Prediction Error of the Future Claims Component of Premium Liabilities under the Loss Ratio Approach

(accepted to be published in Variance)

CAS Annual Meeting
November 8, 2010

Jackie Li PhD, FIAA
Nanyang Business School
Nanyang Technological University, Singapore

International Regulatory Changes

- International Accounting Standards Board (IASB) proposed that both outstanding claims liabilities and premium liabilities should be assessed at their 'fair values'
- it is generally perceived that :
  fair value = mean + margin
- this margin allows for different types of variability for insurance liabilities

Australian Regulatory Changes

- Australian Prudential Standard GPS 310 stipulates that insurance liabilities must be valued at 75th percentile
- risk margin = 75th percentile – mean
- risk margin is subject to a minimum of one half of standard deviation
- risk margin is usually expressed as a percentage of mean

Stochastic Reserving Methods

- bootstrapping, stochastic chain ladder, Mack model, Bayesian Markov chain Monte Carlo (MCMC) simulation
- produce both mean and variability
- so far, studies focus mainly on process error and estimation error
- extensive literature on outstanding claims liabilities but relatively few on premium liabilities

Premium Liabilities

- refer to all future claim payments and associated expenses arising from future events after valuation date
- insured under existing unexpired policies
- account for 30% of insurance liabilities for direct insurers and 15-20% for reinsurers in Australia from 2002 to 2004 (Yan 2005)

Premium Liability Assessment

- **prospective** method :

  full actuarial assessment from first principles
- **retrospective** method :

  adjustment of unearned premiums ;

  Canadian and Australian accounting standards require addition of premium deficiency reserve if this is smaller than full actuarial assessment

Prospective Method

- **historical claims** approach :
  number of claims and average claim size ;
  for short-tailed lines with much data ;
  studied thoroughly under risk theory
- **loss ratio** approach :
  most common in practice ;
  an extension of outstanding claims liability valuation ;
  applies loss ratios to unearned premiums ;
  receives relatively little attention

Research Objectives

- prospective method
- loss ratio approach
- future claims component
- weighted / simple average loss ratio
- standard error of prediction
- process error and estimation error

Notation & Assumptions

- assume all claims are settled in n years
- $C_{i,j}$ is cumulative claim amount of accident year i and development year j
- valuation date is as at end of accident year n
- $C_{i,j}$ data is available for $i + j \leq n + 1$
- $C_{i,j}$ for $i + j > n + 1$ and $1 \leq i \leq n$ refers to outstanding claims liabilities
- $C_{n+1,j}$ refers to premium liabilities
- E_i for $1 \leq i \leq n + 1$ is known premiums of accident year i
- $C_{i,n} / E_i$ is ultimate loss ratio of accident year i

Notation & Assumptions

- assume exposure is evenly distributed over each year
- assume exposure distribution of accident year *n* + 1 is the same as past accident years

Claims Run-Off Triangle

- e.g. *n* = 10

	1	2	3	4	5	6	7	8	9	10
1	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$	$C_{1,4}$	$C_{1,5}$	$C_{1,6}$	$C_{1,7}$	$C_{1,8}$	$C_{1,9}$	$C_{1,10}$
2	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$	$C_{2,4}$	$C_{2,5}$	$C_{2,6}$	$C_{2,7}$	$C_{2,8}$	$C_{2,9}$	
3	$C_{3,1}$	$C_{3,2}$	$C_{3,3}$	$C_{3,4}$	$C_{3,5}$	$C_{3,6}$	$C_{3,7}$	$C_{3,8}$		
4	$C_{4,1}$	$C_{4,2}$	$C_{4,3}$	$C_{4,4}$	$C_{4,5}$	$C_{4,6}$	$C_{4,7}$			
5	$C_{5,1}$	$C_{5,2}$	$C_{5,3}$	$C_{5,4}$	$C_{5,5}$	$C_{5,6}$		|		
6	$C_{6,1}$	$C_{6,2}$	$C_{6,3}$	$C_{6,4}$	$C_{6,5}$			|		
7	$C_{7,1}$	$C_{7,2}$	$C_{7,3}$	$C_{7,4}$				----- OSCL -----		
8	$C_{8,1}$	$C_{8,2}$	$C_{8,3}$					|		
9	$C_{9,1}$	$C_{9,2}$						|		
10	$C_{10,1}$									
11				---------- PL ----------						

Model Assumptions

- from Mack (1993)

$$\mathrm{E}\left(C_{i,j+1}/E_i \mid C_{i,1}, C_{i,2}, \ldots, C_{i,j}\right) = C_{i,j} f_j / E_i$$

$$\mathrm{Var}\left(C_{i,j+1}/E_i \mid C_{i,1}, C_{i,2}, \ldots, C_{i,j}\right) = C_{i,j} \sigma_j^2 / E_i^2$$

$C_{i,j}$ and $C_{g,h}$ are independent for $i \neq g$

- from Schnieper (1991)

$$\mathrm{E}\left(C_{i,1}/E_i\right) = u$$

$$\mathrm{Var}\left(C_{i,1}/E_i\right) = v^2 / E_i$$

Parameter Estimation

- starting from chain ladder structure
- f_j and σ_j^2 are estimated from claims data
- u and v^2 are estimated from claims and premiums data
- unbiased estimators

Loss Ratio Estimator

- expected ultimate loss ratio of accident year $n + 1$ is $q = E(C_{n+1,n} / E_{n+1})$
- weighted average estimator :

$$\hat{q} = \frac{\sum_{i=1}^{n} C_{i,n+1-i} \hat{f}_{n+1-i} \hat{f}_{n+2-i} \cdots \hat{f}_{n-1}}{\sum_{i=1}^{n} E_i}$$

