CHAPTER 5
LOSS RESERVING
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INTRODUCTION

The financial condition of an insurance company can not be adequately assessed without sound loss reserve estimates. A loss reserve is a provision for an insurer's liability for claims. Loss reserving is the term used to describe the actuarial process of estimating the amount of an insurance company's liabilities for loss and loss adjustment expenses. Loss reserving is a major challenge to the casualty actuary because the estimation process involves not only complex technical tasks but considerable judgment as well. No formula will provide the correct answer.

The intent of this chapter is to introduce the topic of loss reserving. We will focus on the purpose, common definitions and basic principles. Once a common framework has been introduced, we will explore the common techniques used by practicing actuaries in estimating loss reserves. A large number of numerical examples will be provided to demonstrate the techniques.

The common methods assume that an insurance company's historical experience can be used to project the future. But in today's business environment, an insurance company's operations are frequently changing. The loss reserve analyst needs to recognize and make adjustments for the changes. Based on experience and judgment, an actuary's opinion will always play a key role in interpreting the results of the numerical techniques. It is important for an actuary to learn about the organization and understand the data before embarking on the task of estimating the loss reserve.

Throughout this chapter, the term **claim** is defined to be the demand for payment by an insured or allegedly injured third party under the terms and conditions of an insurance policy (McClenahan, 1990). We will use the word "claim" and "loss" interchangeably. The term **insurer** represents any risk bearer for property and casualty exposures whether an insurance company, a self-insured entity, a pool or other form of organization.

ACCOUNTING CONCEPTS

To understand the importance of establishing a sound loss reserve estimate, it is helpful to understand the basic accounting principles that apply to insurers. A general reference on accounting principles is Davidson, et al. (1994). Once the basic accounting principles are understood, it will be clearly shown that the loss reserve estimate has a significant impact on the financial strength and stability of an insurance company.

We will focus on two important accounting statements. The balance sheet shows the financial position of a firm. The income statement documents the financial performance of the firm.

The Balance Sheet

The balance sheet reports on the financial position of the firm at a specific point in time. It shows the levels of assets and liabilities, and the status of the shareholders' equity, or surplus, for the insurer. The reporting follows the simple equation:

$$\text{Assets} = \text{Liabilities} + \text{Owners' Equity}$$

An asset is any economic resource that is held by the firm. An asset could be cash, stocks, bonds, real estate, or agents' receivables, for example.

Liabilities are claims on the resources of the firm, to satisfy obligations of the firm. Liabilities could be mortgages, bank debt, bonds issued, premiums received from clients but not yet earned, or benefits payable on behalf of clients due to contractual obligations, for example. Loss and loss expense liabilities are frequently the largest liabilities of an insurer.

Owners' equity is the owners' claim on the assets of the firm. The owners' claim is always subordinate to all other liabilities of the firm. It is actually the balancing item in the equation above. Owners' equity is generally called the surplus of the insurer. For stock companies, the stockholders are the owners, but for mutual insurers, the policyholders are the owners and the surplus belongs to them.

When liabilities exceed assets, the value of the owners' equity is negative, and the firm is insolvent. Another way to write the equation of financial position is then given by:

$$\text{Owners' Equity} = \text{Assets} - \text{Liabilities}$$

The Income Statement

The income statement measures changes in owners' equity during a stated period of time. Owners' equity can be subdivided into the capital contributed by the owners and any retained earnings of the firm from past periods. Thus,

$$\text{Owners' Equity} = \text{Contributed Capital} + \text{Retained Earnings}$$

The income statement measures the firm's performance in the stated period as follows:

$$\text{Income} = \text{Revenue} - \text{Expense}$$

Revenue measures the inflow of assets from providing products or services. Expense measures the outflow of assets that are consumed in providing the firm's products or services.

Income may be either used to increase owners' equity in the firm (i.e., increase retained earnings) or distributed to owners as dividends. This can be written as

$$\text{Income} = \text{Change in Retained Earnings} + \text{Dividends to Owners}$$

This series of equations then defines the relationship between the balance sheet and the income statement. This relationship can be obtained by chaining together the basic accounting equations.

$$\text{Income} = \text{Change in Retained Earnings} + \text{Dividends to Owners}$$

but,

$$\text{Retained Earnings} = \text{Owners' Equity} - \text{Contributed Capital};$$

and,

$$\text{Owners' Equity} = \text{Assets} - \text{Liabilities}.$$

Thus,

$$\text{Retained Earnings} = \text{Assets} - \text{Liabilities} - \text{Contributed Capital};$$

and,

$$\begin{aligned} \text{Income} = {} & \text{Change in Assets} - \text{Change in Liabilities} \\ & - \text{Change in Contributed Capital} \\ & + \text{Dividends to Owners}. \end{aligned}$$

This formula demonstrates the relationship between the balance sheet and the income statement of a firm. In particular, note that any change in a liability account, such as loss reserves, has a direct impact on insurer's income.

Accrual Accounting

The balance sheet and the income statement of an insurance company are prepared on an accrual basis. A cash basis of accounting simply recognizes revenues when they are received and reports expenses when they are made. The accrual basis does a better job of matching revenues with their associated expenses (Davidson et al., 80–84). Under the accrual basis, revenues are recognized when earned. Costs are reported as expenses in the same period as the revenues giving rise to those costs are recognized. The main sources of an insurer's revenue are earned premium, rather than written premium, and the investment income earned on the assets it holds.

Similarly, policyholder benefits are expenses incurred by the firm, which must be matched to the revenues earned on the policies. It would clearly be inappropriate to count only paid losses and paid loss adjustment expense as expenses. The expenses incurred for policy benefits can be computed through use of the "loss reserve" liability account. The formula is:

$$\text{Incurred Losses} = \text{Paid Losses} + \text{Ending Claim Liability} - \text{Beginning Claim Liability}$$

CLAIM DEPARTMENT RESERVING

Before we discuss loss reserving from an actuarial perspective, we will look at the function of a claim department. In its simplest form, an insurance company sells a promise to settle claims that occur from a covered loss. An insurance company incurs a loss liability as soon as the insured incident occurs whether or not the company has even received notice of a loss. A loss liability needs to be established from the time of occurrence to final settlement. For the precise accounting definition of a liability, refer to Appendix A.

After a loss has occurred, an insurance company will receive a report or notice of the claim. Sometimes the delay between the occurrence and the report may be very short such as when a house burns down or a car accident happens. At other times, the delay may be very long. For example, asbestos harmed people many years before any claims were reported to the insurers of the product's manufacturers.

Once a claim has been reported, a claim adjuster establishes an estimate of the settlement amount after taking the facts of the case into consideration. Case reserves are the adjuster's best estimate of the remaining dollars that will eventually be paid on the claim. Taken together, paid loss and case reserves equal the incurred loss value of the claim.

During the process of adjustment, a claims adjuster may revise the case reserve estimate either up or down and make partial payments on the claim. These revisions reflect new information that has been gathered during investigation and settlement of a claim. The term **loss development** is used to describe the changes that take place in the value of a claim over time. Once the claim is settled and no further payments are expected, the claim is closed.

Loss adjustment expenses are also incurred during this process. These expenses include the cost of the claims adjuster and other expenses related to the defense and settlement of a claim. Some claims produce very little loss adjustment expense. For example, the house fire claim may be resolved with a couple of phone calls. Other claims, such as the asbestos example, may revolve around a complex series of issues and involve many interested parties. Frequently, claims of this sort may involve litigation. Attorney's fees and other defense costs may produce very high loss adjustment expenses.

Even after a claim is closed, it is possible for the claim to be reopened if new facts come to light. Also, once an insurer has satisfied its obligation to the policyholder, the company may pursue recoveries from third parties for some part of the indemnity amount paid to the policyholder. This right of recovery is called **subrogation**. An example of subrogation involves the payment of a collision claim by an insurer. If a third party was responsible for the damage, the insurer making the collision coverage payment to its insured has the right to recover the amount of damages from the responsible party.

In addition to subrogation situations, the payment of first party benefits may be accompanied by the insurer's taking of title to the damaged property. This property can often be disposed of for a partial recovery of the amount paid to the policyholder for the loss, and is called **salvage**. An example of a common salvage situation involves an automobile accident in which the insured's vehicle is a total loss. The insurer reimburses the insured for the value of the vehicle and takes title to the vehicle. The auto is then disposed of for any scrap value and the proceeds reduce the amount of loss. Note that both salvage and subrogation serve to reduce the insurer's net payout.

The Claims Inventory

At any point in time, an insurer will have many claims at various stages of development in its claim inventory. A look at the structure of a typical loss reserve case inventory will aid in understanding the processes at work when we observe loss development. For the moment, we'll consider loss development as simply the change in the value of a loss reserve inventory during a specified period of time.

The accompanying table on Case Reserve Activity (see page 204) tracks the types of activity that may occur in a loss reserve inventory during any one time period. Initially, there are 1,015 claims that are open and in the process of adjustment by the insurer's claim department. Losses will enter either as new claims (line 2), reopened claims (line 3), or zero reserves (line 4). A reopened claim is one that has previously been closed, but requires a pending claim file, because further adjusting activity is needed. This must be distinguished from a closed claim that simply requires an additional payment after closure, i.e., a prematurely closed claim (included in line 15). Such a claim is not reopened because no further adjusting effort is expected to be necessary after the single payment.

A particular type of new claim that should be distinguished from others is the precautionary reserve claim or zero reserve claim. This is used to establish a file as a means of monitoring a potential liability situation. No dollar value, or a nominal amount, is put up on the claim file because there is not yet a strong enough fact situation that a liability of the company exists. However, there is potential for liability and the situation must be monitored by the company. The use of these precautionary files is most often found on excess or reinsurance losses. When the primary carrier is another company, there may be very little information in the file initially established, other than the mandatory notice required by the excess policy wording.

