Integrating Reserve Risk Models into Economic Capital Models

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Recent Developments in EC Models

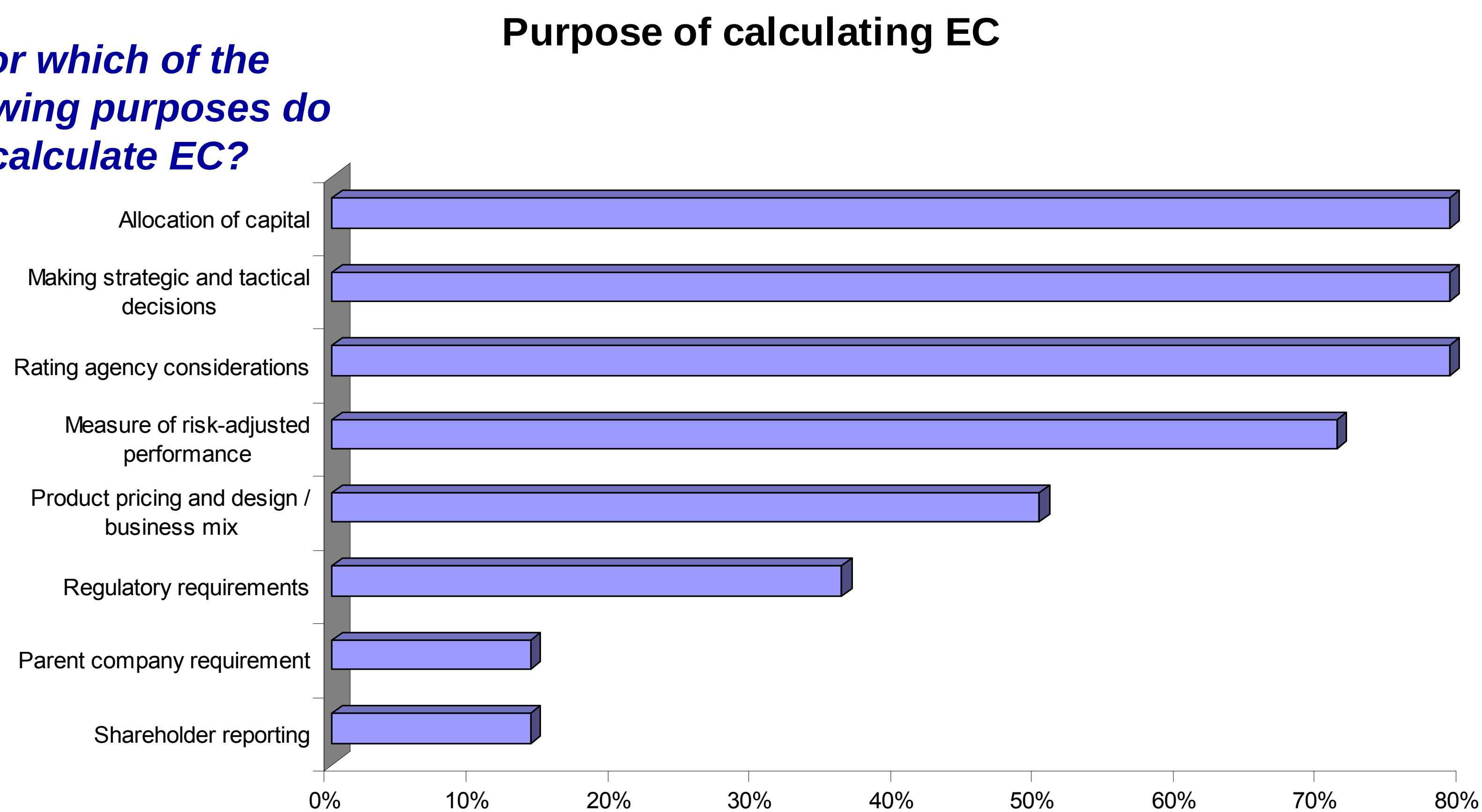
Economic capital survey – Background and results

- In April 2007, Towers Perrin conducted a targeted survey focusing on economic capital (EC) approaches used by North American Property and Casualty insurance companies.
  - This survey was conducted as a supplement to prior surveys of insurance company chief financial officers, chief actuaries and chief risk officers to learn more about their approaches to enterprise risk management (ERM). The Tillinghast insurance consulting business of Towers Perrin conducted these global surveys in 1999, 2002, 2004 and 2006.
  - Organizations found this research valuable to benchmark their risk management activities against their competitors and against industry best practices
- Key Findings
  - EC and Statutory Capital are the financial measures most often used for decision-making purposes, closely followed by GAAP Capital and Rating Agency Capital
  - EC is viewed as a useful risk management tool that has a wide variety of business decision applications, including strategic and tactical decision-making, allocation of capital, rating agency considerations, and risk-adjusted performance measurement
  - Risk tolerances range significantly across organizations and also internally depending on the application
  - Stochastic modeling of multiple risks is the preferred quantification methodology

Companies use the EC model for multiple applications, including capital allocation, decision-making, and rating agency considerations

- When asked what the purpose is for calculating EC, most frequently cited are allocation of capital, making strategic and tactical decisions, and rating agency considerations, each identified by nearly 80% of the respondents

Q: For which of the following purposes do you calculate EC?



