Changes to Exams 2/FM, 3, and 4/C for the 2007 Administrations

Listed below is a summary of the changes, transition rules, and syllabus listings for the 2007 administrations of Exams 2/FM, 3, and 4/C. The final 2007 *Syllabus of Basic Education* will be posted on the CAS Web Site in November 2006.

Summary of Changes

Exam 2/FM:

- Add an introduction to financial derivatives, (forwards, options, futures, swaps) and their use in risk management.
- Add an introduction to the concept of no-arbitrage as a fundamental concept in financial mathematics.
- To accommodate the additional material, expand Exam 2/FM to a 2.5-hour multiple-choice examination.

CAS Exam 3:

- Add learning objectives on option pricing: put-call parity, the binomial model, and Black-Scholes formula. A study note introducing actuarial applications of option pricing is under development.
- Add learning outcomes on interpretation of option Greeks and delta-hedging.
- Add learning outcomes on the features of exotic options.
- Add an introduction to Brownian motion and Itô's lemma.
- To accommodate the additional material, move loss models, including risk theory, to Exam 4/C.
- The exam will remain a 4-hour multiple-choice examination.

Exam 4/C:

- Add lognormal models for asset prices and its relationship to the Black-Scholes formula.
- Include Monte-Carlo valuation of derivative securities with the current material on simulation.
- Add characteristics and calculation of risk measures such as value at risk and conditional tail expectation.
- Add the loss models and risk theory material moved from Exam 3.
- Remove interpolation and smoothing.
- The exam will remain a 4-hour multiple-choice examination.

Transition Rules

There will be no special transition rules. Candidates with credit for any of Exams 2/FM, 3, or 4/C before 2007 will retain credit for those exams in 2007.

2007 Syllabus Listings

Exam 2
Financial Mathematics

This two and one-half hour, multiple-choice examination is administered by Preliminary Actuarial Examinations and is identical to SOA Course FM.

The goal of the Financial Mathematics course of reading is to provide an understanding of the fundamental concepts of financial mathematics, and how those concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting and valuing contingent cash flows. The candidate will also be given an introduction to financial instruments, including derivatives, and the concept of no-arbitrage as it relates to financial mathematics.

The following learning outcomes are presented with the understanding that candidates are allowed to use specified calculators on the exam. The education and examination of candidates should reflect that fact. In particular, such calculators eliminate the need for candidates to learn and be examined on certain mathematical methods of approximation.

LEARNING OBJECTIVES

1. Candidates will know definitions of key terms of financial mathematics: inflation; rates of interest [simple, compound (interest and discount), real, nominal, effective, dollar-weighted, time-weighted, spot, forward], term structure of interest rates; force of interest (constant and varying); equivalent measures of interest; yield rate; principal; equation of value; present value; future value; current value; net present value; accumulation function; discount function; annuity certain (immediate and due); perpetuity (immediate and due); stocks (common and preferred); bonds (including zero-coupon bonds); other financial instruments such as mutual funds, and guaranteed investment contracts.

   Specifically, candidates are expected to demonstrate the ability to:
   a. Choose the term, given a definition.
   b. Define a given term.
   c. Determine an equation of value, given a valuation problem involving one or more sets of cash flows at specified times.

2. Candidates will understand key procedures of the financial mathematics: determining equivalent measures of interest; discounting; accumulating; determining yield rates; estimating the rate of return on a fund; and amortization.

   Specifically, candidates are expected to demonstrate the ability to:
   a. Calculate the equivalent annual effective rate of interest, given a nominal annual rate and a frequency of interest conversion, discrete or continuous, other than annual.
   b. Calculate the equivalent effective rate of interest per payment period given a payment period different from the interest conversion period.
   c. Estimate the interest return on a fund.
   d. Calculate the appropriate equivalent single value (present value, net present value, future (accumulated) value or combination), given a set of cash flows (level or varying), an appropriate term structure of interest rates, the method of crediting interest (e.g., portfolio or investment year) as necessary, an appropriate set of inflation rates as necessary, and accounting for reinvestment interest rates as necessary; for example:
      i. Calculate the loan amount or outstanding loan balance, given a set of loan payments (level or varying) and the desired yield rate (level or varying).

ii. Calculate the price of a bond (callable or non-callable), given the bond coupons, the redemption value, the term of the bond (constant or varying), the coupon interest rate, and the desired yield rate (level or varying).
iii. Calculate the value of a stock, given the pattern of dividends and the desired yield rate (level or varying).
iv. Calculate the net present value, given a set of investment contributions and investment returns.