Case Reserve Activity

	Counts	Amounts
1. Beginning Outstanding	1,015	$5,673,633
2. New Reserves	80	270,850
3. Reopened Reserves	29	84,472
4. Zero Reserves	2	0
5. Reserve Increases	28	163,995
6. Subtotal: Increases (2 + 3 + 4 + 5)	139	$ 519,317
7. Reserve Decreases	81	(57,433)
8. Closed with Payment	30	(713,281)
9. Closed without Payment	71	(147,291)
10. Subtotal: Decreases (7 + 8 + 9)	182	($ 918,005)
11. Total Reserve Change (Counts 2 + 3 + 4 – 8 – 9) (Amounts: 6 + 10)	10	($ 398,688)
12. Final Payments	30	793,180
13. Partial payments	82	60,514
14. Fast Track Payments	8	29,281
15. All Other	51	32,943
16. Total Payments (12 + 13 + 14 + 15)	171	$ 915,918
17. Salvage/Subrogation	28	(3,269)
18. Incurred Loss (11 + 16 + 17)		513,961
19. Ending Outstanding (1 + 11)	1,025	$5,274,945

Many companies also use a "fast track" claim category (line 14). This is simply a claim that is paid without a claim file ever being established. This procedure is often used on small property claims, such as auto physical damage or homeowners.

For reserve analysis purposes, we record only the financial changes that result from the claim adjusters' activities.

Referring to the Case Reserve Activity chart on page 204, we have categorized the types of financial actions we are interested in recording. Reserve increases (line 5) or decreases (line 7) are changes in open claim file valuations, and the file remains open after the change in valuation. These changes in reserve valuation may be accompanied by a loss or expense payment.

Incurred loss is the measure used to calculate the insurance company's liability.

$$\text{Incurred Loss} = \text{Paid Loss} + \text{Ending Loss Reserve} - \text{Beginning Loss Reserve}$$

Partial payments may be split into payments with and without incurred effect for the file. A payment may have no incurred effect if the remaining reserve is reduced by the amount of the payment. Thus if the claim adjuster reduces the case reserve by the amount of payment, there is no incurred loss effect. Often, automated claim systems will reduce the case reserve by the amount of the payment. This requires the adjuster to take specific reserve action only if the intent is to change the total valuation of the claim file.

One of the most important statistics to monitor for any claim inventory is the number of claim closings (lines 8 and 9). Note the distinction between claims closed without payment and those closed with some payment of loss or expense. In terms of simple monitoring of reserve activity, the rate of claim closings should be carefully watched. Often, a change in the closing rate can lead the analyst to discover an important operational change in claims administration.

The incurred effect of reserve closings can be calculated from the same formula:

$$\text{Incurred Loss} = \text{Paid Loss} - \text{Beginning Reserve}$$

Note that the ending reserve on a closed claim is zero, hence the formula simplifies as above. For our example, for the 101 closing payments (30 closed with payment, 71 without payment) we can calculate that:

$$\text{Incurred Loss} = \$793{,}180 - \$713{,}281 - \$147{,}291$$

$$\text{Incurred Loss} = (\$67{,}392)$$

Note that for this period, there is actually a savings on claims closed. This can often be the case, especially for lines of business that generate a high proportion of claims closed with no payment. If we only consider claims that close with payment, there is no savings:

$$\text{Incurred Loss} = \$793{,}180 - \$713{,}281 = \$79{,}899$$

In addition to claim payments associated with reserve files, we also can have payments to which no currently open files are attached. Fast Track payments have already been mentioned. There are also other miscellaneous payments, including payments on files already closed.

Note that almost all of the 82 partial claim payments on line 13 are associated with reserve decreases on line 7. This may be a result of the automatic decrease of case reserves to offset the amount of partial payments.

Note how salvage and subrogration serve to reduce the insurer's net payout. Then the total amount paid in the period is

$$\text{Paid loss} = \$915{,}918 - \$3{,}269 = \$912{,}649$$

A more theoretical approach to loss development can be found in Appendix B: An Actuarial Model of Loss Development.

A loss reserving analyst should be familiar with how the claim department of their organization functions. Any changes in how the department operates, such as reorganization, changes in claim handling practices or defense strategies, could affect the claim reserve inventory and loss development. Loss reserve estimates should reflect the impact of such changes. Understanding what a claim adjuster does and frequent communication with the claim department will aid the actuary in this task.

LOSS RESERVE DEFINITIONS

Before we discuss how to estimate loss reserves, we need to define some basic loss reserve terminology. First, a **total loss reserve** consists of 5 elements:

1. Case reserves assigned to specific claims,
2. A provision for future development on known claims,
3. A provision for claims that re-open after they have been closed,
4. A provision for claims that have occurred but have not yet been reported to the insurer, and
5. A provision for claims that have been reported to the insurer but have not yet been recorded.

A loss reserve can be divided into two categories: known claims vs. unknown claims. The reserve for **known claims** represents the amount that will be required for future payments of claims that have already been reported to the insurer (the sum of (1), (2), (3)). This amount includes the case reserves, the aggregate of the individual estimates made by the adjusters. Some case reserves may be set by formula.

The reserve for **unknown claims** is the amount for claims that have been incurred but not reported to the insurer. The reserve for unknown claims is commonly called an IBNR reserve. Often, it is not possible to distinguish between claims in the last two categories. These "pure" IBNR claims (4) and the claims in transit (5) are frequently combined together and called IBNR. This is the strict definition of IBNR, but in practice, future development on known claims, a provision for re-open claims, unreported claims and unrecorded claims are often combined together and called IBNR.

Over time, losses develop and the IBNR claims emerge. One of the reasons loss reserves need to be estimated is due to the delay between when a loss occurs, when it is reported to an insurer, and when it is finally settled. Therefore, dates become very important in the data organization and loss reserve estimation process. The list below defines the five key dates.

1. **Accident Date**—the date on which the loss occurred.
2. **Report Date**—the date on which the loss is first reported to an insurer.
3. **Recorded Date**—the date on which the loss is first recorded in the insurer's statistical information.
4. **Accounting Date**—the date used to define the group of claims to be included in the liability estimate. A loss reserve is an estimate of the liability for unpaid claims as of a given date, called the accounting date. An accounting date may be any date and is generally a date for which a financial statement is prepared such as a month end, quarter end or year end.
5. **Valuation Date**—the date as of which the evaluation of the loss liability is made. The valuation date defines the point in time through which all transactions are to be included for the group of claims. The valuation date can be before, after or the same as the accounting date.

Since the loss reserve liability is always an estimate, and the amount of the estimate will change as of successive valuation dates, we should establish some conventional terminology to discuss the results of the loss reserve process.

The **required loss reserve** as of a given accounting date is the amount that must ultimately be paid to settle all claim liabilities. The value of the required loss reserve can only be known when all claims have been finally settled. Thus, the required loss reserve as of a given accounting date is a fixed number that does not change at different valuation dates. However, the value of the required loss reserve is generally unknown for an extremely long period of time.

The **indicated loss reserve** is the result of the actuarial analysis of a reserve inventory as of a given accounting date conducted as of a certain valuation date. This indicated loss reserve is the analyst's opinion of the amount of the required loss reserve. This estimate will change with successive valuation dates and will converge to the required loss reserve as the time between valuation date and the accounting date of the inventory increases.

The **carried loss reserve** is the amount of unpaid claim liability shown on external or internal financial statements.

The **loss reserve margin** is the difference between the carried reserve and the required reserve. Since the required reserve is an unknown quantity we only have an indicated margin. The indicated loss reserve margin is defined to be the carried loss reserve minus the indicated loss reserve. One should not generally expect the margin to be zero, since for any subset of an entity's business it is unlikely that the carried loss reserve will be identical to either the indicated or required loss reserve.

LOSS RESERVING PRINCIPLES

The Casualty Actuarial Society (CAS) has published a statement of principles regarding property and casualty loss and loss adjustment expense reserves. There are four basic principles. The first principle says that an actuarially sound loss reserve "for a defined group of claims as of a given valuation date is a provision, based on estimates derived from reasonable assumptions and appropriate actuarial methods, for the unpaid amount required to settle all claims, whether reported or not, for which liability exists on a particular accounting date."

The second principle says an actuarially sound loss adjustment expense reserve has the same characteristics except that the liability is for "the unpaid amount required to investigate, defend and effect the settlement of all claims."

The third principle states that since there is inherent uncertainty in the estimation process, a range of reserves can be actuarially sound. The final principle states that the "most appropriate reserve within a range of actuarially sound estimates depends on both the relative likelihood of estimates within the range and the financial reporting context in which the reserve will be presented."

THE LOSS RESERVING PROCESS

Now that we have defined the basic loss reserving terminology and the key loss reserving principles, we are ready to begin the loss reserving process. The loss reserve estimation process can only be properly applied to grouped data. We will be estimating loss reserves in the aggregate as opposed to the claim adjusters' perspective that tends to be on an individual claim basis.

Data Availability

The availability of proper data is essential to the loss reserving process. The actuary is responsible for informing management of the need for sufficiently detailed and quality data to obtain reliable reserve estimates.

Data must be presented that clearly displays the development of losses. One of the most common ways to organize the data is in a loss development triangle. Each row in the triangle represents a given accident year. The data in each row represents a fixed group of claims. The columns keep track of the losses at subsequent evaluations for an individual accident year. Each column represents the age of the accident year. The usual convention is that an accident year is age 12 months at the end of the last day of the accident year.