Solvency II is around the corner for (European) P&C insurers

- The European insurance community is heading towards a new regime for solvency regulation.
  - 'Solvency II' will introduce **economic risk-based solvency requirements** across all EU Member States for the first time.
  - The new requirements move away from a crude "one-model-fits-all" way of estimating capital requirements to more entity-specific requirements.
- Valuation of assets and liabilities on a "market consistent" basis
  - Liabilities = technical provisions + other liabilities
  - Technical provision calculated on a **current exit value** basis
    - — Hedgeable risks: market consistent valuation
    - — Non-hedgeable risks: Best estimate + Risk margin (**Cost-of-capital**)

Implementing EC Models

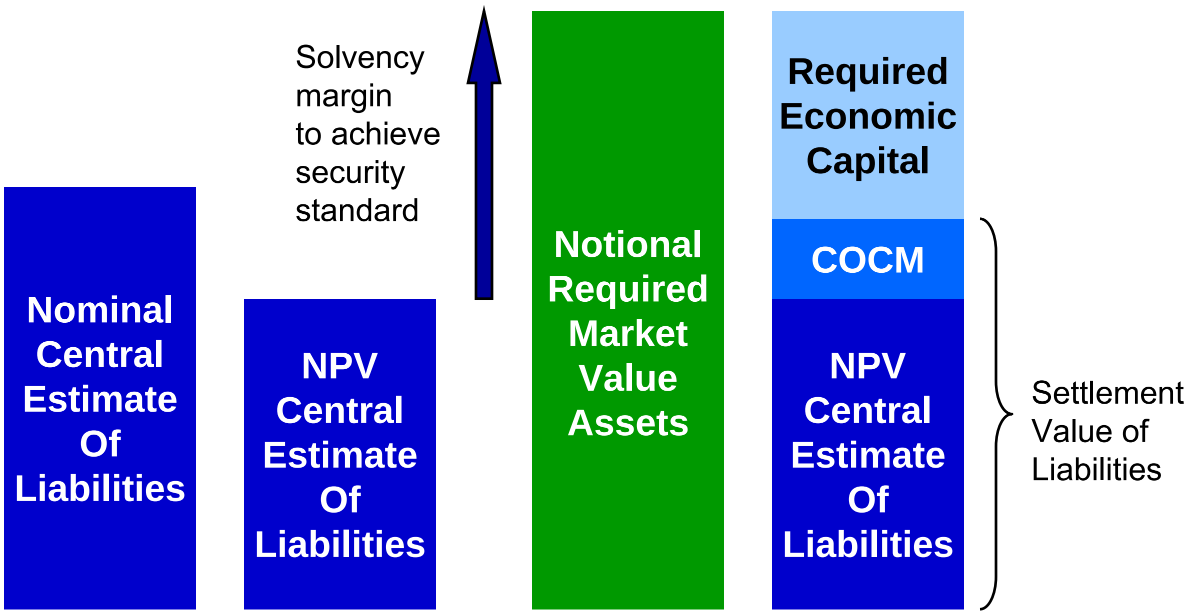
A number of key decisions need to be made when implementing EC

Decision 1	Decision 2	Decision 3	Decision 4	Decision 5	Decision 6
Period for assessment	**Valuation of liabilities**	**Measure of risk**	**Risks to include**	**Quantification method**	**Aggregation**
One year n years Run off of portfolio	Statutory GAAP Economic	Risk of ruin TVAR/CTE ECOR	Market Credit Insurance Operational Liquidity	Stochastic modelling Stress testing Factor based	Additive Variance/ covariance Stochastic

These decisions must balance multiple objectives of simplicity, reliability and practicality

To measure required EC, we must construct a notional economic balance sheet

- **Settlement Value Liabilities** (SVL) includes
  - The NPV **Central Estimate of Liabilities** (CEL), equal to the present value of the expected liabilities discounted at the risk free rate
  - A **Cost-of-Capital Margin** (COCM) that compensates the liability holder for the capital required to support the associated risk over the life of the liabilities
- **Economic Capital** is the difference between the notional required Market Value Assets (MVA) and SVL

