Casualty Loss Reserve Seminar
Technical Provisions in Solvency II
What EU Insurers Could Do with Schedule P

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ISO Innovative Analytics
September 21, 2010
• **Overall Objectives**
  – Treat as a statistical problem
  – Use realistic distributions in fitting model
    • Various versions of the collective risk model
  – Use Bayesian methodology
    • Compelling case in “Thinking Outside the Triangle”
  – Aggressive retrospective testing on subsequent data
    • How good are statistical models in predicting ultimates?
  – Apply results to insurer risk management
    • Given the limits of our ability to predict ultimate losses (whatever the are), how do we manage insurer risk?
    • IAA and Solvency II
Outline of Presentation

• Technical Provisions in Solvency II
  – As described in the EU Framework Directive
• American NAIC Schedule P Loss Triangles
• How to use data in Schedule P
  – Calculate best estimate
  – Compare predictions against experience
  – Calculate risk margin (Market Value Margin)
Background

• **Solvency II adopted by European Parliament**
  – Originally effective October 31, 2012
• **Objectives include:**
  – Increased focus on effective risk management, control and governance
  – Market consistent valuation of assets & liabilities
  – Increased disclosure and transparency
Focus of the Paper
Technical Provisions

Available capital

Assets covering technical provisions

Solvency Capital Requirement

Risk margin for non-hedgeable risks
Best estimate
Technical Provisions
Market consistent valuation for hedgeable risks

Conceptually in line with IFRS
**Principles Underlying Technical Provisions Described in Articles 76-83 of EU Framework Directive**

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<tr>
<th>THE EUROPEAN PARLIAMENT</th>
<th>THE COUNCIL</th>
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**LEGISLATIVE ACTS AND OTHER INSTRUMENTS**

Subject: DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast)
1. The value of technical provisions shall be equal to the sum of a best estimate and a risk margin as set out in paragraphs 2 and 3.

2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

The best estimate shall be calculated gross, without deduction of the amounts recoverable from reinsurance contracts and special purpose vehicles. Those amounts shall be calculated separately, in accordance with Article 81.
4. Insurance and reinsurance undertakings shall value the best estimate and the risk margin separately.

5. Where insurance and reinsurance undertakings value the best estimate and the risk margin separately, the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.
Insurance and reinsurance undertakings shall segment their insurance and reinsurance obligations into homogeneous risk groups, and as a minimum by lines of business, when calculating their technical provisions.
Insurance and reinsurance undertakings shall have processes and procedures in place to ensure that best estimates, and the assumptions underlying the calculation of best estimates, are regularly compared against experience.

Where the comparison identifies systematic deviation between experience and the best estimate calculations of insurance or reinsurance undertakings, the undertaking concerned shall make appropriate adjustments to the actuarial methods being used and/or the assumptions being made.
Schedule P Loss Triangles

• Part of American NAIC Annual Statement
  – All American insurers must submit to regulators
  – Data is available to the public
    • At a price (academic discounts available)
    • [http://www.naic.org/store_financial_home.htm](http://www.naic.org/store_financial_home.htm)
  – Multiples lines of business
  – Paid and incurred loss triangles
From Data Reported in Schedule P
Assemble a Triangle of Incremental Paid Losses

- Include Earned Premium
- This data is for a real insurer Insurer #1 in Paper
- Commercial Auto Liability

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<tr>
<th>AY</th>
<th>Premium</th>
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<th>Lag 2</th>
<th>Lag 3</th>
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Amounts in Thousands
Using Schedule P Data

- We of course have the reported earned premium and incremental paid loss
- But we also have similar data for every American insurer for each line of business
  - A great source of “prior information” that can be used in a Bayesian analysis
  - Use Schedule P data for 50 large insurers to create 5000 “benchmark scenarios” for each line of insurance.
    - Select 100 parameter sets from each of the 50 large insurers with likelihood “close” to the maximum likelihood.
    - Fit a gamma distribution to the combined set for each parameter.
Loss Model

- Expected Loss

\[ \mu_{AY,Lag} = \text{Premium}_{AY} \cdot \text{ELR}_{AY} \cdot \text{Dev}_{Lag} \cdot t^{AY+Lag-1} \]