Groupings other than accident year can be used. In descending order of preference, the rows could represent report year, policy year, or underwriting year (Berquist and Sherman). The rows could also represent quarterly, monthly, or other fixed grouping of data. The columns are typically shown in annual intervals, but quarterly intervals are also common.

To show how a loss development triangle is constructed, we will do an example with paid losses. Suppose the paid losses during calendar year 2000 for a line of business totaled $71,273,000. This information can be readily obtained from an accounting exhibit. It would be useful to know how that amount is split by accident year. In this example, suppose we knew that $11,346,000 of the paid losses came from claims that had accident dates during 2000. Similarly, $16,567,000 of the calendar year 2000 payments were from losses with 1999 dates of loss. By date of loss occurrence, we would find:

	(000 omitted)
Paid on 2000 losses:	$11,346
Paid on 1999 losses:	16,567
Paid on 1998 losses:	19,935
Paid on 1997 losses:	11,956
Paid on 1996 losses:	5,985
Paid on 1995 losses:	3,211
Paid on 1994 losses:	2,274
Total paid loss in 2000	$71,274

Since we now know that $11,346,000 was paid on 2000 losses during the year 2000, we would like to know the comparable amount paid on 1999 losses during 1999. We can find that a total of $73,972,000 was paid in 1999 on this line of business, and that $17,001,000 is for losses that occurred during 1999. Further, the full 1999 paid amount can be split into amounts (in thousands) paid on accidents from different years as was done for 2000 payments:

	(000 omitted)
Paid on 1999 losses:	$17,001
Paid on 1998 losses:	22,343
Paid on 1997 losses:	13,036
Paid on 1996 losses:	9,098
Paid on 1995 losses:	6,235
Paid on 1994 losses:	4,693
Paid on 1993 losses:	1,566
Total paid loss in 1999	$73,972

Comparison of these amounts by loss year for several calendar years would quickly become awkward, which is why we construct loss development triangles that facilitate the comparisons we want to make between the accident year components of calendar year paid amounts.

For instance, the incremental payments in thousands, on accident years 1994 through 2000 can be displayed as follows:

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	22,603	17,461	14,237	9,813	7,143	4,693	2,274
1995	22,054	21,916	14,767	13,104	6,235	3,211	
1996	20,166	18,981	12,172	9,098	5,985		
1997	19,297	18,058	13,036	11,956			
1998	20,555	22,343	19,935				
1999	17,001	16,567					
2000	11,346						

Now we see that the loss payments of the 2000 calendar year appear on the lowest diagonal of the triangle. Similarly, the 1999 calendar year payments appear on the second lowest diagonal. This data organization greatly facilitates comparison of the development history experienced by an accident year.

While this arrangement shows the amount paid in each 12-month period, it is often convenient to accumulate the payments on a given loss year. This would result in the following triangle of cumulative loss payments:

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224
1995	22,054	43,970	58,737	71,841	78,076	81,287	
1996	20,166	39,147	51,319	60,417	66,402		
1997	19,297	37,355	50,391	62,347			
1998	20,555	42,898	62,832				
1999	17,001	33,568					
2000	11,346						

The cumulative paid loss development triangle shows how the losses in each accident year are developing to their ultimate value. The number of years of development should be great enough so that further developments will be negligible (Berquist and Sherman). A paid loss triangle is only one of several types of triangles that can be constructed. We will discuss other types in the Exploratory Data Analysis section.

Reserve Estimation Strategy

The overall approach to a reserve valuation problem can be broken into four phases:

1. **Exploratory analysis of the data** to identify its key characteristics and possible anomalies. Balancing of data to other verified sources should be undertaken at this point.

2. Application of appropriate **reserve estimation techniques**.

3. **Evaluation** of the conflicting results of the various reserve methods used, with an attempt to reconcile or explain the different outcomes. At this point the proposed reserving ultimate amounts are evaluated in contexts outside their original frame of analysis.

4. **Monitor** projections of reserve development over subsequent calendar periods. Deviations of actual development from projected development of counts or amounts are one of the most useful diagnostic tools in evaluating accuracy of reserve estimates.

PHASE 1: EXPLORATORY DATA ANALYSIS

Considerations

Exploring the data begins by understanding the trends and changes affecting the database. Understanding the data is a prerequisite to estimating sound loss reserves. This exploration will help the analyst select appropriate loss reserving methods and interpret the results of the methods.

Knowledge of the business being evaluated is a key consideration. Changes in the classes of business written or geographical focus may affect the data. Changes in policy provisions such as policy limits and deductibles will impact observed development patterns. Reinsurance purchased by the insurance company will impact net loss development patterns. Changes in the reinsurance limits and attachment points can alter future net development.

Changes that occur in the external environment may impact the data. The legal/social environment can change abruptly. This is a particular concern if the data is concentrated in one state. For example, the introduction of no-fault auto insurance clearly changes the business environment. Changes in the economic environment, such as the inflation rate, may also cause changes in the database.

Operational changes within the company can affect the data over time. The installation of a new computer system may impact the time it takes to process a loss payment. Claims adjusting practices may change because of office consolidation or a new claim management team.

How the data is divided into groups for review is essential if meaningful loss reserve estimates are going to be produced. In order to draw valid conclusions about future loss development, the groups should be homogeneous i.e., display similar characteristics. In loss reserving, the focus is on grouping data that displays similar:

1. loss emergence patterns—the length of time between loss occurrence and report, and
2. loss settlement patterns—the length of time between the loss report and settlement.

Some examples of common divisions include separating data by line of business, coverage type (property or liability), and policy limits. The groupings should be made of business with similar frequency and severity characteristics. Business that produces a high frequency, low severity of losses could mask the development of a low frequency, high severity line.

Actuaries use the term credibility to mean the measure of predictive value given to a block of data. Increasing the homogeneity of the block of data or increasing the volume of data in the block increases credibility. The two means of increasing credibility are conflicting goals and the analyst needs to strike an appropriate balance between the two. If the volume of data is too small to produce meaningful loss development patterns, use of industry development patterns may be necessary.

Issues of credibility and homogeneity for loss reserving should most often be thought of in terms of their impact on loss development patterns. Credibility and homogeneity of data from even the same line of business is greatly enhanced if the policy limits or layers of loss are very similar. For example, loss development data on General Liability excess reinsurance may not be very stable or useful if it is not grouped by underlying limits and layer widths. Similarly, history on a book of basic limits auto liability business is of little value in evaluating a new book with $1 million limits. Aggregate limits of liability or even large losses that are capped at policy limits are very important facts for the loss reserve analyst to take into account in the analysis.

There are many other factors that an analyst should take into consideration when deciding how to group the data. A thorough discussion of the factors can be found in the *CAS Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves.*

Data Analysis

Before the analyst begins to project immature loss data to ultimate loss estimates, it is important to review the data. The objective of this review is to understand the data in terms of:

1. rate of development,
2. smoothness of development,
3. presence of large losses, and
4. volume of data.

Review of the data will allow the analyst to form conclusions about:

1. appropriate projection methodologies,
2. anomalies in the data, and

3. appropriate questions to ask management concerning issues that manifest themselves in the data, that will further the analyst's understanding of the book of business that generated the data.

Some of the more common data displays that should be reviewed by the loss reserve analyst are discussed here.

Cumulative Incurred Losses: An incurred loss triangle contains the history of combined paid losses and case reserves. The example below is an "accident year triangle," a history of losses incurred organized with losses from the same year of loss in the same row. A review of this incurred loss triangle points to a fluctuating level of losses since 1994. Note that the dip in losses reported on the 1997 accident year as of 12 months did not result in less loss reported through 48 months of development. This should alert the analyst to search for possible processing slowdowns at year-end 1997, or major fluctuations in case reserve adequacy. In light of this, the analyst must consider how to interpret the low level of 2000 accident year incurred losses. Clearly, some measure of exposure is called for—whether earned exposures, earned premiums, or even policies in force. This will help determine whether a level of ultimate incurred losses proportional to the low reported 2000 incurred is reasonable. The situation with loss processing as well as case reserve adequacy needs to be probed in order to decide on the proper 2000 accident year reserve.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	58,641	74,804	77,323	77,890	80,728	82,280	82,372
1995	63,732	79,512	83,680	85,366	88,152	87,413	
1996	51,779	68,175	69,802	69,694	70,041		
1997	40,143	67,978	75,144	77,947			
1998	55,665	80,296	87,961				
1999	43,401	57,547					
2000	28,800						

Cumulative Paid Losses: A paid loss triangle contains the history of paid losses. Small variations in paid loss as of 12 months can be seen to be indicative of very large differences in ultimate accident year losses. The low reported incurred on 1997 accident year is also paralleled by a lower paid loss amount on the 1997 accident year. The 20% drop from 1996 to 1997 incurred losses seen above produces only a 5% drop in paid claims from 1996 to 1997 accident years, as of the initial reporting. The large drop in payments in accident year 2000 indicates a much more severe change in claims environment than the drop of 5% in 1997. The analyst would look for evidence of lower 2000 exposure levels to explain the paid losses.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224
1995	22,054	43,970	58,737	71,841	78,076	81,287	
1996	20,166	39,147	51,319	60,417	66,402		
1997	19,297	37,355	50,391	62,347			
1998	20,555	42,898	62,832				
1999	17,001	33,568					
2000	11,346						