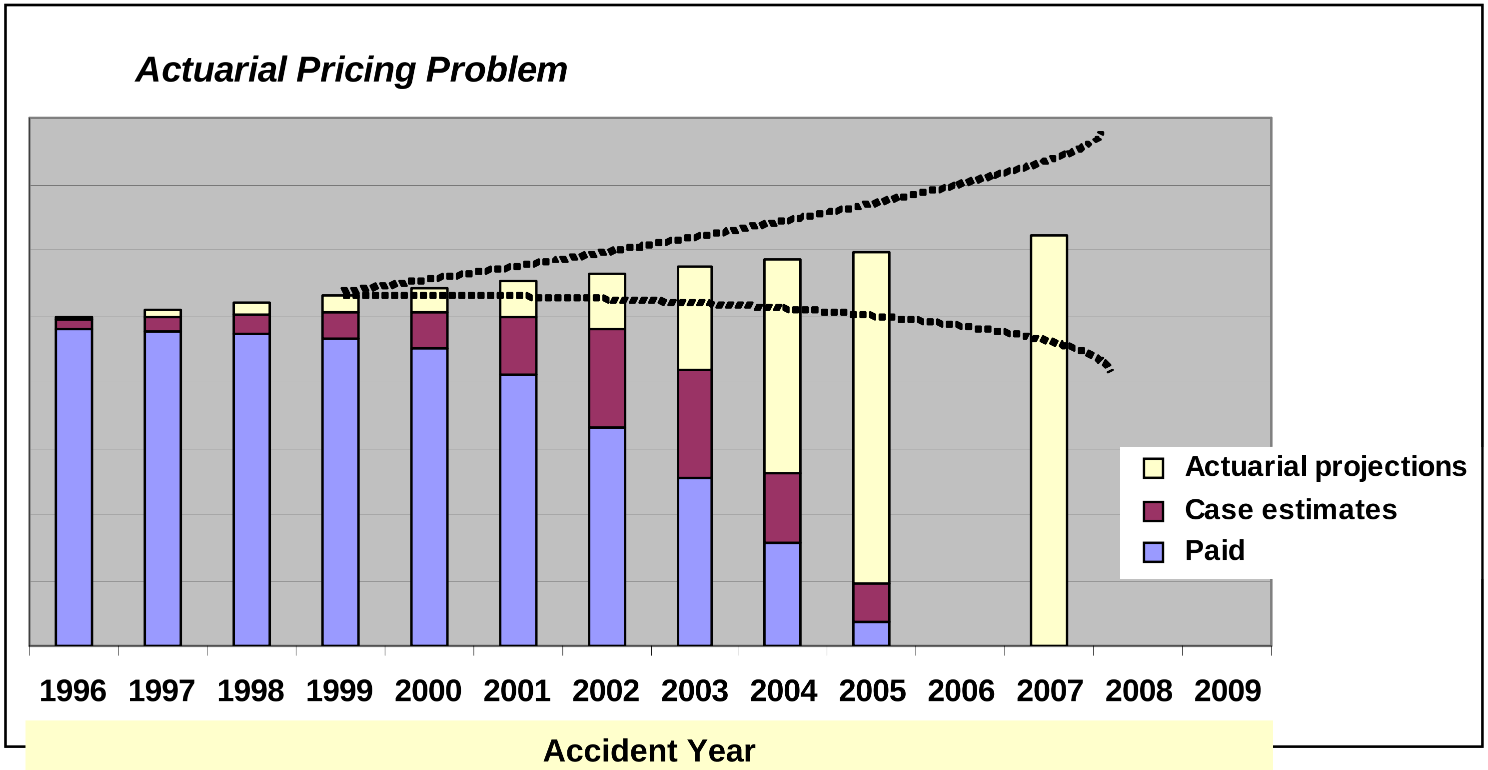


There are two divergent views on the appropriate risk horizon for EC

Horizon	Where Used?	Principal Criticisms	Application
One-Year	■ Solvency II ■ UK ICA ■ Swiss Solvency Test	■ Assumes that risk position can be reduced or additional capital can be raised at the end of the year ■ No secondary market for claim liabilities exists	Companies with: ■ Active capital management ■ Strong earnings momentum ■ Significant franchise value
Run-Off	■ NAIC P&C RBC ■ Rating Agency P&C Capital Adequacy Formulae	■ Requires capital to be held now against possible future consumption later; inefficient ■ Inconsistent with real-world annual balance sheet reviews ■ Makes proper aggregation of risks very difficult	■ Companies that build capital primarily through retained earnings ■ Troubled companies

Reserve Risk Models

Insurance risk stems from the uncertainty of estimated future claim payments



- Risk can be measured from historical data in two ways
  - Empirically – from hindsight review of historical performance
  - Statistically – by comparing the goodness of fit of the projection models

