Casualty Actuarial Society / Society of Actuaries

Project Regarding the

Modeling of Economic Series Coordinated with Interest Rate Scenarios

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Progress Report: Overview

The researchers continue to be actively engaged on this project, and we remain scheduled for completion as specified in our letter of agreement. Our progress to-date is described in the project outline below. The outline is excerpted from our original proposal; each step is noted briefly as to current status in parentheses, followed by a more in-depth description of the step and our progress.

1) ***Literature review***. (Completed, but being enhanced on an ongoing basis.) A comprehensive survey and review of the relevant literature – involving actuarial, financial, and other articles – has largely taken place. Brief summaries of many of these articles were included in our previous progress report. We continue to add summaries to this list, as additional articles are suggested or discovered (see "Responses to Prior Comments" section below). Several new summaries are included in Appendix I at the end of this report.
2) ***Development and presentation of a financial scenario model***.
    a) *Development of a model to represent economic and financial series.* (Significant progress made, and ongoing.) Data sources for several of the key economic and financial series were documented in our previous progress report. Our primary efforts in this area over the last two months have involved specifying a model for interest rates and equity returns. Once these key processes have been satisfactorily modeled, the remaining economic series (e.g., inflation, real estate rates) will be modeled via appropriate relationships to those processes.

        Although there are a variety of model structures that have been proposed for interest rates, one way to categorize these is according to whether they are "equilibrium" or "no-arbitrage" models. Understanding the differences between these approaches is a key aspect of selecting an appropriate interest rate model. Therefore, we have provided a discussion of this important issue in Appendix II below.
    b) *Parameter estimation*. (Progress made, and ongoing.) We continue to examine econometric approaches to the development of parameter values associated with each series' model structure.
    c) *Provision for extreme conditions*. (Initial efforts underway.) There will be a provision for the representation of extreme financial and economic conditions. This provision might involve the selection of appropriate parameter values, or other means (e.g., through a stochastic jump process). Some of the literature which we have reviewed addresses this issue and may help to guide us in this process.
3) ***Creation of software which allows users to model economic and financial series***. (Progress made, and ongoing.) A current working version of the software under development accompanies this report as an Excel spreadsheet ("Financial Scenario Model"). The model takes as inspiration the approach in the Hibbert-Mowbray-Turnbull paper (discussed in our previous progress report). Currently, the parameters populating the model are taken from that paper, and thus are non-U.S.-specific; we are in the process of re-parameterizing this model for U.S. applications. (For example, we will attempt to re-parameterize so that the interest rate path does not explode upward the way it currently does in some scenarios.) When completed, this model will be made available to the sponsoring organizations for general use and comment. (This model is designed to be used with @Risk simulation

software; even without that add-on package, however, the structure, formulas, and relationships in the model can be examined.)

4) ***Documentation***. (Progress made, and ongoing.) A comprehensive report describing the literature review, the model, and the parameterization will be written. This report will also include a description of how the parameter estimation can be updated, and a brief discussion of the implications of the model for specific areas of actuarial interest, such as dynamic financial analysis, asset-liability management, risk-based capital, and insurance pricing. In addition, slide presentations summarizing this research will be provided for posting on the sponsoring organizations' websites. To-date, the literature review portion of the report has largely been documented, and pieces of other modeling issues have been written.
5) ***Additional articles***. (To be done.) The researchers will write a paper discussing this research, with the intention of publishing it in the *CAS Proceedings.* In addition, other articles may be written, possibly for publications such as the *North American Actuarial Journal*, the *Journal of Actuarial Practice*, or the *Journal of Risk and Insurance*.

Responses to Prior Comments: We appreciate the comments and suggestions provided by various people in response to our first progress report, and we hope and trust that additional constructive thoughts will be forthcoming relative to this report. We continue to search for additional relevant information and articles based on suggestions that were made.

With respect to specific issues raised by reviewers of our first progress report:

(1) *Regarding the inflation – real interest rate relationship*: We have included summaries of two relevant articles in our literature review update. The model we are developing allows for correlation, and we are investigating the proper parameters to use to reflect this correlation.

(2) *Regarding regime-shifting values for equity returns*: The accompanying Excel spreadsheet ("S and P 500 data") shows that a normal distribution does not adequately reflect the outliers in historical S&P 500 equity returns. A better fit has been found to involve a regime-switching model (incorporated in our Financial Scenario Model spreadsheet) in which one regime has a relatively low standard deviation and the other a relatively high standard deviation. Although Hibbert, Turnbull and Mowbry selected a lower mean value to correspond with the higher standard deviation, we may select different parameters. The key point is that no one knows which regime they are in at any given time, and they cannot elect to invest only when the higher expected values are occurring.

(3) *Regarding other issues and suggestions*:

- We have identified, and continue to identify, life insurance articles of relevance to this project, based on both suggested references and our own searches. These will be summarized for future literature review listings and incorporated, as appropriate, into our analyses.
- We intend to enhance the model to make it capable of very long-term projections – e.g., 50 or 60 years. This is one factor in our equilibrium vs. no-arbitrage

discussion – an equilibrium interest rate model may be better suited to handle such time horizons.

- The model will have the ability to accept, as inputs, nominal interest rate scenarios (for example, as specified in NY Regulation 126).