Incremental Incurred Losses: This triangle shows the incremental incurred losses in each successive period. It is useful for the analyst to gauge the reasonability of yearly aggregate loss accumulations on an accident year. Note the "speedup" of incurred losses in 12 to 24-months aging of the 1997 accident year (calendar year 1998), when incurred losses increased $27,835,000. It appears that the second annual development on the 1998 accident year of $24,632,000 is also unusually large when compared to accident years 1996 and prior. Thus, the analyst must suspect that processing problems were also apparent in the organization at year-end 1998. Questions to key managers in Claims and Underwriting should help the analyst gather information to confirm this suspicion.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	58,641	16,163	2,519	567	2,838	1,552	92
1995	63,732	5,779	4,168	1,686	2,786	−739	
1996	51,779	16,396	1,627	−107	347		
1997	40,143	27,835	7,166	2,803			
1998	55,665	24,632	7,665				
1999	43,401	14,147					
2000	28,800						

Incremental Paid Losses: This triangle shows the incremental paid losses in each successive 12-month period. We see immediately that payments during the second annual development period on an accident year are roughly equal to the amount paid in the first annual development period. Thus, we form an expectation that any reasonable projected development to ultimate must yield at least $10–12 million projected paid losses during the 12 to 24-month development on the 2000 accident year.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	22,603	17,461	14,237	9,813	7,143	4,693	2,274
1995	22,054	21,916	14,767	13,104	6,235	3,211	
1996	20,166	18,981	12,172	9,098	5,985		
1997	19,297	18,058	13,036	11,956			
1998	20,555	22,343	19,935				
1999	17,001	16,567					
2000	11,346						

Paid Loss as a Percent of Incurred Loss: This triangle divides paid losses at each development age by reported losses as of the same development age. This statistic tests the consistency of the development of paid and reported losses. It also may give warning of case reserve inadequacies. This statistic clearly flags the 1997 accident year as being inconsistent with history. The high ratio at age 12 indicates that the case reserve portion of the 1997 accident year incurred losses at that age was much weaker than it was historically. One benefit of this statistic is that it appears concurrent with the analysis, and does not rely on hindsight. The crucial 2000 accident year looks normal with regard to case reserves.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	38.5%	53.6%	70.2%	82.3%	88.3%	92.3%	95.0%
1995	34.6%	55.3%	70.2%	84.2%	88.6%	93.0%	
1996	38.9%	57.4%	73.5%	86.7%	94.8%		
1997	48.1%	55.0%	67.1%	80.0%			
1998	36.9%	53.4%	71.4%				
1999	39.2%	58.3%					
2000	39.4%						

Reported Claim Counts: Claim count history is extremely important in any loss reserve analysis. This triangle simply displays all reported claims by development period. We can conclude that for this particular line of business, essentially all claims are reported within 24 months. The 2000 accident year does have a much lower level of reported claims than prior accident years. A radical change in volume, such as 2000 accident year displays, should alert us to possible changes in business mix, reporting patterns or exposures.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	32,751	41,201	41,618	41,755	41,773	41,774	41,774
1995	33,736	39,528	39,926	40,044	40,072	40,072	
1996	27,067	32,740	33,084	33,183	33,209		
1997	24,928	29,796	30,074	30,169			
1998	25,229	31,930	32,281				
1999	17,632	21,801					
2000	15,609						

Closed with Payment Claim Counts: Here, the cumulative number of claims closed with payment are displayed. Note that this does not include all closed claims, as many claims may be closed with no payment. The possible processing problem at the end of 1997 and 1998 also appears in this statistic, which again suggests the need to understand changes in company operations that may have occurred during 1997–1998. The closing counts for 1999 and 2000 support the hypothesis of reduced exposures.

Accident			Age in months:				
Year	12	24	36	48	60	72	84
1994	23,355	31,940	33,288	33,860	34,091	34,247	34,294
1995	22,662	30,294	31,588	32,129	32,323	32,433	
1996	18,951	25,197	26,214	26,582	26,777		
1997	16,631	22,894	23,806	24,229			
1998	17,381	24,581	25,765				
1999	12,666	16,669					
2000	10,592						

Closed with No Payment Claim Counts: Claims may also be closed without any payment. The claims closed with no payment could easily account for over half or more of all claims reported for some lines such as medical malpractice. In this case, the display does not show any unusual patterns.

Accident			Age in months:				
Year	12	24	36	48	60	72	84
1994	2,646	6,285	6,935	7,240	7,353	7,393	7,412
1995	3,142	6,529	7,053	7,308	7,411	7,465	
1996	2,752	5,366	5,840	6,050	6,185		
1997	2,343	4,744	5,132	5,400			
1998	2,238	4,666	5,375				
1999	1,749	3,458					
2000	1,246						

Closed Claims as Percent of Reported Claims: Total closed claims can be related to claims reported. This is a monitor on closing activities. We can see a very steady performance of this ratio as of 12 and 24 months of development for accident years 1994 to 1999. Note the slightly lower closing ratio on the 2000 accident year. This should be explored with the claims department. On a property line this could be the result of a catastrophe in December, but in a liability line it indicates potential processing problems.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	79.4%	92.8%	96.6%	98.4%	99.2%	99.7%	99.8%
1995	76.5%	93.2%	96.8%	98.5%	99.2%	99.6%	
1996	80.2%	93.4%	96.9%	98.3%	99.3%		
1997	76.1%	92.8%	96.2%	98.2%			
1998	77.8%	91.6%	96.5%				
1999	81.8%	92.3%					
2000	75.8%						

Open Claim Counts: This triangle displays open claims, i.e., claims reported less all claims closed. Note that there were almost 2,000 fewer claims reported on the 2000 accident year than on the 1998 accident in the initial 12 months of reporting. However, at the end of 2000, there are almost 500 more claims open in the 2000 accident year than the 3,217 claims open on the 1999 accident year at the end of 1999. This could be the result of some very significant changes in the organization.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	6,750	2,976	1,395	655	329	134	68
1995	7,932	2,705	1,285	607	338	174	
1996	5,364	2,177	1,030	551	247		
1997	5,954	2,158	1,136	540			
1998	5,610	2,683	1,141				
1999	3,217	1,674					
2000	3,771						

Average Open Claim Amount: This triangle tracks the average amount reserved on open claims. Note the accident year 2000 average open case reserve has dropped from the prior year's level. This should be investigated with the claim department. Combined with the greater number of 2000 open reserves, a change in case reserving practices or a processing problem should be suspected. Note that the 1997 accident year showed a similar drop in its open reserve amount.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	5,339	11,671	16,499	21,029	28,782	47,240	60,722
1995	5,254	13,137	19,405	22,285	29,820	35,209	
1996	5,894	13,334	17,939	16,832	14,722		
1997	3,501	14,190	21,798	28,896			
1998	6,258	13,941	22,026				
1999	8,206	14,324					
2000	4,629						

Change in Average Open Claim: This triangle charts the change in the average open reserve between accident periods as of each development age. This is useful for determining if case reserves are keeping up with reasonable inflationary increases.

Accident Year	Age in months: 12	24	36	48	60	72
1994–95	−1.6%	12.6%	17.6%	6.0%	3.6%	−25.5%
1995–96	12.2%	1.5%	−7.6%	−24.5%	−50.6%	
1996–97	−40.6%	6.4%	21.5%	71.7%		
1997–98	78.7%	−1.8%	1.0%			
1998–99	31.1%	2.7%				
1999–00	−43.6%					
Average	6.0%	4.3%	8.2%	17.7%	−23.5%	−25.5%

Average Closed Claim: This triangle shows paid losses divided by closed with payment counts. Note that these averages are very regular with no reversals across accident years, until 2000 as of 12 months.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	968	1,254	1,631	1,894	2,090	2,218	2,281
1995	973	1,451	1,859	2,236	2,415	2,506	
1996	1,064	1,554	1,958	2,273	2,480		
1997	1,160	1,632	2,117	2,573			
1998	1,183	1,745	2,439				
1999	1,342	2,014					
2000	1,071						

Change in Average Closed Claim: This triangle shows the change in the average paid claim amount between accident years as of identical development periods. The multi-year average increase in closed claim values is also shown. It is useful to compare the annual increases in open case reserves to these increases in closed claim amounts. This allows the analyst to evaluate if the claim department reserves are keeping pace with inflationary increases in settlements. The analyst might suspect a serious backlog in paid claims for the 2000 accident year.

Accident	Age in months:					
Year	12	24	36	48	60	72
1994–95	0.6%	15.7%	14.0%	18.1%	15.6%	13.0%
1995–96	9.3%	7.0%	5.3%	1.6%	2.7%	
1996–97	9.0%	5.0%	8.1%	13.2%		
1997–98	1.9%	7.0%	15.2%			
1998–99	13.5%	15.4%				
1999–00	–20.2%					
Average	2.4%	10.0%	10.7%	11.0%	9.1%	13.0%

Closed Claims as a Percent of Open Claims: The rate of claim closure is one of the most important indicators of the condition of the claim department. This statistic measures the ratio of claims closed in the period to claims open at the beginning of the period. Our example shows some deterioration in claims performance in calendar year 1999. Only 60.2% of the claims occurring in 1997 that were open at 12/31/1998 were closed during the 1999 calendar year. Some level of recovery is evident during calendar year 2000 when 60.8% of 1997 occurrences that were open at 12/31/1999 were closed in the calendar year. However, the analyst might surmise that the closing activity on the 1999 inventory may have slipped in order to allow the catch up activity on the older inventories. These indications of processing problems should be probed with questions for claims management.