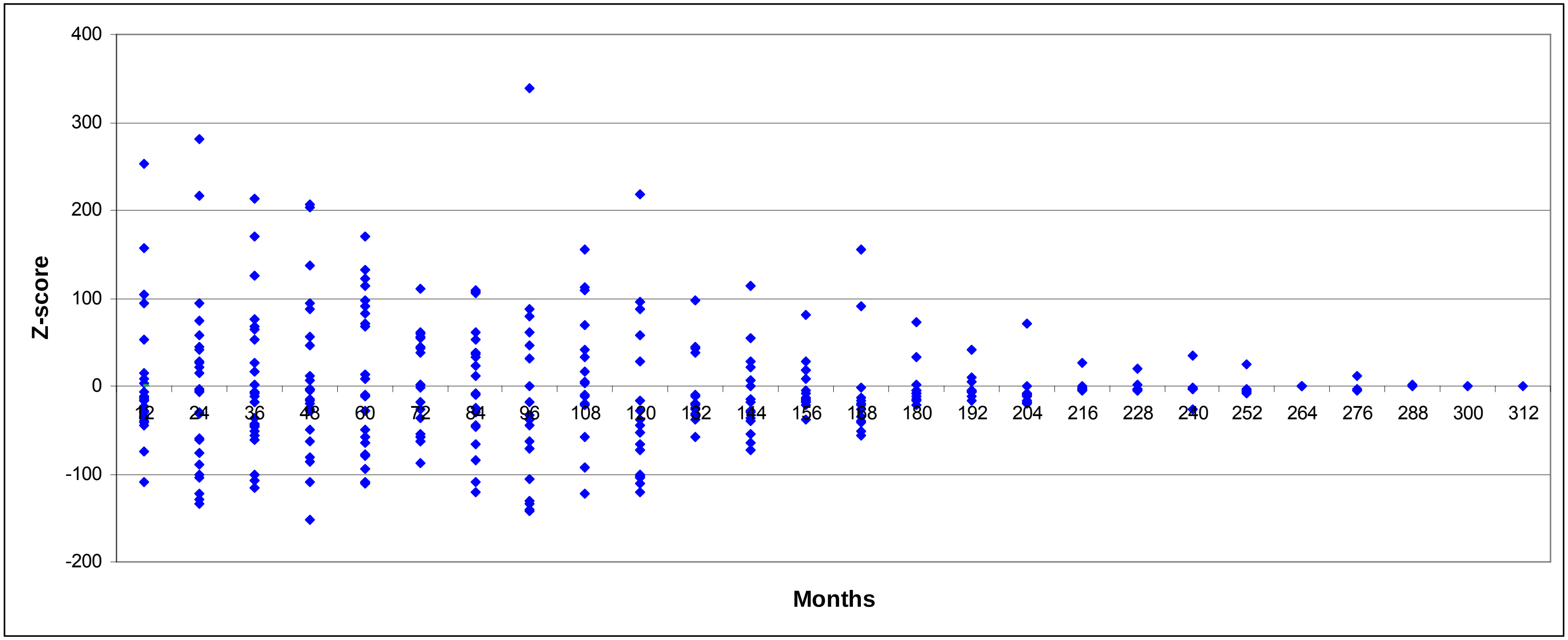
Mack-Murphy Method: Overview

- Mack-Murphy derives formulas for the standard error of the reserves projected by the chain ladder method
- The formulas provide for process and parameter risk, separately and in total
- The method can be extended to generate risk estimates given a wide array of possible selected link ratio inputs (unpublished)
- A normal or lognormal distribution is fit to the mean and variance of the reserve to yield a distribution of reserves

Bootstrap Method: Overview

- Bootstrapping is a simulation technique that generates empirical probability distributions of complex functions
- A triangle of cumulative fitted values for the past triangle is obtained by backwards recursion on the most recent diagonal using standard chain ladder link ratios
- A set of Pearson residuals is calculated from the fitted and actual data
- Bootstrapping utilizes the sampling-with-replacement technique on the residuals of the historical data
- Each simulated sampling scenario produces a new “realization” of triangular data that has the same statistical characteristics as the actual data
- The sampling of residuals can be restricted by development period