Appendix I

UPDATES TO LITERATURE REVIEW

Brief Summaries of Articles Reviewed and Relevant to the Economic Scenario Generator Project

- Hardy, Mary, 2001, "A Regime-Switching Model of Long-Term Stock Returns," *North American Actuarial Journal*, 5 (2), 41-53.
  - ***Using monthly data from the S&P 500 and the Toronto Stock Exchange, a regime-switching lognormal model is parameterized and compared with other models. The author finds the performance of the regime-switching model to be favorable.***

- Pennacchi, George G., 1991, "Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data," *Review of Financial Studies*, 4: 53-86.
  - ***Over the period 1968-1988, there is evidence that the instantaneous real interest rates and expected inflation are significantly negatively correlated. The inflation expectations are based on surveys of professional economic forecasters, which may not necessarily correspond with market expectations.***

- Risa, Stefano, 2001, "Nominal and Inflation Indexed Yields: Separating Expected Inflation and Inflation Risk Premia," Columbia University Working Paper.
  - ***This paper provides an excellent review of the literature on the relationship between inflation and interest rates. Based on nominal and inflation indexed bonds from the United Kingdom from 1983-1999, the nominal and inflation indexed interest rates are derived. The inflation risk premium is determined based on a four factor pricing model.***

Appendix II

Equilibrium vs. Arbitrage Free Models

One of the primary processes in a financial scenario generator is a term structure model. A tremendous variety of models is available for both practitioners and researchers. (For a discussion of many of the available models see Yan (2001)). No single term structure model has yet proven itself worthy for all possible applications (see the discussion in Chapman and Pearson (2001)). In virtually all cases, the user of a term structure model has a tradeoff to consider: complexity of the model vs. accuracy. This tradeoff depends on the specific application of the term structure model.

There are two important issues to consider when choosing among term structure models. The first consideration is related to the theoretical background of the model. Specifically, there are equilibrium models and arbitrage-free models. Equilibrium models typically begin with an assumption for short-term interest rates, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for short-term rates, one can determine the yield on longer-term bonds by looking at the expected path of interest rates until the bond's maturity. One of the primary advantages of equilibrium models is that the prices of many popular securities have closed-form analytic solutions. Another advantage is that equilibrium models are fairly easy to use. On the negative side, equilibrium term structure models generate yield curves which are inconsistent with current market prices. While the parameters of these models may be selected carefully, there is no guarantee that the resulting term structure will generate observed market prices.

Contrary to equilibrium models, arbitrage-free term structure models project future interest rate paths that emanate from the existing yield curve. For applications using arbitrage-free term structure models, resulting prices will be based on the concept of arbitrage. Unfortunately, arbitrage-free term structure models are frequently more difficult to use than their equilibrium counterparts.

Outside of the previously mentioned items, there are other benefits and costs associated with equilibrium and arbitrage-free term structure models.

- Pricing accuracy
  - Arbitrage-free models are more useful for pricing derivatives. Since derivatives are priced against the underlying assets, a model that explicitly captures the market prices of those underlying assets is superior to models that more or less ignore market values. Jegadeesh (1998) looks at the pricing of interest rate caps and determines that arbitrage-free models price interest rate caps more accurately than equilibrium models. Hull (2000) and Tuckman (1994) also comment that given that arbitrage-free term structure models are founded upon the absence of arbitrage, using these models to price derivative contracts is more plausible then equilibrium approaches.
  - Unfortunately, comments revolving around the pricing accuracy of arbitrage-free term structure models are based on a short pricing horizon. There have been no formal long-term tests of accuracy.

- Internal consistency
  - Exploding models – Arbitrage-free models can "explode" over long periods of time. With many arbitrage-free models, the forward rate plays a central role in the expected path of interest rates. Forward rates are related to the slope of the yield curve. Depending on the existing slope of the yield curve, forward rates may exhibit strange behavior impacting projections of interest rate paths in arbitrage-free term structure models. For steeply sloped yield curves, the forward rate may become very large. For inverted yield curves, the forward rate may even become negative. Especially for long-term projections, simulation paths may become extreme.
  - Arbitrage-free models also suffer from inconsistency across time. (see Wilmott (1998) and Tuckman (1994)). The underlying assumption of many arbitrage-free term structure models is that the risk-free rate is closely related to the forward rate curve. If the model were correct, the forward rates would be the perfect predictors of future spot rates. At time 0, the known term structure implies future spot rates and volatilities throughout the projection period; all of the mean reversion levels and volatilities in the future are known on the projection date, without any regard to any risk premium that may be contained in these values. Equilibrium models provide more consistent statements about interest rates over time.
- Data issues
  - Isolating the term structure – Determining the "true" term structure for input into an arbitrage-free model is difficult. One usually considers risk-free securities such as U.S. Treasuries. There are several difficulties in looking at U.S. Treasury data. First, market data gathered from strip data is noisy; term structures that are created from this data are not smooth. An alternative source for long-term interest rate data is to look at long-term U.S. Treasury coupon bonds. Even when there was a regular history of issuing 30-year bonds, liquidity affected long-term rates. When on-the-run Treasuries were issued, the securities typically had higher liquidity and therefore higher price, forcing down long-term yields. The result is a forward rate curve that initially increased until liquidity issues dominated. The end of the forward rate curve dipped, leading to a strange forward rate curve. Aside from liquidity concerns, the future of 30-year bonds is uncertain, given the federal government's termination of 30-year bond issues.
  - Noisy data and interpolation - When there is sparse data available (which is typically the case for long-term Treasuries), there are fewer points to interpolate the term structure. This makes arbitrage-free models very sensitive to the market data or any inefficiencies in market quotes that is due to noisy data.
  - Market price of risk – While these issues affect the input for arbitrage-free models, equilibrium models require some measure of the market price of risk. This information can be harder to obtain than spot rates.

Wilmott, Paul, 1998, *Derivatives: The Theory and Practice of Financial Engineering,* John Wiley & Sons: Chichester, England.

Yan, Hong, 2001, "Dynamic Models of the Term Structure," *Financial Analysts Journal,* July/August: 60-74.