Accident			Age in months:			
Year	12	24	36	48	60	72
1994	181.1%	67.1%	62.9%	52.5%	59.6%	49.3%
1995	138.9%	67.2%	61.9%	48.9%	48.5%	
1996	165.2%	68.5%	56.1%	59.9%		
1997	145.5%	60.2%	60.8%			
1998	171.6%	70.6%				
1999	177.6%					

PHASE 2: LOSS RESERVE ESTIMATION METHODS

There are many methods for the projection of reserves to ultimate values. The most common simply project the growth behavior of the reserve inventory. These methods take little advantage of the fact that we are estimating loss reserves. The methods can equally well be used to project ultimate values of any growth process. Some examples might be the ultimate response rate to a direct mail campaign, or the loan defaults in a loan portfolio. All reserve estimation work must be supplemented with the experience of the reserve analyst.

Triangular Methods

The most common methods used to estimate ultimate loss levels consist of tracking the history of a group of claims with similar definitional groupings. The data for this purpose are arranged in a triangular loss format as discussed above. Each undeveloped loss year is projected to its expected ultimate level based on the assumption that each loss year will be completed in a way that is analogous to prior years. Several of these methods are described below.

Paid Loss Development

Suppose cumulative paid losses are as described by the triangle below.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224
1995	22,054	43,970	58,737	71,841	78,076	81,287	
1996	20,166	39,147	51,319	60,417	66,402		
1997	19,297	37,355	50,391	62,347			
1998	20,555	42,898	62,832				
1999	17,001	33,568					
2000	11,346						

A review of the data in the first column "as of 12 months" indicates that some rather extreme fluctuations in loss volume seem to have taken place over the last seven years. This should be checked out by a review of the historical claim count triangles, and also the earned exposure history, or earned premiums at a uniform exposure level as a proxy. These concerns are discussed above under the data exploration topics.

These shifts in the volume of losses make it difficult to reach any conclusion about this development. Thus we would like to "normalize" the development history by removing this volume effect. This is accomplished by studying the aging effect within each accident year as follows:

Accident	Age in months:					
Year	12–24	24–36	36–48	48–60	60–72	72–84
1994	1.773	1.355	1.181	1.111	1.066	1.030
1995	1.994	1.336	1.223	1.087	1.041	
1996	1.941	1.311	1.177	1.099		
1997	1.936	1.349	1.237			
1998	2.087	1.465				
1999	1.974					

This triangular display represents the historical development of each accident year. For instance, the development from 12 to 24 months on accident year 1999 is an increase in paid losses of 97.4% ($33,568/$17,001). Now the range of variation is considerably reduced. One can see that 12–24 month development seems to vary from 1.773 to 2.087. The high 1998 development of 2.087 seems to be outside a reasonable range of development factors observed in recent time periods. If we can predict the next 12–24 month development that we expect to see take place during 2001, we would be able to forecast the 2000 accident year paid losses at 24 months of development.

Thus our next task should consist of predicting the 2000 accident year 12–24 month paid loss development factor. One common technique is to inspect several averages of the age-to-age factors. The averages should provide a guide in selecting the next calendar period's development on that accident year.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84
1994	1.773	1.355	1.181	1.111	1.066	1.030
1995	1.994	1.336	1.223	1.087	1.041	
1996	1.941	1.311	1.177	1.099		
1997	1.936	1.349	1.237			
1998	2.087	1.465				
1999	1.974					
Average	1.951	1.363	1.205	1.099	1.053	1.030
Average Last 3	1.999	1.375	1.213	1.099		
Average Last 4	1.985	1.365	1.205			
Avg Exc Hi & Lo	1.961	1.347	1.202	1.099		
Weighted Average	1.948	1.364	1.205	1.099	1.053	1.030
Harmonic Mean	1.949	1.362	1.204	1.099	1.053	1.030

There is a practically unlimited number of ways to average the historical development factors. The key point to remember is that these averages are only guides to selection of the next reasonable development, based on all information that the loss reserve actuary has developed from reviews with management as well as the historical loss development.

Let's review the averages displayed above. The "Average" line is simply the arithmetic average of all historical loss development factors at that stage of development. Similarly, the "Average Last 3" and "Average Last 4" are simple arithmetic averages of the latest 3 and latest 4 development factors respectively at a given point of development. The "Avg Excluding Hi & Lo" is the arithmetic average of the development factors other than the highest and lowest. The "Weighted Average" is weighted by the amount of incurred loss. In this case the weighted average for age 36 to 48 is calculated as follows:

$$\frac{54,301 \times 1.181 + 58,737 \times 1.223 + 51,319 \times 1.177 + 50,391 \times 1.237}{54,301 + 58,737 + 51,319 + 50,391}$$

$$= 1.205$$

The harmonic mean is the n^{th} root of the n historical development factors. Here, the harmonic mean for age 36 to 48 was calculated as follows:

$$(1.181 \times 1.223 \times 1.177 \times 1.237)^{1/4} = 1.204$$

Notice that we are generally interested in examining the averages of the latest few development periods, because any forces that are impacting our current experience are more likely to be seen in the most recent past, rather than in earlier years. Hence we calculate averages of the latest three or four factors. Like many actuarial procedures, the analyst is asked to make a judgment of the most appropriate trade-off between stability (i.e., more development periods in the average computation) and responsiveness (i.e., only include the latest few development periods in the average).

The familiar triangular format also becomes a convenient way to inspect the addition of more development points in the averages. For instance, assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months (in bold).

Accident	Age in months:						
Year	12–24	24–36	36–48	48–60	60–72	72–84	84–Ult
1994	1.773	1.355	1.181	1.111	1.066	1.030	**1.053**
1995	1.994	1.336	1.223	1.087	1.041	**1.030**	
1996	1.941	1.311	1.177	1.099	**1.060**		
1997	1.936	1.349	1.237	**1.100**			
1998	2.087	1.465	**1.210**				
1999	1.974	**1.350**					
2000	**1.960**						

Often the selected developments are not identical to any of the averages. For instance the 12 to 24 month expected development is 1.960. Review of this history with claims and systems managers indicates that a new claim processing system was installed during 1998 that necessitated a longer installation period than anticipated. Thus the analyst views the 1998 accident year development of 2.087 as an anomaly. This is supported by the recovery of the 1999 accident year 12–24 months development to 1.974, a value that seems to fit with the prior developments. It is extremely important to realize that once the suspected anomalous development pattern on 1998 accident year has been identified, further information must be sought about company operations that may have caused this anomaly. In most cases, this information cannot be acquired through further study of the numbers, but requires the actuary to gather additional information, often of a qualitative nature.

The search for an explanation of recent unusual historical developments is critical to the reserve estimate because the analyst must decide whether the situation causing the abnormal loss development is still a factor that can affect future developments. In this case the new claim processing system has been in place since the end of 1998, and the 1999 accident year shows a return to more normal paid loss development patterns in the 12–24 months development. This leads the analyst to discount the 1998 development from 12 to 24 months as a nonrecurring situation. In general, the analyst should not use averages if the data show a clear trend that makes averaging inappropriate, or if an unusual accident year distorts the average.

On the other hand, suppose the analyst had determined that the unusual development on the 1998 accident year was due to several large losses that required unexpected large payments in 1999. This is clearly a situation that could happen again in any particular year, absent any changes in policy limit profiles or reinsurance retentions, and should receive some weight in future scenarios. In this situation, the averages including the 1998 accident year development from 12 to 24 months should be used as a guide to the 12 to 24 months paid development on the 2000 accident year.

The choice of the next projected development of 24 to 36 months is difficult because of the unusually high development of +46.5% on the 1998 reported paid losses from 24 to 36 months. At this point the analyst must regard the 1998 accident year as very unusual. A note should be made to inspect the projected ultimate on this accident year very carefully once development factors have been selected.

Investigation of events in the 2000 calendar year indicates that several large payments were evident on 1998 accident year cases, but this was on classes of business no longer written by the company. Accordingly the 1998 development cannot be used again in the choice of the 24 to 36 months development on the 2000 accident year.

Once the development factors for calendar year 2001 have been chosen for accident years 1995 through 2000, the analyst must forecast development for the 2002 and subsequent calendar years. The forecasted development for each subsequent calendar year is not necessarily the same as the 2001 calendar year forecast. In the triangle below, initial development factors have been chosen for future periods.

Accident			Age in months:					Dev to
Year	12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Ult
1994	1.773	1.355	1.181	1.111	1.066	1.030	**1.053**	**1.053**
1995	1.994	1.336	1.223	1.087	1.041	**1.030**	**1.053**	**1.085**
1996	1.941	1.311	1.177	1.099	**1.060**	**1.030**	**1.053**	**1.150**
1997	1.936	1.349	1.237	**1.100**	**1.060**	**1.030**	**1.053**	**1.265**
1998	2.087	1.465	**1.210**	**1.100**	**1.060**	**1.030**	**1.053**	**1.622**
1999	1.974	**1.350**	**1.210**	**1.100**	**1.060**	**1.030**	**1.053**	**2.066**
2000	**1.960**	**1.350**	**1.210**	**1.100**	**1.060**	**1.030**	**1.053**	**4.049**

The analyst then learns that the 1998 accident year contained more hazardous classes than the preceding years. These classes were assumed to account for the higher development on the 1998 accident year from 24 to 36 months. After discussions with the

Underwriting Department, the analyst feels that future development will also be higher due to this more hazardous business mix; however, the amount of the adjustment is in question.

The analyst finds that the 24 to 36 month development on 1998 at 1.465 is higher than the selected development of 1.350 which was based on the observed development in accident years 1994–97. It is also assumed that this differential of 1.085 (the relativity of the observed 1.465 to the “projected” 1.350) should dampen back to unity with the passage of time. The analyst decides to reflect this dampening effect by taking the square root of the differential with the passage of each year. Thus, the differential for the 36 to 48 month development should be the square root of 1.085 or 1.042. This results in a development from 36 to 48 months of 1.261, given by 1.210 multiplied by 1.042. Carried out an additional year, the adjusted development factor from 48 to 60 months is the square root of 1.042, or 1.021, times the otherwise selected 1.100. This method of dampening the observed differential in 1998 accident year development is driven more by the analyst’s experience than a set of mathematical formulae.