e. Calculate a unique yield rate, when it exists, given a set of investment cash flows.
f. Calculate the amount(s) of investment contributions, given there is more than one contribution, and given a set of yield rates, the amount(s) and timing of investment return(s), and the desired timing of the investment contributions.
g. Calculate the amount(s) of investment returns, given there is more than one return, and given a set of yield rates, the amount(s) and timing of investment contribution(s) and the desired timing of the investment returns; for example:

    i. Calculate loan payments, given the loan amount(s), the term of the loan, and the desired yield rate (level or varying).
    ii. Calculate the principal and interest portions of a loan payment, given the loan amount, the set of loan payments (level or varying), and a set of interest rates (level or varying).
    iii. Calculate bond coupons or redemption values, given the bond price, the term of the bond, and the desired yield rate (level or varying).

h. Calculate the term of an investment, given a set of cash flows (level or varying), and a set of interest rates (level or varying); for example:
    i. Calculate the length of time required to accumulate a given amount, given the yield rate and an initial amount.
    ii. Calculate the length of time to repay a given loan amount, given the loan payments and the loan interest rate(s).
    iii. Calculate the time to maturity of a bond, given the price of the bond, the coupon payments, redemption value, and yield rate.

3. Candidates will know <u>definitions</u> of key terms of modern financial analysis at an introductory and intuitive level, and be able to complete basic calculations involving such terms: yield curves, spot rates, forward rates, duration, convexity, and immunization.

   Specifically, candidates are expected to demonstrate the ability to:
   a. Choose the term, given a definition.
   b. Write the definition, given a term.
   c. Perform calculations such as:
       i. measuring interest rate risk using duration and convexity
       ii. basic immunization calculations

4. Candidates will know <u>definitions</u> of key terms of financial economics at an introductory level: derivatives, forwards, futures, short and long positions, call and put options, spreads, collars, hedging, arbitrage, and swaps.

   Specifically, candidates are expected to demonstrate the ability to:
   a. Explain why firms might care about risk management.
   b. Evaluate the risk/return characteristics of the basic building blocks of financial derivatives: forward contracts; call and put options.
   c. Identify associated hedging and investment strategies.
   d. Explain the use of derivatives as risk management tools.
   e. Explain the cash-flow characteristics of forwards, futures and swaps.

f. Use the concept of no-arbitrage to determine the theoretical value of forwards, futures and swaps.
g. Manage financial risk through use of forwards, futures and swaps.

Note that probability-based calculations for applications of financial mathematics are in Exam 3.

READINGS

Option A

- *Mathematics of Investment and Credit* (Third Edition), 2004, by Broverman, S.A., Chapter 1 (1.1-1.6); Chapter 2 (2.1-2.4 excluding 2.4.2 and 2.4.3); Chapter 3 (3.1-3.3 excluding pages 188–189), Chapter 4 (4.1- 4.3.1), Chapter 5 (5.1-5.3 excluding 5.1.3, 5.1.4 and 5.3.2), Chapter 6 (6.1-6.3 excluding 6.2), Chapter 7 (7.1- 7.2), Chapter 8 (8.2.1, 8.2.4, 8.3.1–8.3.3).

- *Derivatives Markets* (Second Edition), 2006 by McDonald, R. Chapter 1 (1.1-1.4); Chapter 2 (2.1-2.6); Chapter 3 (3.1-3.5), Chapter 4 (4.1-4.4), Chapter 5 (5.1-5.4), Chapter 8 (8.1-8.2), Appendices 2A and 5B.

Option B

- *Financial Mathematics – A Practical Guide for Actuaries and other Business Professionals* (Second Edition), 2005, by C. Ruckman and J. Francis, Chapters 1, 2, Chapter 3 (3.1-3.7), Chapter 4 (4.2), Chapter 5, Chapter 6 (6.1-6.3), Chapter 7 (7.1-7.8), Chapter 8 (8.1-8.3).

- *Derivatives Markets* (Second Edition), 2006 by McDonald, R. Chapter 1 (1.1-1.4); Chapter 2 (2.1-2.6); Chapter 3 (3.1-3.5), Chapter 4 (4.1-4.4), Chapter 5 (5.1-5.4), Chapter 8 (8.1-8.2), Appendices 2A and 5B.