- Variance of Loss

\[ \text{Var}\left[ X_{AY,Lag} \right] = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot \left(1 + 1 / \alpha\right) + c \cdot \mu_{AY,Lag}^2 \]

\[ \tau_{Lag} = \text{Sev} \cdot \left(1 - \left(1 - \frac{Lag}{10}\right)^3\right) \text{ for Lag} = 1, 2, ..., 10. \]

- \{ELR_{AY}\}, \{Dev_{Lag}\}, t, c, and Sev are unknown parameters,
Tweedie Model of Losses in Each (AY,Lag) Cell

Tweedie - Compound Poisson with Gamma Severity

\[
\mu_{AY,Lag} = \text{Premium}_{AY} \cdot \text{ELR}_{AY} \cdot \text{Dev}_{Lag} \cdot t^{AY+Lag-1}, \quad p = \frac{\alpha + 2}{\alpha + 1}
\]

\[
\phi_{AY,Lag} \cdot \mu_{AY,Lag}^p = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot \left(1 + 1/\alpha\right) + c \cdot \mu_{AY,Lag}^2
\]

• Pick a parameter set \{\text{ELR}_{AY}\}, \{\text{Dev}_{Lag}\}, t, c, Sev
• Translate parameters into Tweedie parameters
  • \(\mu_{AY,Lag}, p\) and \(\phi_{AY,Lag}\)
• Calculate sample parameter sets from posterior distribution with Metropolis-Hastings Algorithm
  • Generate thousands of parameter sets
  • Thinning - Keep a sample of 500 parameter sets
Graphical Representation of Metropolis-Hastings Sample

Note that the posteriors are tighter, showing how the data narrows the range of results.

“Information Reduces Uncertainty”

Claude Shannon
Graphical Representation of Metropolis-Hastings Sample

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Calculating the Best Estimate
From Article 77 – Framework Directive

2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

- Expected loss for $n^{th}$ Metropolis-Hastings scenario

$$\mu_{n,AY,Lag} = \text{Premium}_{AY} \cdot \text{ELR}_{AY} \cdot \text{Dev}_{Lag} \cdot t^{AY+Lag-1}$$

- Best Estimate $i = 4\%$

$$\sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} \left( \frac{1}{500} \sum_{n=1}^{500} \mu_{n,AY,Lag} \right) \cdot \frac{1}{(1+i)^{AY+Lag-11.5}} = 91,220$$
Comparisons against experience

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Distribution of Reserve Outcomes

Incremental Paid Losses

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Predictive Distribution of Reserve Outcomes – 1 Year

\[ R₁ = \sum_{AY=2}^{10} X_{AY,12-AY} \]
1. Select a random parameter set from the list
\[
\left\{ \text{sev}_n, t_n, c_n, \{ \text{ELR}_{n,AY} \}, \{ \text{Dev}_{n,Lag} \} \right\}
\]

2. For each \((AY, Lag)\) cell in next calendar year 
\((AY = 2, \ldots, 10, \ Lag = 12 - AY)\): 
- Calculate \(\mu_{AY,Lag}\) from Equation 1. 
- Calculate \(\phi_{AY,Lag}\) from Equation 6. 
- Select a random loss \(X_{AY,Lag}\) from a Tweedie distribution with parameters \(p = 1.67, \mu_{AY,Lag}, \) and \(\phi_{AY,Lag}\).

3. Set 
\[
X = \sum_{AY=2}^{10} X_{AY,12-AY}
\]
Histogram of $\{\mu_{AY,Lag}\}$ from Metropolis-Hastings output

Can be done by Simulations or FFT

I used FFT
Where the Actual Data Lies on the Predictive Distribution

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<tr>
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<th>Actual</th>
<th>Expected</th>
<th>Ratio</th>
<th>p-value</th>
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Insurers 3 and 4 Need Close Examination
Technical Provisions, Risk Margins and the IAA

• International Association of Insurance Supervisors (IAIS) requested help from the International Actuarial Association (IAA) to work on the issues of risk based capital and risk margins for loss reserves.