In a report on the loss reserve analysis, the analyst should disclose the assumption that the 1998 year will have higher loss development due to the more severe business mix.

The final selected development factors can be seen below.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.773	1.355	1.181	1.111	1.066	1.030	**1.053**	**1.053**
1995	1.994	1.336	1.223	1.087	1.041	**1.030**	**1.053**	**1.085**
1996	1.941	1.311	1.177	1.099	**1.060**	**1.030**	**1.053**	**1.150**
1997	1.936	1.349	1.237	**1.100**	**1.060**	**1.030**	**1.053**	**1.265**
1998	2.087	1.465	**1.261**	**1.123**	**1.060**	**1.030**	**1.053**	**1.622**
1999	1.974	**1.350**	**1.210**	**1.100**	**1.060**	**1.030**	**1.053**	**2.066**
2000	**1.960**	**1.350**	**1.210**	**1.100**	**1.060**	**1.030**	**1.053**	**4.049**

As can be seen from the process of selection of development factors the analyst should be able to recognize:

1. normal levels of random fluctuation in developments,
2. aberrations in development patterns and be able to isolate their causes, and determine if they are ongoing or one time changes in development,
3. trends in loss development patterns.

The analyst was also required to make a projection of paid loss from 84 months of age to ultimate settlement. Generally, the historical data should extend back far enough to make the projected development to ultimate, or tail factor as it is often called, very small. The selection of a tail factor is made difficult by two factors:

1. there is generally little relevant data available,
2. the tail factor affects all accident years reserve needs, thus has a disproportionate leverage on the total reserve need.

In this case, the analyst selects a tail factor of 5.3% additional development after 84 months of age, based on a subjective judgment and a review of cases over seven years old that were settled in the last calendar year. In addition to judgment, the analyst should include a review of any available industry data.

This triangle can be then transformed into its dollar equivalents, by successively multiplying the selected development factors by the last actual report of incurred losses.

Accident			Age in months:						Paid	
Year	12	24	36	48	60	72	84	Ultimate	to Date	Reserve
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224	**82,370**	78,224	4,146
1995	22,054	43,970	58,737	71,841	78,076	81,287	**83,726**	**88,163**	81,287	6,876
1996	20,166	39,147	51,319	60,417	66,402	**70,386**	**72,498**	**76,340**	66,402	9,938
1997	19,297	37,355	50,391	62,347	**68,582**	**72,697**	**74,878**	**78,846**	62,347	16,499
1998	20,555	42,898	62,832	**79,231**	**88,977**	**94,315**	**97,145**	**102,293**	62,832	39,461
1999	17,001	33,568	**45,317**	**54,833**	**60,317**	**63,936**	**65,854**	**69,344**	33,568	35,776
2000	11,346	**22,238**	**30,022**	**36,326**	**39,959**	**42,356**	**43,627**	**45,939**	11,346	34,593
									Total Reserve	147,289

This particular analysis of the paid loss data indicates that a reserve of about $147 million is necessary to provide for unpaid loss reserve liabilities from accident years 1994 through 2000.

While an estimate is available for a reserve need for this book of business, we can note at least two deficiencies in our knowledge at this point. First, we have not made any use of other available information, such as claim counts or case reserve values. Second, we have no means of evaluating prospectively the confidence we should have in this single forecast of the future. Both of these concerns can be addressed by alternative forecasts of the ultimate loss reserve need using other information available to us.

Incurred Loss Development

A triangular development analysis can also be developed using paid losses plus case reserves. Case reserves could be either adjuster determined or set by use of average values. Assume incurred losses and calculated age-to-age development factors as presented below.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	58,641	74,804	77,323	77,890	80,728	82,280	82,372
1995	63,732	79,512	83,680	85,366	88,152	87,413	
1996	51,779	68,175	69,802	69,694	70,041		
1997	40,143	67,978	75,144	77,947			
1998	55,665	80,296	87,961				
1999	43,401	57,547					
2000	28,800						

	12–24	24–36	36–48	48–60	60–72	72–84
1994	1.276	1.034	1.007	1.036	1.019	1.001
1995	1.248	1.052	1.020	1.033	0.992	
1996	1.317	1.024	0.998	1.005		
1997	1.693	1.105	1.037			
1998	1.443	1.095				
1999	1.326					

	Age in months:					
	12–24	24–36	36–48	48–60	60–72	72–84
Average	1.384	1.062	1.016	1.025	1.006	1.001
Average Last 3	1.487	1.075	1.018	1.025		
Average Last 4	1.445	1.069	1.016			
Avg Exc Hi & Lo	1.341	1.060	1.014	1.033		
Weighted Average	1.367	1.062	1.016	1.026	1.005	1.001
Harmonic Mean	1.376	1.062	1.015	1.025	1.005	1.001

Assume the analyst chose the following development patterns as the most likely in future calendar periods:

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.276	1.034	1.007	1.036	1.019	1.001	**1.010**	**1.010**
1995	1.248	1.052	1.020	1.033	0.992	**1.000**	**1.010**	**1.010**
1996	1.317	1.024	0.998	1.005	**1.000**	**1.000**	**1.010**	**1.010**
1997	1.693	1.105	1.037	**1.020**	**1.000**	**1.000**	**1.010**	**1.030**
1998	1.442	1.095	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.051**
1999	1.326	**1.095**	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.151**
2000	**1.350**	**1.095**	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.553**

Based on development factors chosen above the complete projection of incurred losses by accident year may be completed.

Accident Year	Age in months: 12	24	36	48	60	72	84	Ultimate	Paid to Date	Reserve
1994	58,641	74,804	77,323	77,890	80,728	82,280	82,372	**83,196**	78,224	4,972
1995	63,732	79,512	83,680	85,366	88,152	87,413	**87,413**	**88,287**	81,287	7,000
1996	51,779	68,175	69,802	69,694	70,041	**70,041**	**70,041**	**70,741**	66,402	4,339
1997	40,143	67,978	75,144	77,947	**79,506**	**79,506**	**79,506**	**80,301**	62,347	17,954
1998	55,665	80,296	87,961	**89,720**	**91,515**	**91,515**	**91,515**	**92,430**	62,832	29,598
1999	43,401	57,547	**63,014**	**64,274**	**65,560**	**65,560**	**65,560**	**66,215**	33,568	32,647
2000	28,800	**38,880**	**42,574**	**43,425**	**44,294**	**44,294**	**44,294**	**44,737**	11,346	33,391
									Total Reserve	129,901

This particular analysis of the incurred loss data indicates that a reserve of about $130 million is necessary to provide for unpaid loss and loss adjustment expenses from accident years 1994 through 2000. This is substantially different from the $147 million reserve estimate obtained through a paid loss projection. Any difference in estimates clearly raises questions that need to be investigated by the analyst in an attempt to reconcile the reserve estimate using two sets of loss data.

Claim Count Development

The pattern of claim reporting should be reviewed in the same fashion. The number of reported claims and calculated development factors are shown below.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	32,751	41,201	41,618	41,755	41,773	41,774	41,774
1995	33,736	39,528	39,926	40,044	40,072	40,072	
1996	27,067	32,740	33,084	33,183	33,209		
1997	24,928	29,796	30,074	30,169			
1998	25,229	31,930	32,281				
1999	17,632	21,801					
2000	13,609						

	12–24	24–36	36–48	48–60	60–72	72–84
1994	1.258	1.010	1.003	1.000	1.000	1.000
1995	1.172	1.010	1.003	1.001	1.000	
1996	1.210	1.011	1.003	1.001		
1997	1.195	1.009	1.003			
1998	1.266	1.011				
1999	1.236					
Average	1.223	1.010	1.003	1.001	1.000	1.000
Average Last 3	1.232	1.010	1.003	1.001		
Average Last 4	1.227	1.010	1.003			
Avg Exc Hi & Lo	1.225	1.010	1.003	1.001		
Weighted Average	1.221	1.010	1.003	1.001	1.000	1.000
Harmonic Mean	1.222	1.010	1.003	1.001	1.000	1.000

Assume the analyst chose the following development patterns as the most likely characterizations of development of total claims on those accident years to their ultimate reported level.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.258	1.010	1.003	1.000	1.000	1.000	**1.000**	**1.000**
1995	1.172	1.010	1.003	1.001	1.000	**1.000**	**1.000**	**1.000**
1996	1.210	1.011	1.003	1.001	**1.000**	**1.000**	**1.000**	**1.000**
1997	1.195	1.009	1.003	**1.001**	**1.000**	**1.000**	**1.000**	**1.001**
1998	1.266	1.011	**1.003**	**1.001**	**1.000**	**1.000**	**1.000**	**1.004**
1999	1.236	**1.012**	**1.003**	**1.001**	**1.000**	**1.000**	**1.000**	**1.016**
2000	**1.200**	**1.012**	**1.003**	**1.001**	**1.000**	**1.000**	**1.000**	**1.219**

Based on factors chosen above, the complete projection of incurred loss counts by accident year may be completed.