Knowledge and understanding of financial mathematics concepts are significantly enhanced through working out problems based on those concepts. Thus in preparing for the Financial Mathematics examination, whichever of the source of textbooks students choose to use, students are encouraged to work out the textbook exercises related to the listed readings.

STUDY NOTES

Study Notes for the preliminary examinations will be linked [in JANUARY 2007] from the "Study Tools" in the "Admissions" section of the CAS Web Site as well as on the SOA Web site under Exams and Jobs/Candidate and Exam Information/Spring Exam Session/Spring 2007 Basic Education Catalog – Study Notes Information. Hard copies may be purchased by downloading the form from the SOA Spring Exam Session Web page.

Code	Title
FM-05-07	FM Introductory Study Note
FM-09-05	FM Sample Exam Questions and Solutions
FM-10-05	May 2005 FM Exam Questions and Solutions
FM-12-05	November 2005 FM Exam Questions and Solutions
FM-22-05	Review of Calculator Functions for the Texas Instruments BA-35
FM-23-05	Review of Calculator Functions for the Texas Instruments BA II Plus

Exam 3
Statistics and Actuarial Models

Before commencing study for this four-hour, multiple-choice examination, candidates should read the introduction to "Materials for Study" of this *Syllabus* for important information about learning objectives, knowledge statements, readings, and the range of weights. Items marked with a bold **w** are available at no charge under Web Notes in the "Admissions" section of the CAS Web Site (www.casact.org).

This examination develops the candidate's knowledge of the theoretical basis of actuarial models and the application of those models to insurance and other financial risks. A thorough knowledge of calculus, probability, and interest theory is assumed. Knowledge of risk management at the level of Exam 1 is also assumed.

The candidate will be required to develop an understanding of contingent payment models. The candidate will be expected to understand what important results can be obtained from these models for the purpose of making business decisions, and what approaches can be used to determine these results.

A variety of tables will be provided to the candidate with the exam. Copies of the specific tables are available on the CAS Web Site under Web Notes. They include values for the standard normal distribution, illustrative life tables, abridged inventories of discrete and continuous probability distributions, Chi-square Distribution, *t*-Distribution, and *F*-Distribution. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

The CAS will test the candidate's knowledge of the material, but may decide not to include questions from every reading on a particular exam. A guessing adjustment will be used in scoring Exam 3.

A. Statistics

Range of weight for Section A: 20-25 percent

Candidates should be able to apply statistical theory to solve business problems.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Perform point estimation of statistical parameters using the following statistical methods: • Maximum likelihood estimation ("MLE") • Method of moments Apply criteria to the estimates such as: • Consistency • Unbiasedness • Minimum variance • Mean square error Range of weight: 5-10 percent	a. Equations for MLE of mean, variance from a sample b. Estimation of mean and variance based on sample c. General equations for MLE of parameters d. Equations for estimation of parameters using method of moments for means, variances, and higher moments e. Recognition of consistency property of estimators and alternative measures of consistency f. Application of criteria for measurement when estimating parameters through minimization of variance, mean square error g. Definition of statistical bias and recognition of estimators that are unbiased or biased

2. Test statistical hypotheses including Type I and Type II errors using: • Neyman-Pearson lemma • Likelihood ratio tests Apply Neyman-Person lemma to construct likelihood ratio equation. Range of weight: 5-10 percent	a. Presentation of fundamental inequalities based on general assumptions and normal assumptions b. Definition of Type I and Type II errors c. Significance levels d. One-sided versus two-sided tests e. Estimation of sample sizes under normality to control for Type I and Type II errors f. Determination of critical regions g. Definition and measurement of likelihood ratio tests h. Determining parameters and testing using tabular values i. Recognizing when to apply likelihood ratio tests versus chi-square or other goodness of fit tests (statistics)
3. Calculate order statistics of a sample and use critical values from a sampling distributions to test means and variances. Range of weight: 0-5 percent	a. General form for distribution of n^{th} largest element of a set b. Application to a given distributional form c. Recognition of random variables from sample that behave as t-stat or F-stat d. Determination of parameters when applying these tests and obtaining tabular values e. Presentation of hypotheses testing from above for mean and variances
4. Perform a linear regression using the least squares method. Range of weight: 0-5 percent	a. Presentation and calculation of equations for regression statistics

READINGS

There is no single required text for Section A. The texts listed below may be considered as representative of the many texts available to cover the material on which the candidate may be examined based on the learning objectives and knowledge statements:

Hoel
Hogg and Tanis
Hogg et al.
Mood et al.