• Available on IIA Website
  

• Refer to risk margin as Market Value Margin (MVM)
Focus of Presentation
Technical Provisions

Available capital

Assets covering technical provisions

Solvency Capital Requirement

Risk margin for non-hedgeable risks

Best estimate

Technical Provisions

Market consistent valuation for hedgeable risks

Conceptually in line with IFRS
1. The less that is known about the current estimate and its trend; the higher the risk margins should be.
2. Risks with low frequency and high severity will have higher risk margins than risks with high frequency and low severity.
3. For similar risks, contracts that persist over a longer timeframe will have higher risk margins than those of shorter duration.
4. Risks with a wide probability distribution will have higher risk margins than those risks with a narrower distribution.
5. To the extent that emerging experience reduces uncertainty, risk margins will decrease, and vice versa.
Possibilities

- **Undiscounted reserves**
  - Only satisfy Property 3
- **Percentile method**
  - Does not satisfy Property 3
- **Cost of Capital Method**
  - Satisfies all properties
- **So – What is the Cost of Capital Method?**
What is Capital?

- **Sufficient for time horizon of one year**
  - Controversial – Many prefer a longer time horizon
- **Capital = TVaR@99% – Expected Loss**
  - Calculate for each future payment year
    - i.e. sub diagonal
  - Remember the time value of money
- **FFT methods on Tweedie distributions allow for reliable calculation of TVaR**
### Statistics of Interest for Risk Margin

#### Distribution of Reserve Outcomes

#### Incremental Paid Losses

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<td>X₁₀,5</td>
<td>X₁₀,6</td>
<td>X₁₀,7</td>
<td>X₁₀,8</td>
<td>X₁₀,9</td>
<td>X₁₀,10</td>
<td></td>
</tr>
</tbody>
</table>

**Predictive Distribution of Reserve Outcomes – 1 Year**

\[
R_1 = \sum_{AY=2}^{10} X_{AY,12-AY}
\]

**Predictive Distribution of Reserve Outcomes – 10 Year**

\[
R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}
\]
Statistics of Interest for Risk Margins
Predictive Distributions of Reserve Outcomes

• Simulation
  - Randomly select \{ELR_i\},\{Dev_j\}, t, c, Sev from the posterior
  - Simulate \( R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag} \) where \( X_{AY,Lag} \sim \text{Tweedie} \)

• Simulate \( R_1 \) Similarly
• Use the Fast Fourier Transform
  - Faster, more accurate, but uses some math
<table>
<thead>
<tr>
<th>$t$</th>
<th>$L_t^{Nom}$</th>
<th>$\Delta L_t^{Nom}$</th>
<th>$L_t^{Disc}$</th>
<th>TVaR$_t^{Nom}$</th>
<th>$\Delta$ TVaR$_t^{Nom}$</th>
<th>TVaR$_t^{Disc}$</th>
<th>$C_t$</th>
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<td>37,526</td>
<td>52,875</td>
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<td>10,889</td>
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<td>3,661</td>
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<td>451</td>
<td>442</td>
<td>4,097</td>
<td>4,097</td>
<td>4,017</td>
<td>3,575</td>
</tr>
</tbody>
</table>
Calculating Capital Needs in the Future
10 Year Time Horizon - Discount @ 4%

<table>
<thead>
<tr>
<th></th>
<th>$L_{t}^{Nom}$</th>
<th>$\Delta L_{t}^{Nom}$</th>
<th>$L_{t}^{Disc}$</th>
<th>TVaR$_{t}^{Nom}$</th>
<th>$\Delta$ TVaR$_{t}^{Nom}$</th>
<th>TVaR$_{t}^{Disc}$</th>
<th>$C_{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>40,375</td>
<td>91,220</td>
<td>128,894</td>
<td>48,491</td>
<td>118,529</td>
<td>27,309</td>
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<tr>
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<td>7,622</td>
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<td>31,528</td>
<td>9,412</td>
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<td>13,504</td>
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<td>15,891</td>
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<td>1,276</td>
<td>2,407</td>
<td>11,570</td>
<td>3,673</td>
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<td>8,493</td>
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<tr>
<td>7</td>
<td>1,243</td>
<td>792</td>
<td>1,202</td>
<td>7,898</td>
<td>3,801</td>
<td>7,590</td>
<td>6,388</td>
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<tr>
<td>8</td>
<td>451</td>
<td>451</td>
<td>442</td>
<td>4,097</td>
<td>4,097</td>
<td>4,017</td>
<td>3,575</td>
</tr>
</tbody>
</table>
Explanation of $C_t$ Calculation (Documentation)

(1) The time, $t$, after the liability is set.