Accident Year	Age in months: 12	24	36	48	60	72	84	Ultimate	Reported	Unreported
1994	32,751	41,201	41,618	41,755	41,773	41,774	41,774	**41,774**	41,774	0
1995	33,736	39,528	39,926	40,044	40,072	40,072	**40,072**	**40,072**	40,072	0
1996	27,067	32,740	33,084	33,183	33,209	**33,209**	**33,209**	**33,209**	33,209	0
1997	24,928	29,796	30,074	30,169	**30,199**	**30,199**	**30,199**	**30,199**	30,169	30
1998	25,229	31,930	32,281	**32,378**	**32,410**	**32,410**	**32,410**	**32,410**	32,281	129
1999	17,632	21,801	**22,063**	**22,129**	**22,151**	**22,151**	**22,151**	**22,151**	21,801	350
2000	13,609	**16,331**	**16,527**	**16,576**	**16,593**	**16,593**	**16,593**	**16,593**	13,609	2,984
								Total Unreported		3,493

This analysis implies almost 3,500 claims remain to be reported. By itself, this analysis is useful as an indicator of true IBNR reporting. However, the projected ultimate claims may also be used to reduce the paid and incurred triangles to an average basis. Note that these incurred counts include those claims closed without loss payment.

It is possible to project the net claim count after claims closed without payment are excluded. Let the following triangle represent the history of claims reported that are closed with no loss payment (CNP).

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	2,646	6,285	6,935	7,240	7,353	7,393	7,412
1995	3,142	6,529	7,053	7,308	7,411	7,465	
1996	2,752	5,366	5,840	6,050	6,185		
1997	2,343	4,744	5,132	5,400			
1998	2,238	4,666	5,375				
1999	1,749	3,458					
2000	1,246						

Below is the percentage of claims without payment relative to total reported claims. For this line of business, it appears about 18% of the claims reported will be closed with no indemnity payment.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	8.1%	15.3%	16.7%	17.3%	17.6%	17.7%	17.7%
1995	9.3%	16.5%	17.7%	18.2%	18.5%	18.6%	
1996	10.2%	16.4%	17.7%	18.2%	18.6%		
1997	9.4%	15.9%	17.1%	17.9%			
1998	8.9%	14.6%	16.7%				
1999	9.9%	15.9%					
2000	9.2%						

This triangular display may be completed to obtain an ultimate estimate of the percent of reported claims closed with no indemnity payment, or alternatively, as shown below, the reported claims could be reduced for the claims closed with no indemnity.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	30,105	34,916	34,683	34,515	34,420	34,381	34,362
1995	30,594	32,999	32,873	32,736	32,661	32,607	
1996	24,315	27,374	27,244	27,133	27,024		
1997	22,585	25,052	24,942	24,769			
1998	22,991	27,264	26,906				
1999	15,883	18,343					
2000	12,363						

Age-to-age development factors may be calculated for the data representing claims reported net of claims closed with no payment.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84
1994	1.160	0.993	0.995	0.997	0.999	0.999
1995	1.079	0.996	0.996	0.998	0.998	
1996	1.126	0.995	0.996	0.996		
1997	1.109	0.996	0.993			
1998	1.186	0.987				
1999	1.155					
Average	1.136	0.993	0.995	0.997	0.999	0.999
Average Last 3	1.150	0.993	0.995	0.997		
Average Last 4	1.144	0.994	0.995			
Avg Exc Hi & Lo	1.138	0.995	0.996	0.997		
Weighted Average	1.133	0.994	0.995	0.997	0.999	0.999
Harmonic Mean	1.135	0.993	0.995	0.997	0.998	0.999

Assume the analyst chose the following development patterns as the most likely to ultimate settlement.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.160	0.993	0.995	0.997	0.999	0.999	**0.999**	**0.999**
1995	1.079	0.996	0.996	0.998	0.998	**0.999**	**0.999**	**0.998**
1996	1.126	0.995	0.996	0.996	**0.998**	**0.999**	**0.999**	**0.996**
1997	1.109	0.996	0.993	**0.996**	**0.998**	**0.999**	**0.999**	**0.992**
1998	1.186	0.987	**0.994**	**0.996**	**0.998**	**0.999**	**0.999**	**0.986**
1999	1.155	**0.996**	**0.994**	**0.996**	**0.998**	**0.999**	**0.999**	**0.982**
2000	**1.120**	**0.996**	**0.994**	**0.996**	**0.998**	**0.999**	**0.999**	**1.100**

Based on the development factors selected above, the complete projection of the net number of incurred claims by accident year may be obtained.

Accident Year	Age in months: 12	24	36	48	60	72	84	Ultimate	Reported	Unreported
1994	30,105	34,916	34,683	34,515	34,420	34,381	34,362	**34,328**	34,362	−34
1995	30,594	32,999	32,873	32,736	32,661	32,607	**32,574**	**32,542**	32,607	−65
1996	24,315	27,374	27,244	27,133	27,024	**26,970**	**26,943**	**26,916**	27,024	−108
1997	22,585	25,052	24,942	24,769	**24,670**	**24,621**	**24,596**	**24,571**	24,769	−198
1998	22,991	27,264	26,906	**26,745**	**26,638**	**26,584**	**26,558**	**26,531**	26,906	−375
1999	15,883	18,343	**18,270**	**18,160**	**18,087**	**18,051**	**18,033**	**18,015**	18,343	−328
2000	12,363	**13,847**	**13,791**	**13,701**	**13,654**	**13,626**	**13,613**	**13,599**	12,363	1,236
								Total Unreported (Net of CNP)		128

A negative number of unreported claims means the number of reported claims that will eventually close without payment is larger than the gross number of unreported claims. Only accident year 2000 will have a net increase in the number of reported claims net of closed with no payment claims.

Average Paid Claim Projection

A history of average paid claim amounts can be obtained by dividing the paid amounts by the number of claims closed with payments.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	968	1,254	1,631	1,894	2,090	2,218	2,281
1995	973	1,451	1,859	2,236	2,415	2,506	
1996	1,064	1,554	1,958	2,273	2,480		
1997	1,160	1,632	2,117	2,573			
1998	1,183	1,745	2,439				
1999	1,342	2,014					
2000	1,071						

Note that these average paid amounts will represent both average closing payment, plus any amounts paid on claims before closure. In some lines, such as workers compensation, these interim payments can be substantial. In those cases, this technique may not be as useful as for lines that close with a single payment.

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult
1994	1.296	1.300	1.161	1.104	1.061	1.029	
1995	1.491	1.281	1.203	1.080	1.038		
1996	1.460	1.260	1.161	1.091			
1997	1.406	1.297	1.216				
1998	1.476	1.397					
1999	1.500						
Average	1.438	1.307	1.185	1.092	1.050	1.029	
Average Last 3	1.461	1.318	1.193	1.092			
Average Last 4	1.461	1.309	1.185				
Avg Exc Hi & Lo	1.458	1.293	1.182	1.091			
Weighted Average	1.442	1.310	1.186	1.091	1.048	1.029	
Harmonic Mean	1.436	1.306	1.185	1.092	1.049	1.029	

Assume the analyst chose the following development patterns below. Note that the 12–24 factor on the 2000 year has been chosen to be quite high to compensate for the low average amount of $1,071 paid during 2000.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.296	1.300	1.161	1.104	1.061	1.029	**1.010**	**1.010**
1995	1.491	1.281	1.203	1.080	1.038	**1.020**	**1.010**	**1.030**
1996	1.460	1.260	1.161	1.091	**1.045**	**1.020**	**1.010**	**1.077**
1997	1.406	1.297	1.216	**1.090**	**1.045**	**1.020**	**1.010**	**1.173**
1998	1.476	1.397	**1.200**	**1.090**	**1.045**	**1.020**	**1.010**	**1.408**
1999	1.500	**1.400**	**1.200**	**1.090**	**1.045**	**1.020**	**1.010**	**1.971**
2000	**1.600**	**1.400**	**1.200**	**1.090**	**1.045**	**1.020**	**1.010**	**3.154**

Based on the factors chosen above the complete projection of average paid loss by accident year may be completed.

Accident Year	Age in months: 12	24	36	48	60	72	84	Ultimate Average
1994	968	1,254	1,631	1,894	2,090	2,218	2,281	**2,304**
1995	973	1,451	1,859	2,236	2,415	2,506	**2,556**	**2,582**
1996	1,064	1,554	1,958	2,273	2,480	**2,591**	**2,643**	**2,670**
1997	1,160	1,632	2,117	2,573	**2,805**	**2,931**	**2,990**	**3,020**
1998	1,183	1,745	2,439	**2,926**	**3,190**	**3,333**	**3,400**	**3,434**
1999	1,342	2,014	**2,819**	**3,383**	**3,688**	**3,854**	**3,931**	**3,970**
2000	1,071	**1,714**	**2,399**	**2,879**	**3,138**	**3,280**	**3,345**	**3,379**

Accident Year	Ultimate Average	Ultimate Counts	Ultimate Cost	Paid to Date	Reserve
1994	2,304	34,328	79,092	78,224	868
1995	2,582	32,542	84,023	81,287	2,736
1996	2,670	26,916	71,866	66,402	5,464
1997	3,020	24,571	74,204	62,347	11,857
1998	3,434	26,531	91,107	62,832	28,275
1999	3,970	18,015	71,520	33,568	37,952
2000	3,379	13,599	45,951	11,346	34,605
				Total Reserve	121,757

The 2000 average paid claim at ultimate appears to be completely unreasonable at $3,379 per claim, compared to past values projected. A review of annual increases in the ultimate average paid amount follows:

Accident Year	Ultimate Average	Change
1994	2,304	
1995	2,582	12.1%
1996	2,670	3.4%
1997	3,020	13.1%
1998	3,434	13.7%
1999	3,970	15.6%
2000	3,379	−14.9%

This indicates that a 12% increase from the 1999 value might be more in line with past experience. Using this increase yields an average payment of $4,446 per claim closed.