B. Financial Economics

Range of weight for Section B: 30-45 percent

Candidates should be able to apply statistical theory to solve business problems.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Use put-call parity to determine the relationship between prices of European put and call options and to identify arbitrage opportunities. Range of weight: 5-10 percent	a. Put-call parity of European Options with and without dividends b. Early exercise of American puts and calls, with and without dividends c. Use of put-call parity relationships to develop synthetics stocks, T-bills and options d. Currency and bond options e. Generalized options: strike assets other than cash f. Relation of option values with respect to style, time to expiration, and strike price

READINGS
McDonald, Chapter 9

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
2. Construct single and multiple period binomial trees. Value European and American options using the binomial model. Range of weight: 5-10 percent	a. Single period and multi-period binomial stock price trees. Recombining and nonrecombing trees. b. Selecting parameters (u and d) for the binomial option pricing model based on the stock volatility c. Risk neutral probabilities d. Early exercise of American options e. Binomial model for options on dividend-paying stocks, indices, currencies and futures f. Binomial trees for options on currencies, future contracts, commodities and bonds
READINGS	
McDonald, Chapters 10 and 11	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Value European and American options using the Black-Scholes option-pricing model. Interpret the option Greeks and elasticity measures Range of weight: 5-10 percent	a. Black-Scholes Formulas, assumption of Black-Scholes Formulas, Consideration of Discrete Dividends and Options on Currencies and Futures b. Implied volatility c. Option Greeks, Option elasticity, option risk premiums, and Sharpe ratios d. Option elasticity, option risk premiums, and Sharpe ratios e. Profit Diagrams and calendar spreads
READINGS	
McDonald, Chapter 12.1-12.5	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
4. Describe how market –makers control risk by delta-hedging. Calculate profits and losses for hedged positions. Analyze analogous relationships between option and insurance contracts. Range of weight: 3-7 percent	a. Delta-hedging b. Marking-to-market c. Delta-gamma approximation d. Black-Scholes equation e. Gamma hedging
READINGS	
McDonald, Chapter 13	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
5. Explain the cash-flow characteristics of the following exotic options: Asian options; barrier options, compound options, gap options, exchange options. Range of weight: 3-7 percent	a. Asian option: averages, asset prices and strike prices b. Barrier options: knock-outs, knock-ins and rebates c. Compound Options d. Barrier options and compound options as currency hedges e. Gap Options f. Exchange Options
READINGS	
McDonald, Chapter 14	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
6. Explain what it means to say that stock prices follow a diffusion process and apply Itô's lemma in the one-dimensional case only. Range of weight: 3-7 percent	a. Brownian Motion and mean reversion b. Geometric Brownian Motion c. The Sharpe Ratio d. Risk-neutral processes e. Itô's lemma
READINGS	
McDonald, Chapter 20	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
7. Evaluate features of the Vasicek and Cox-Ingersoll-Ross bond price models 8. Construct a Black-Derman-Toy binomial model matching a given time-zero yield curve and a set of volatilities. Range of weight: 0-5 percent	a. Short-rate Bond Price Models: Rendelman-Barter, Vasicek, Cox-Ingersoll-Ross b. Bond options: caps and caplets c. The Binomial Interest Rate Model: Black-Derman-Toy Model
READINGS	
McDonald, Chapter 24	

C. Survival Models

Range of weight for Section C:15-20 percent

Candidates should be able to work with discrete and continuous univariate probability distributions for failure time random variables. They will be expected to set up and solve equations in terms of life table functions, cumulative distribution functions, survival functions, probability density functions, and hazard functions (e.g., force of mortality), as appropriate. They should have similar facility with models of the joint distribution of two failure times (multiple lives) and the joint distribution of competing risks (multiple decrement).