(2) The expected value of all future payments, $L_t^{\text{Nom}} = \sum_{AY=2-t}^{10} \sum_{Log=12-t}^{10} \mu_{AY,Log} \cdot$.

(3) $\Delta L_t^{\text{Nom}} = L_t^{\text{Nom}} - L_{t+1}^{\text{Nom}}$.

(4) The discounted liability, $L_t^{\text{Disc}} = \sum_{i=t}^{8} \frac{\Delta L_t^{\text{Nom}}}{(1+i)^{k-t+0.5}}$.

(5) The Tail-Value-at-Risk, i.e., the conditional expected value of the random loss,

$$\sum_{AY=2-t}^{10} \sum_{Log=12-t}^{10} X_{AY,Log} \cdot$$
given that the loss exceeds the 99th percentile.

(6) $\Delta TVaR_t^{\text{Nom}} = TVaR_t^{\text{Nom}} - TVaR_{t+1}^{\text{Nom}}$.

(7) The discounted $TVaR_t^{\text{Disc}} = \sum_{i=t}^{8} \frac{\Delta TVaR_k^{\text{Nom}}}{(1+i)^{k-t+0.5}}$.

(8) The needed capital at time $t$ is expected to be $C_t = TVaR_t^{\text{Disc}} - L_t^{\text{Disc}}$. 

ISO 35
Risk Margin Version 1 – Capital Cash Flow (CCF)

- $C_t =$ required capital at time $t$ for one year.
  - TVaR@99% - Expected Loss (discounted)
- $i =$ risk-free rate of return
- $r =$ risky rate of return due to insurer’s investors
- $MVM_{CCF} = C_0 - PV(Released \ Capital \ @ \ rate \ r)$

\[
MVM_{CCF} = C_0 - \sum_{t=0}^{\infty} \frac{C_t \cdot (1+i) - C_{t+1}}{(1+r)^{t+1}}
\]

\[
MVM_{CCF} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}
\]

After some algebra
Versions 2 and 3

- **Capital Cash Flow (CCF)**
  \[ MVM_{CCF} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1 + r)^{t+1}} \]

- **Swiss Solvency Test (SST)**
  - Starts at \( t = 1 \). Ignores capital raised in first year.
  - Discounts at rate \( i \) instead of rate \( r \).
  \[ MVM_{SST} = (r - i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1 + i)^{t+1}} \]

- **Solvency II/QIS4 (SII)**
  - Starts at \( t = 0 \)
  \[ MVM_{SII} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1 + i)^{t+1}} \]

All three versions satisfy the IAIS criteria.
Rationale Behind $MVM_{SST}$

\[ MVM_{SST} = (r - i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1 + i)^{t+1}} \]

“The risk margin can be expressed as the expected present value of the cost of capital necessary to buffer the nonhedgeable risk of insurance liabilities during the entire lifetime of the insurance liabilities.”
Rationale Behind $MVM_{SII}$

$$MVM_{SII} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

Both Solvency II and SST require capital to cover risk over a one year time horizon. SST says that you don’t need a risk margin to cover the first year. Solvency II says you do.
The Results

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>$MVM_{CCF}$</th>
<th>%</th>
<th>$MVM_{SST}$</th>
<th>%</th>
<th>$MVM_{QIS4}$</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,994</td>
<td>2.2%</td>
<td>1,854</td>
<td>2.0%</td>
<td>2,411</td>
<td>2.6%</td>
</tr>
<tr>
<td>10</td>
<td>5,082</td>
<td>5.6%</td>
<td>4,736</td>
<td>5.2%</td>
<td>6,129</td>
<td>6.7%</td>
</tr>
</tbody>
</table>

Best Estimate = 91,220

$r = 10\%$  
$i = 4\%$
Summary

• Used Bayesian analysis and likelihood of a triangle of data based on the Tweedie model to calculate posterior probabilities of scenarios

• Used posterior probability of scenarios to calculate
  – Current Estimate
  – Compare Actual with Prediction
  – Risk margin