Scaled Pearson residuals should not be sampled randomly throughout the triangle

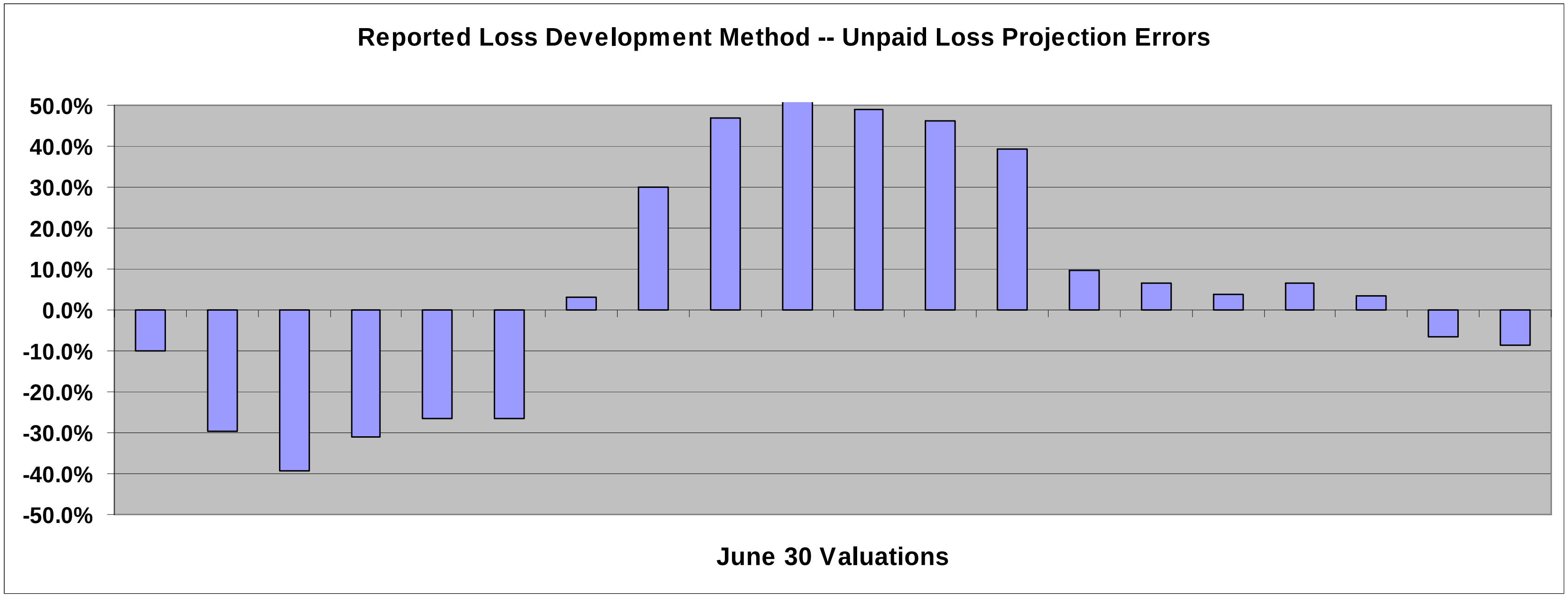


- Pearson residuals vary considerably by development age for a given line
- Can be grouped into three distinct homoskedastic groups
  - Residuals from ages 0-120 months have a relatively wide spread
  - Residuals from ages 120-204 months have a moderate spread
  - Residuals from ages 204+ have a relatively small magnitude
- Sampling large, immature residuals into the tail and the small tail residuals into the early periods invariably may overstate the bootstrap indications

Practical Method: Overview

- The Practical Method uses Monte Carlo simulation to estimate reserve distributions based on the three most popular deterministic methods – chain ladder, loss ratio, and B-F – and a different method can be used by accident year
- Practical simulates link ratios as normal or lognormal random variables and loss ratios as normal random variables
  - Means and variances of those distributions are selected inputs
  - For B-F method, loss development factors (LDFs) can be "fixed" based on the link ratio means, or "variable" based on the link ratio simulations
- Explicitly reflects process risk only, but parameter can be incorporated judgmentally

Hindsight Method: Overview



- Consists of testing the performance of past estimates of ultimate losses by comparing them to actual emergence with the benefit of hindsight
- Uses best estimates from actual past reserve reviews; for older periods it is usually necessary to imitate current reserving methods to obtain past best estimates
- Method is non-parametric; captures all sources of risk, including model risk
  - Indicated Hindsight CVs are generally higher than those produced by stochastic methods

Christofides Method: Overview

- The model:

$$P_{ij} = OverallMean \cdot AYLevel \cdot DevelopmentAgeFactor \cdot NoiseFactor$$

or

$$Y_{ij} = \log(P_{ij}) = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

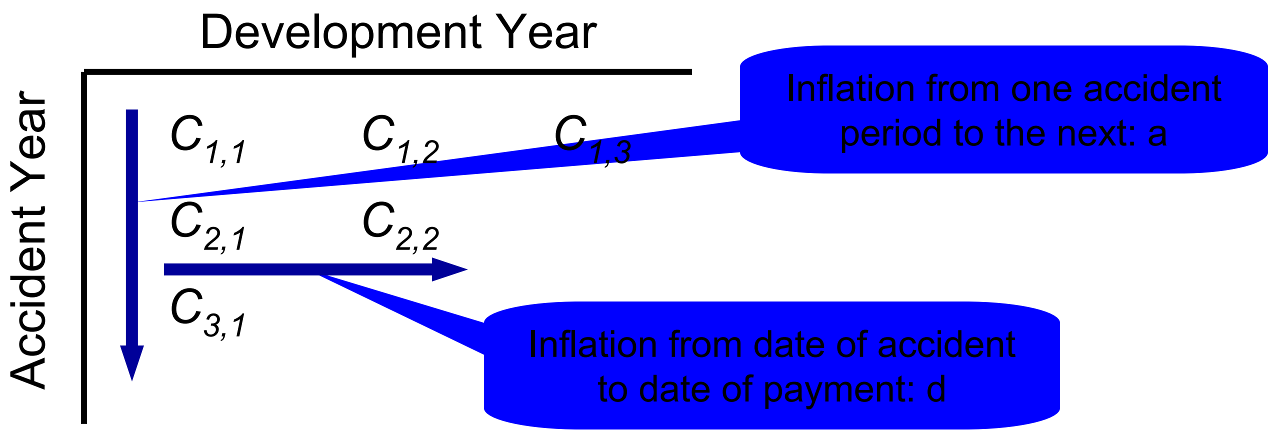
  where the error terms are assumed to be independently and identically distributed $N(0,\sigma^2)$ random variables