- simple average estimator :

$$\hat{q}^* = \frac{1}{n} \sum_{i=1}^{n} \frac{C_{i,n+1-i} \hat{f}_{n+1-i} \hat{f}_{n+2-i} \cdots \hat{f}_{n-1}}{E_i}$$

- these two estimators are unbiased

Prediction Error

- mean square error of prediction :

$$\mathrm{MSEP}(\hat{q}) = \mathrm{E}\left(\left(\frac{C_{n+1,n}}{E_{n+1}} - \hat{q} \right)^2 \right) = \mathrm{Var}\left(\frac{C_{n+1,n}}{E_{n+1}} \right) + \mathrm{Var}(\hat{q})$$

  i.e. process error + estimation error
- process error is related to future only
- estimation error is related to past only
- standard error of prediction :

$$\mathrm{SEP}(\hat{q}) = \sqrt{\mathrm{MSEP}(\hat{q})}$$

Process Error Component

- variance of ultimate loss ratio of accident year n + 1 :

$$\mathrm{Var}\left(\frac{C_{n+1,n}}{E_{n+1}}\right)=\frac{1}{E_{n+1}}\mathrm{E}\left(\frac{C_{n+1,n}}{E_{n+1}}\right)\sum_{j=1}^{n-1}\frac{\sigma_j^2}{f_j}f_{j+1}f_{j+2}\cdots f_{n-1}+\frac{v^2}{E_{n+1}}f_1^2f_2^2\cdots f_{n-1}^2$$

Estimation Error Component

- variance of loss ratio estimator :

$$\mathrm{Var}(\hat{q})\approx\frac{1}{\left(\sum_{i=1}^{n}E_i\right)^2}\sum_{j=1}^{n-1}\left(\sum_{i=n+1-j}^{n}\frac{\mathrm{E}(C_{i,n})}{f_j}\right)^2\mathrm{Var}(\hat{f}_j)$$

$$+\frac{1}{\left(\sum_{i=1}^{n}E_i\right)^2}\sum_{i=1}^{n}f_{n+1-i}^2f_{n+2-i}^2\cdots f_{n-1}^2\mathrm{Var}(C_{i,n+1-i})$$

$$+\frac{2}{\left(\sum_{i=1}^{n}E_i\right)^2}\sum_{j=1}^{n-1}\sum_{i=1}^{n-j}\left(\sum_{r=n+1-j}^{n}\frac{\mathrm{E}(C_{r,n})}{f_j}\right)(f_{n+1-i}f_{n+2-i}\cdots f_{n-1})\mathrm{Cov}(\hat{f}_j,C_{i,n+1-i})$$

- estimators of variance and covariance terms

Australian Public Liability (Gross)

$i \setminus j$	1	2	3	4	5	6	7	8	9	10	Premiums
1981	15,898	20,406	17,189	19,627	35,034	12,418	8,922	12,555	8,965	6,693	289,732
1982	16,207	21,518	17,753	18,780	19,113	18,634	15,857	13,050	9,362		319,216
1983	14,141	20,315	16,458	25,473	16,427	**92,888**	18,698	15,295			314,607
1984	14,649	21,162	19,084	23,857	20,171	15,098	17,637				344,446
1985	21,949	26,455	23,285	25,251	22,286	23,424					418,358
1986	18,989	28,741	32,754	30,240	28,443			Outstanding			535,658
1987	19,367	36,420	31,204	27,487				Claims			639,130
1988	26,860	39,550	33,852					Liabilities			751,897
1989	23,738	52,683									780,669
1990	34,567										719,181
1991					Premium Liabilities						334,566

Australian Public Liability (Net)

$i \setminus j$	1	2	3	4	5	6	7	8	9	10	Premiums
1981	13,451	16,801	12,947	13,752	13,802	8,583	6,847	9,237	5,641	3,784	168,975
1982	13,533	17,489	13,111	13,541	13,603	11,937	10,524	8,609	5,987		186,990
1983	11,808	17,525	12,644	15,609	11,821	17,305	10,524	11,061			200,475
1984	13,309	17,806	14,777	17,295	15,340	12,060	11,752				222,843
1985	19,546	22,786	19,686	21,860	19,268	18,692					262,748
1986	17,865	25,888	28,194	25,578	22,985			Outstanding			333,716
1987	17,797	33,517	24,182	24,337				Claims			410,429
1988	24,591	33,398	28,512					Liabilities			502,869
1989	21,567	46,146									532,298
1990	30,343										545,218
1991					Premium Liabilities						234,659

Results

- expected ultimate loss ratio of accident year 1991 :
  49.2% (gross) 53.6% (net)
- standard error of prediction (% of mean) :
  47.1% (gross) 33.1% (net)
- gross liability variability
  > net liability variability

Concluding Remarks

- starting point for assessing premium liability variability
- insurance cycle, claims at tail, catastrophes, superimposed inflation, multi-year policies, expenses, recoveries, reinsurance, retrospectively rated polices, unclosed business, refund claims, claims management, underwriting

Current Research Projects

- regression-like estimators
- coherent modeling of two or more lines of business
- over-dispersed Poisson (ODP) and gamma models
- Bayesian analysis and Markov chain Monte Carlo (MCMC) simulation

References

- Mack, T., "Distribution-free Calculation of the Standard Error of Chain Ladder Reserve Estimates," *ASTIN Bulletin* 23(2), 1993, pp. 213-225.
- Schnieper, R., "Separating True IBNR and IBNER Claims," *ASTIN Bulletin* 21(1), 1991, pp. 111-127.
- Yan, M., "Premium Liabilities," *The Institute of Actuaries of Australia XVth General Insurance Seminar*, 2005.