Candidates should be able to use Markov Chains in order to determine state probabilities and transition probabilities.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. For discrete and continuous univariate probability distributions for failure time random variables, develop expressions in terms of the life table functions, l_x, q_x, p_x, ${}_nq_x$, ${}_np_x$, and ${}_{m\|n}q_x$, for the cumulative distribution function, the survival function, the probability density function and the hazard function (force of mortality), and be able to: • Establish relations between the different functions • Develop expressions, including recursion relations, in terms of the functions for probabilities and moments associated with functions of failure time random variables, and calculate such quantities using simple failure time distributions • Express the effect of explanatory variables on a failure time distribution in terms of proportional hazards and accelerated failure time models The distributions may be left-truncated, right-censored, both, or neither. Range of weight: 3-7 percent	a. Failure time random variables b. Life table functions c. Cumulative distribution functions d. Survival functions e. Probability density functions f. Hazard functions g. Relationships between the above variables in the above functions
READINGS	
Bowers, Chapter 3 (excluding 3.6)	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
2. Assuming a uniform distribution of deaths, define the continuous survival time random variable that arises from the discrete survival time random variable. Range of weight: 0-5 percent	a. Life table function forms under uniform distribution of deaths assumption
READINGS	
Bowers, Chapter 3.6	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Given the joint distribution of two failure times: • Calculate probabilities and moments associated with functions of these random variables' variances. • Characterize the distribution of the smaller failure time (the joint life status) and the larger failure time (the last survivor status) in terms of functions analogous to those in the Learning Objective above, as appropriate. • Develop expressions, including recursion relations, for probabilities and moments of functions of the joint life status and the last survivor status, and express these in terms of the univariate functions in Learning Objective above (assuming independence of the two failure times). Range of weight: 3-7 percent	a. Joint distribution of failure times b. Probabilities and moments
READINGS	
Bowers, Chapter 9.1-9.5	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
4. Based on the joint distribution (pdf and cdf) of the time until failure and the cause of failure in the competing risk (multiple decrement) model and in terms of the functions ${}_{}l_x^{(t)}$, ${}_{t}q_x^{(t)}$, ${}_{t}p_x^{(t)}$, ${}_{t}d_x^{(t)}$, ${}_{t}m_x^{(t)}$ (t): • Establish relations between the functions. • Calculate probabilities and moments associated with functions of these random variables, given the joint distribution of the time of failure and the cause of failure. Range of weight: 3-7 percent	a. Time until failure b. Competing risk (multiple decrement) models
READINGS	
Bowers, Chapter 10.1-10.3	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
5. For homogenous and non-homogenous discrete-time Markov chain models: • Define each model. • Calculate probabilities of being in a particular state at a particular time. • Calculate probabilities of transitioning between states.	a. Markov chains b. Transition probability matrix c. Discrete-time Markov chains

Range of weight: 3-7 percent	
READINGS	
Daniel, Chapters 1 and 3	

D. Stochastic Processes

Range of weight for Section D: 3-7 percent

Candidates should be able to solve problems using stochastic processes. They should be able to determine the probabilities and distributions associated with these processes. Specifically, candidates should be able to use a Poisson process in these applications.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Describe the properties of Poisson processes: • For increments in the homogeneous case • For interval times in the homogeneous case • For increments in the non-homogeneous case • Resulting from special types of events in the Poisson process • Resulting from sums of independent Poisson processes Range of weight: 0-5 percent	a. Poisson process b. Non-homogeneous Poisson process
2. For any Poisson process and the interarrival and waiting distributions associated with the Poisson process, calculate: • Expected values • Variances • Probabilities Range of weight: 0-5 percent	a. Probability calculations for Poisson process
READINGS	
Ross, Chapters 5.3.1-5.3.4 and 5.4.1	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. For a compound Poisson process, calculate moments associated with the value of the process at a given time Range of weight: 0-5 percent	a. Compound Poisson process
READINGS	
Ross, Chapter 5.4.2 preceding Example 5.25	

E. Life Contingency Models

Range of weight for Section E: 13-17 percent

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Apply a principle to a present value model to associate a cost or pattern of costs (possibly contingent) with a set of future contingent cash flows. Range of weight: 5-10 percent	a. Principles include: equivalence, exponential, standard deviation, variance, and percentile b. Models including those listed in Learning Objectives B2-B4 c. Principle applications include: life insurance, annuities, health care, credit risk, environmental risk, consumer behavior (e.g., subscriptions), and warranties
READINGS	
Bowers, Chapters 4.1-4.3, 5.1-5.3, 6.1-6.3, 9.7	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
2. Analyze present value of future loss random variables for life insurances and annuities and determine net liabilities using prospective and retrospective methods. Range of weight: 3-7 percent	a. Life insurance liability calculations b. Prospective and retrospective methods
READINGS	
Bowers, Chapter 7.1-7.4	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Using present-value-of-benefit random variables and present-value-of-future-loss random variables extended to discrete time Markov chains, calculate: • Actuarial present values of cash flows at transitions between states • Actuarial present values of cash flows while in a state • Considerations (premiums) using the Equivalence Principle • Liabilities (reserves) using the prospective method Range of weight: 0-5 percent	a. Cash flows at transition b. Triple product summation c. Transition probabilities
READINGS	
Daniel, Chapters 2 and 3	

Complete Text References for Exam 3 for 2007

Text references are alphabetized by the citation column.