- Regression yields estimates of the parameters μ, α_i, β_j, σ.
- The lower, southeast corner of the triangle is predicted using a future design matrix $\boldsymbol{X_f}$ that “indicates” which parameters make up the expected value of the future observations
- The variances and covariances are obtained via matrix calculations
- Those log-space means, variances, and covariances are transformed back into dollar value means, variances, and covariances
- Variances of sums – accident year cumulative payments and accident year totals – are calculated using the variance/covariance estimates
- Parameters deemed “insignificant” can be dropped from the model, potentially addressing the criticism of over-parameterization

Structural Method: Overview

- Assumes that claim payments are a function of
  - — Stationary emergence pattern
  - — Social and economic effects
  - — Noise
- A claim cost model is built around the paid or reported data.
- First step is to separate out the social and economic effects, and then use standard stochastic models on the adjusted data
- Second step is to build a stochastic time series model that represents the social and economic effects
- The two are then combined in a simulation

Both monetary and social inflation can have accident year and development year effects



- Some components of cost are fixed at the data of the accident, for example wage loss benefits in workers compensation
- Some components of cost are not fixed until the time of the actual claim payment, for example medical benefits in workers compensation
- The "Butsic" approach can be used to model these effects — for both economic and social inflation
  - For example $C_{2,2} = C_{1,2} \times (1+d)^{(1-\alpha)} \times (1+a)^{\alpha}$
  - Where *d* and *a* are the development year and accident year inflation rates and alpha is the Butsic factor

Converting stochastic reserving methods from runoff to a one-year risk horizon requires additional calculations

- Under Solvency II, the one-year risk horizon is defined as the change in the estimate, one year hence.
  - It is not equivalent to the paid loss during the calendar year
  - One-year risk is generally measured as the sample variance of the difference between the deterministic ultimates one year from now and the deterministic ultimates today
- For the Mack-Murphy Method, it requires an analytic calculation of the mean square error
  - Define the **change in the estimate** as $$\Delta\hat{C}_{iJ}^{(1)} \equiv \hat{C}_{iJ}^{(1)} - \hat{C}_{iJ}^{(0)}$$
  - Define the **change in the noise** as $$\Delta\varepsilon_{iJ}^{(1)} \equiv \varepsilon_{iJ}^{(1)} - \varepsilon_{iJ}^{(0)}$$
  - Define the **calendar year mean square error** (CYMSE) as the sum of two components: the variance arising from the change in noise plus the variance arising from the change in the estimate $$CYMSE_{ij}^{(1)} = Var(\Delta\varepsilon_{ij}^{(1)}) + Var(\Delta\hat{C}_{ij}^{(1)})$$
- For the Bootstrap method, simulation of the prospective diagonal is required
- For Practical, the method must be reapplied to the simulated payment in one year

Reserve risk models
should have the ability to incorporate pricing risk

- The upcoming exposure year can be thought of as an extension of prior periods
- For the Mack/Murphy method, there are no published articles that we are aware of that address an analytic calculation of the CV of a loss ratio based on the assumptions underlying the chain ladder method
- For bootstrap, an accident year's 12-month paid/incurred loss ratio or pure premium can be "bootstrapped"
  - The simulated 12-month losses can be combined with the simulation of the developed triangle in the reserve risk model to yield simulated ultimate loss ratios or pure premiums
  - The pricing risk is the CV of the simulated ultimate loss ratios
- The results of the reserve risk Practical method can be used to simulate ultimate loss ratios for prior accident years
  - Monte Carlo simulation is employed to simulate next accident year's loss ratio based on a time series ARMA model.

For most of the stochastic models, pricing risk is slightly higher than reserve risk

Which Model to Use?

Many criteria should be used in the selection of an EC model for insurance risk

Technical

- Data requirements?
- Ease of implementation?
- Degree of judgment required / allowed?
- Types of risk measured: process / parameter / model?
- Robust to violations of underlying assumptions?
- Robust to outliers in the data?
- Can accommodate loss development tail?

