1220</td>
<td>3.771</td>
<td>0.080%</td>
<td>2.634%</td>
<td>2.100%</td>
<td>2.434%</td>
</tr>
<tr>
<td>22.28</td>
<td>680</td>
<td>1390</td>
<td>3.667</td>
<td>0.055%</td>
<td>2.476%</td>
<td>2.147%</td>
<td>2.451%</td>
</tr>
<tr>
<td>27.85</td>
<td>615</td>
<td>1460</td>
<td>3.610</td>
<td>0.052%</td>
<td>2.443%</td>
<td>2.154%</td>
<td>2.445%</td>
</tr>
</tbody>
</table>

- If the overall profit rate increases 2% and \( \theta \) remains at 22.28, the optimal layers becomes (740, 1420)

6. Conclusions

- The overall profitability (both cat and noncat losses) impacts optimal insurance decision
- Risk appetites are difficult to measure by a single parameter.
- DRAP capture risk appetites comprehensively though \( \theta \) (risk aversion coefficient), \( T \) (downside benchmark), and moment \( k \) (increasing perception toward large loss)
- DRAP provides an alternative approach to calculate optimal layers.