Citation	Abbreviation	Learning Objectives
Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; and Nesbitt, C.J., *Actuarial Mathematics* (Second Edition), 1997, Society of Actuaries.	Bowers et al.	C1-C4, E1, E2
Daniel, J.W., "Multi-state Transition Models with Actuarial Applications," Study Note, 2004.	Daniel	C5, E3
Hoel, P.G.; Port, S.C.; and Stone, C.J., *Introduction to Statistical Theory*, 1971, Houghton Mifflin Company.	Hoel	A1-A4
Hogg, R.V.; Craig, A.T.; and McKean, J.W., *Introduction to Mathematical Statistics* (Sixth Edition), 2004, Prentice Hall.	Hogg et al.	A1-A4
Hogg, R.V.; and Tanis, E., *Probability and Statistical Inference* (Seventh Edition), 2006, Prentice Hall.	Hogg and Tanis	A1-A4
McDonald, R.L., *Derivatives Markets* (Second Edition), 2006, Addison Wesley.	McDonald	B1-B8
Mood, A.M.; Graybill, F.A.; and Boes, D.C., *Introduction to the Theory of Statistics* (Third Edition), 1974, McGraw-Hill [out of print].	Mood et al.	A1-A4
Ross, S.M., *Introduction to Probability Models* (Eighth Edition), 2003, Academic Press, San Diego, Sections 5.3.1-5.3.4, 5.4.1-5.4.2 preceding Example 5.25. [Candidates may also use the seventh edition with the following citation: Sections 5.3.1-5.3.4, 5.4.1-5.4.2 preceding Example 5.25.]	Ross	D1-D3

Exam 4
Construction and Evaluation of Actuarial Models

This four-hour, multiple-choice examination is administered by Preliminary Actuarial Examinations and is identical to SOA Course C.

This material provides an introduction to modeling and covers important actuarial methods that are useful in modeling. A thorough knowledge of calculus, probability and mathematical statistics is assumed.

The candidate will be introduced to useful frequency and severity models beyond those covered in Exam 3. The candidate will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The candidate should be able to: 1) analyze data from an application in a business context; 2) determine a suitable model including parameter values; and 3) provide measures of confidence for decisions based upon the model. The candidate will be introduced to a variety of tools for the calibration and evaluation of the models.

A variety of tables will be provided to the candidate in the study note package and at the examination. These include values for the standard normal distribution, chi-square distribution, and abridged inventories of discrete and continuous probability distributions. These tables are also available on the SOA and CAS Web sites. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

LEARNING OBJECTIVES

The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below.

A. Severity Models

1. Calculate the basic distributional quantities:
   a. Moments
   b. Percentiles
   c. Generating functions
2. Describe how changes in parameters affect the distribution.
3. Recognize classes of distributions and their relationships.
4. Apply the following techniques for creating new families of distributions:
   a. Multiplication by a constant
   b. Raising to a power
   c. Exponentiation
   d. Mixing
5. Identify the applications in which each distribution is used and reasons why.
6. Apply the distribution to an application, given the parameters.
7. Calculate various measures of tail weight and interpret the results to compare the tail weights.
8. Explain the properties of the lognormal distribution.
9. Explain the Black-Scholes formula as a limited expected value for a lognormal distribution.

B. Frequency Models

For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof (as well as compound distributions):

1. Describe how changes in parameters affect the distribution.
2. Calculate moments.

3. Identify the applications for which each distribution is used and reasons why.
4. Apply the distribution to an application given the parameters.

C. Aggregate Models

1. Compute relevant parameters and statistics for collective risk models.
2. Evaluate compound models for aggregate claims.
3. Compute aggregate claims distributions.

D. Severity, Frequency, and Aggregate Models

1. Evaluate the impacts of coverage modifications:
   a. Deductibles
   b. Limits
   c. Coinsurance
2. Calculate Loss Elimination Ratios.
3. Evaluate effects of inflation on losses.