Strategic

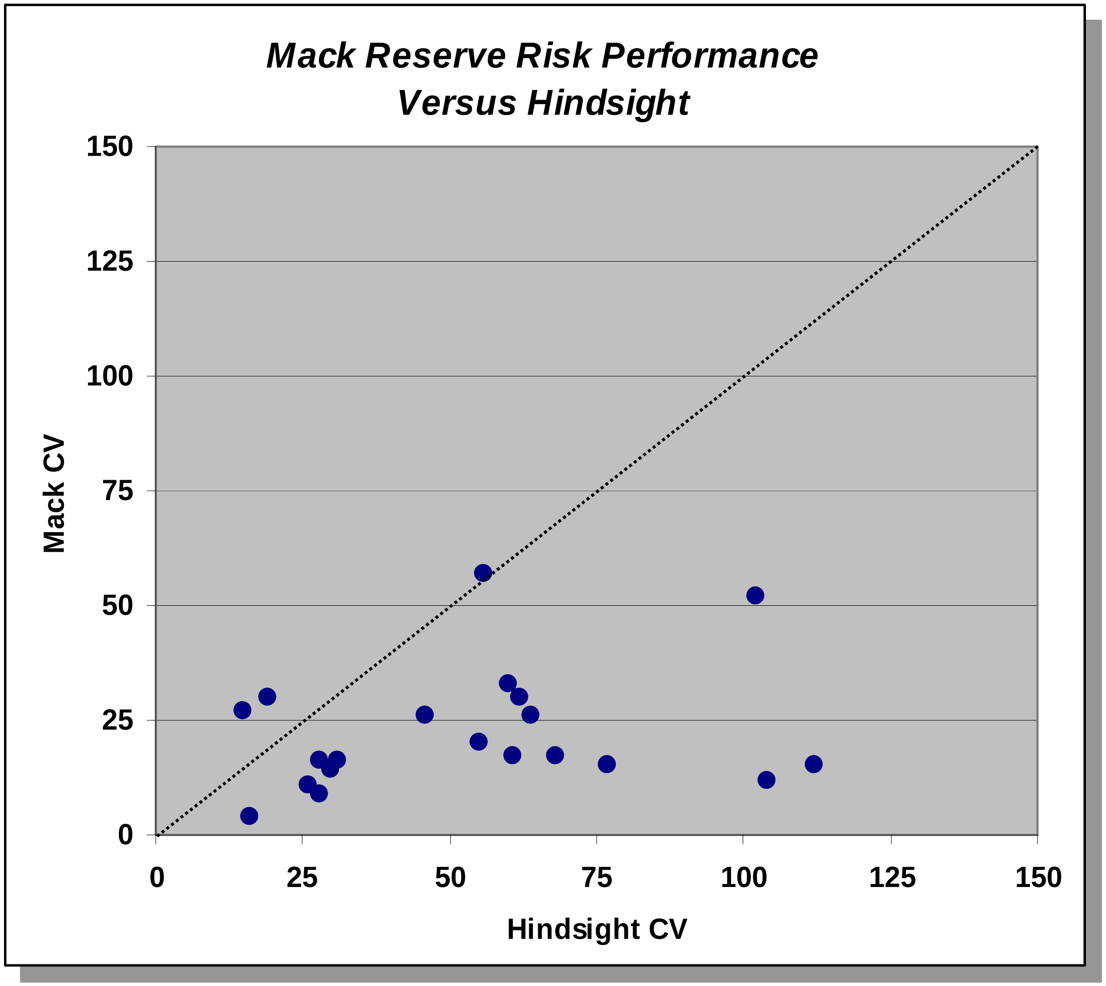
- Extendable to measuring both reserving and pricing risk?
- Output can be used to measure correlations between lines?
- Amenable to one-year risk horizon?

Our experience with various methods for measuring insurance risk have identified some strengths and weaknesses

Method	Strengths	Weaknesses
Mack	■ Published, generally accepted, grounded in statistical theory ■ Relatively robust to outliers and assumption violations	■ Assumes stationarity of the chain ladder process ■ User must assume a distribution to obtain tail values ■ GIRO testing indicates that it may understate risk
Bootstrap	■ Published, generally accepted, grounded in statistical theory ■ Outliers can be detected and restricted ■ Accomodates B-F as well as chain ladder method	■ Assumes stationarity of the chain ladder process ■ Data outliers have a leveraged effect on results ■ Method does not work well with negative loss development (due to underlying theoretical model)
Practical Simulation	■ Most flexible approach; user judgements reflected via inputs ■ Easy to understand and explain	■ Not as well-known within the actuarial community ■ Requires more input assumptions than other methods
Structural Simulation	■ Does not assume stationarity of actual historical development; more robust ■ Provides natural linkages to externalities	■ Complex method, requiring more calibration ■ Not as well-known within the actuarial community
Hindsight	■ Non-parametric; captures all sources of risk: process, parameter, and model ■ Easy to explain and defend	■ Requires historical database of past estimates; reconstruction can be labor intensive

Empirical hindsight data indicates that Mack understates reserve risk

- Sample of 20 lines of business, "more difficult" casualty lines
- Experience over a 15-20 year period
- Mack includes parameter risk and tail factor volatility



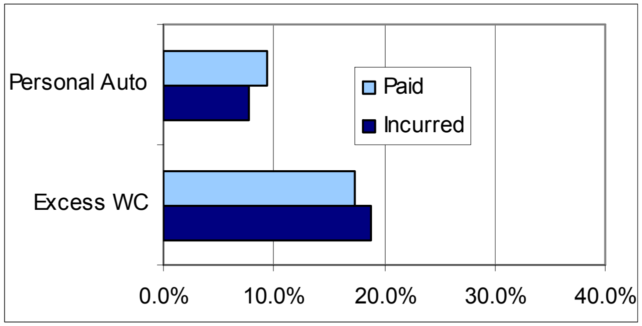
The methods for measuring insurance risk do not all meet strategic criteria

Method	Reserving and Pricing?	One-Year and Run-Off?	Correlation Between Lines?
Mack	Not an effective method for pricing risk	Yes, one-year requires redoing the math	Not directly in the model
Bootstrap	Not an effective method for pricing risk	One-year not easily accommodated	Not directly in the model
Practical Simulation	Yes, requires trend as well as development simulation	Yes	Not directly in the model
Structural Simulation	Yes, requires trend as well as development simulation	Yes	Yes
Hindsight	Yes, if historical expected loss ratios are available	Yes	Yes

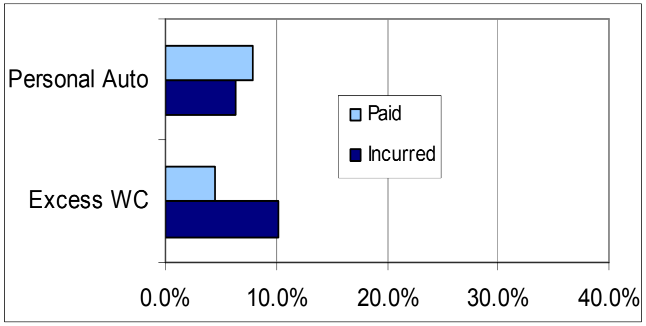
There are benefits in comparing the results from various models before making a final determination of the overall insurance risk

Indicated CVs vary by line and method

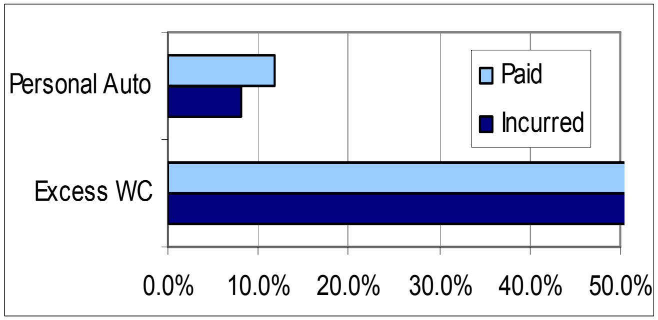
Mack Run-off



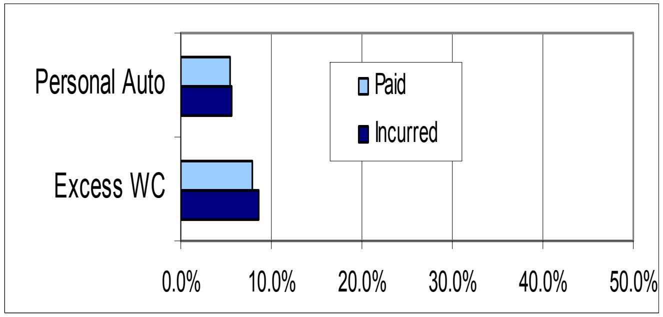
Mack One-year



Bootstrap Run-off



Bootstrap One-year



Workers' Compensation
Run-off horizon – Total Risk

Method	Paid				Incurred			
	Mean ($M)	CV	99% ($M)	99.95% ($M)	Mean ($M)	CV	99% ($M)	99.95% ($M)
Mack	392	5.2%	455	483	430	7.0%	526	574
Bootstrap	393	6.9%	483	523	431	11.0%	583	694
Practical	394	6.3%	473	514	431	7.3%	535	589
Hindsight	520	130.9%	3,747	9,981				
Christofides								
Structural								
Selected **	520	10.0%	678	757				

Illustration

** Mean = Reserves implied by company's selected ultimate losses