E. Risk Measures

1. Calculate risk measures VaR, CTE and explain their use and limitations.

F. Ruin Theory

1. Calculate survival and ruin probabilities using discrete models.
2. Describe the considerations included in a ruin model.

G. Construction of Empirical Models

1. Estimate failure time and loss distributions using:
   a. Kaplan-Meier estimator, including approximations for large data sets
   b. Nelson-Aalen estimator
   c. Kernel density estimators
2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
3. Estimate failure time and loss distributions with the Cox proportional hazards model and other basic models with covariates.
4. Apply the following concepts in estimating failure time and loss distribution:
   a. Unbiasedness
   b. Consistency
   c. Mean squared error

H. Construction and Selection of Parametric Models

1. Estimate the parameters of failure time and loss distributions using:
   a. Maximum likelihood
   b. Method of moments
   c. Percentile matching
   d. Bayesian procedures
2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.
3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.

4. Apply the following concepts in estimating failure time and loss distributions:
    a. Unbiasedness
    b. Asymptotic unbiasedness
    c. Consistency
    d. Mean squared error
    e. Uniform minimum variance
5. Determine the acceptability of a fitted model using:
    a. Graphical procedures
    b. Kolmogorov-Smirnov test
    c. Anderson-Darling test
    d. Chi-square goodness-of-fit test
    e. Likelihood ratio test

I. Credibility

1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.
2. Perform Bayesian analysis using both discrete and continuous models.
3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.
4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.
5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.

J. Simulation

1. Simulate both discrete and continuous random variables using the inversion method.
2. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.
3. Use simulation to determine the p-value for a hypothesis test.
4. Use the bootstrap method to estimate the mean squared error of an estimator.
5. Apply simulation methods within the context of actuarial models.
6. Simulate lognormal stock prices.
7. Incorporate jumps in stock prices by mixing Poisson and lognormal random variables.
8. Use variance reduction techniques to accelerate convergence.
9. Use the Cholesky decomposition method for simulating correlated random variables.

READINGS

- *Loss Models: From Data to Decisions,* (Second Edition), 2004, by Klugman, S.A., Panjer, H.H. and Willmot, G.E., Chapter 3, Chapter 4, Sections 4.1-4.6.6 only, Chapter 5, Chapter 6, Sections 6.1-6.7, 6.11.1, 6.11.2 only, Chapter 7, Sections 7.1, 7.2.3, 7.3.1, 7.3.2 only, Chapters 9–11, Chapter 12 (excluding 12.5.4, 12.5.5 and 12.6), Chapter 13, and Chapter 17.

- *Derivatives Markets* (Second Edition), 2006, by McDonald, R.L., Chapters 18-19, excluding appendices.

- "An Introduction to Risk Measures for Actuarial Applications," 2006, by Hardy, M.R., Study Note.

Reading Options for Credibility

The candidate may use any of the alternatives shown below.

Option A

- *Loss Models: From Data to Decisions,* (Second Edition), 2004, by Klugman, S.A., Panjer, H.H., and Willmot, G.E., Chapter 16, Sections 16.1 (background only), 16.2 (background only), 16.3, 16.4 (excluding16.4.7), and 16.5 (excluding 16.5.3).

Option B

- *Foundations of Casualty Actuarial Science* (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, "Credibility", by Mahler, H.C., and Dean C.G., Section 1 (background only) and Sections 2–5.
- *Topics in Credibility Theory* (Study Note C-24-05) by Dean, C.G.

Option C

- *Introduction to Credibility Theory* (Third Edition), 1999, Herzog, T.N., Chapter 1-3 (background only)**,** 4–8, and 9 (background only)**.**

Study Notes

Study Notes for the preliminary examinations will be linked [in NOVEMBER 2006] from the "Study Tools" in the "Admissions" section of the CAS Web Site as well as on the SOA Web site under Exams and Jobs/Candidate and Exam Information/Spring Exam Session/Spring 2007 Basic Education Catalog – Study Notes Information. Hard copies may be purchased by downloading the form from the SOA Spring Exam Session Web page.

Code	**Title**
C-05-07	Exam C Introductory Study Note
C-09-05	Exam C Sample Questions and Solutions
C-10-05	May 2005 Exam C Questions and Solutions
C-12-05	November 2005 Exam C Questions and Solutions
C-21-01	Credibility **(to be used with Option B only)**
C-24-05	Topics in Credibility Theory (**to be used with Option B only)**
C-25-07	Risk Measures