Accident Year	Ultimate Average	Ultimate Counts	Ultimate Cost	Paid to Date	Reserve
1994	2,304	34,328	79,092	78,224	868
1995	2,582	32,542	84,023	81,287	2,736
1996	2,670	26,916	71,866	66,402	5,464
1997	3,020	24,571	74,204	62,347	11,857
1998	3,434	26,531	91,107	62,832	28,275
1999	3,970	18,015	71,520	33,568	37,952
2000	4,446	13,599	60,461	11,346	49,115
				Total Reserve	136,267

The resulting reserve estimate based on average paid data has allowed us to correct for an obvious aberration in paid claims.

This aberration was not noticeable from the aggregate paid or incurred histories. This provides an example of the necessity of using several methods in investigating reserve needs.

Average Incurred Claim Projection

A similar projection can be made of the average amount of loss incurred per reported claim less claim counts closed with no payment. This amount and calculated development factors are shown below.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	1,948	2,142	2,229	2,257	2,345	2,393	2,397
1995	2,083	2,410	2,546	2,608	2,699	2,681	
1996	2,130	2,491	2,562	2,569	2,592		
1997	1,777	2,713	3,013	3,147			
1998	2,421	2,945	3,269				
1999	2,733	3,137					
2000	2,330						

	12–24	24–36	36–48	48–60	60–72	72–84
1994	1.100	1.041	1.012	1.039	1.020	1.002
1995	1.157	1.056	1.024	1.035	0.993	
1996	1.170	1.029	1.003	1.009		
1997	1.527	1.110	1.045			
1998	1.216	1.110				
1999	1.148					
Average	1.220	1.069	1.021	1.028	1.007	1.002
Average Last 3	1.297	1.083	1.024	1.028		
Average Last 4	1.265	1.076	1.021			
Avg Exc Hi & Lo	1.173	1.069	1.018	1.035		
Weighted Average	1.210	1.072	1.022	1.027	1.006	1.002
Harmonic Mean	1.212	1.069	1.021	1.028	1.006	1.002

Assume the analyst chose the following development patterns as the most likely outcome.

Accident Year	12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.100	1.041	1.012	1.039	1.020	1.002	**1.000**	**1.000**
1995	1.157	1.056	1.024	1.035	0.993	**1.003**	**1.000**	**1.003**
1996	1.170	1.029	1.003	1.009	**1.000**	**1.003**	**1.000**	**1.003**
1997	1.527	1.110	1.045	**1.025**	**1.000**	**1.003**	**1.000**	**1.028**
1998	1.216	1.110	**1.025**	**1.025**	**1.000**	**1.003**	**1.000**	**1.054**
1999	1.148	**1.100**	**1.025**	**1.025**	**1.000**	**1.003**	**1.000**	**1.159**
2000	**1.300**	**1.100**	**1.025**	**1.025**	**1.000**	**1.003**	**1.000**	**1.507**

Based on factors chosen above, the complete projection of average incurred losses by accident year may be completed.

Accident Year	12	24	36	48	60	72	84	Ultimate Average
1994	1,948	2,142	2,229	2,257	2,345	2,393	2,397	**2,397**
1995	2,083	2,410	2,546	2,608	2,699	2,681	**2,689**	**2,689**
1996	2,130	2,491	2,562	2,569	2,592	**2,592**	**2,600**	**2,600**
1997	1,777	2,713	3,013	3,147	**3,226**	**3,226**	**3,235**	**3,235**
1998	2,421	2,945	3,269	**3,351**	**3,435**	**3,435**	**3,445**	**3,445**
1999	2,733	3,137	**3,451**	**3,537**	**3,626**	**3,626**	**3,637**	**3,637**
2000	2,330	**3,028**	**3,331**	**3,415**	**3,500**	**3,500**	**3,510**	**3,510**

Accident Year	Ultimate Average	Ultimate Counts	Ultimate Cost	Paid to Date	Reserve
1994	2,397	34,328	82,284	78,224	4,060
1995	2,689	32,542	87,505	81,287	6,218
1996	2,600	26,916	69,982	66,402	3,580
1997	3,235	24,571	79,487	62,347	17,140
1998	3,445	26,531	91,399	62,832	28,567
1999	3,637	18,015	65,521	33,568	31,953
2000	3,510	13,599	47,732	11,346	36,386
				Total Reserve	127,904

The ultimate reserve estimate resulting from this average incurred history indicates a reserve of $128 million is needed. However, the same problem noted with the 2000 paid claim averages is noted here. The 2000 average ultimate incurred claim of $3,022 does not look realistic based on the steady increases of average incurred losses since 1994.

Year	Average	Change
1994	2,397	
1995	2,681	11.8%
1996	2,600	−3.0%
1997	3,235	24.4%
1998	3,445	6.5%
1999	3,637	5.6%
2000	3,022	−16.9%

Since the average incurred amounts have been increasing at an average rate of 8.7% from 1994 to 1999, this would imply an average incurred of $3,637 multiplied by 1.087 or $3,953. With 13,599 claims at ultimate settlement, the total incurred for 2000 will be $53,757,000, or almost $6 million dollars higher than our original projection. This implies a reserve of $134 million is required to provide for the ultimate disposition of all claims.

Summary of Results

The triangular loss development factor methods applied to the four different loss statistics above yields the following different sets of ultimate accident year incurred losses.

Accident Year	Paid Devel't	Incurred Devel't	Average Paid	Average Incurred
1994	82,370	83,196	79,092	82,284
1995	88,163	88,287	84,023	87,505
1996	76,340	70,741	71,866	69,982
1997	78,846	80,301	74,204	79,487
1998	102,293	92,430	91,107	91,399
1999	69,344	66,215	71,520	65,521
2000	45,939	44,737	60,461	53,757

Note that there is substantial variation from method to method. The analyst must still choose a point estimate in some fashion. This choice will be dependent on the supporting information the analyst has developed concerning company operations as well as further statistical tests discussed below. However, at this point there is strong reason to suspect that both the aggregate paid and incurred loss projection methods have substantially underestimated the 2000 ultimate loss amounts. The final selection of ultimate losses is discussed in detail after additional methods are reviewed.

Reserve Development Methods

The triangular methods have used either paid or incurred data, exclusively, but have not made any use of historical relationships between paid amounts and case reserved amounts. The reserve development method attempts to analyze the adequacy of case reserves based on the history of payments against those case reserves.

Report Year Approach

Report year data are organized by year of report to the company, as opposed to year of loss for accident year. This array freezes the inventory of loss to study only those cases actually reported to a company during a calendar year. Thus, there can be no late reported claims on a report year compilation. This method is discussed in Marker and Mohl (1980) and Mohl (1987).

Case Loss Reserves by Report Year

Report	Age in months						
Year	12	24	36	48	60	72	84
1994	46,770	31,944	18,832	9,559	4,999	2,821	1,693
1995	53,422	36,588	21,214	11,345	8,049	3,701	
1996	41,802	28,899	15,798	9,560	5,403		
1997	40,334	28,266	18,312	8,724			
1998	47,500	35,455	22,225				
1999	42,219	27,221					
2000	30,416						

Incremental Paid by Report Year

Report Year	Age in months 12	24	36	48	60	72	84
1994	30,001	16,021	14,144	8,238	5,923	3,119	1,145
1995	29,421	18,081	16,904	10,811	4,942	2,930	
1996	26,601	17,078	13,169	7,522	4,739		
1997	24,981	15,251	12,665	9,465			
1998	27,595	18,196	17,687				
1999	25,886	17,700					
2000	15,220						

The fundamental idea of the reserve development method of reserve evaluation is to track the development of a case reserve amount into subsequent paid losses and remaining reserves. For instance, the $42,219 in reserves from report year 1999 cases has developed into $17,700 of paid loss during 2000, with $27,221 remaining in reserve as of the end of 2000. We are then interested in the amount we expect to be paid on the $27,221 reserve during the next 12 months. The entire liquidation pattern of the report year reserves can then be charted and used to evaluate the ultimate liquidation value of the report year case reserves.

Consider the following ratios of the amounts paid to open reserves in the prior calendar period:

Paid on Reserve Ratio by Report Year

Report Year	Age in months 24	36	48	60	72	84
1994	0.343	0.443	0.437	0.620	0.624	0.406
1995	0.338	0.462	0.510	0.436	0.364	
1996	0.409	0.456	0.476	0.496		
1997	0.378	0.448	0.517			
1998	0.383	0.499				
1999	0.419					
Average	0.378	0.461	0.485	0.517	0.494	0.406
Average Last 3	0.393	0.468	0.501	0.517		
Average Last 4	0.397	0.466	0.485			
Avg Exc Hi & Lo	0.378	0.455	0.493	0.496		

The following development patterns were chosen as the most likely to occur for paid on open reserve amounts.

Report Year	Age in months: 24	36	48	60	72	84	Ultimate
1994	0.343	0.443	0.437	0.620	0.624	0.406	**1.000**
1995	0.338	0.462	0.510	0.436	0.364	**0.400**	**1.000**
1996	0.409	0.456	0.476	0.496	**0.500**	**0.400**	**1.000**
1997	0.378	0.448	0.517	**0.500**	**0.500**	**0.400**	**1.000**
1998	0.383	0.499	**0.500**	**0.500**	**0.500**	**0.400**	**1.000**
1999	0.419	**0.500**	**0.500**	**0.500**	**0.500**	**0.400**	**1.000**
2000	**0.420**	**0.500**	**0.500**	**0.500**	**0.500**	**0.400**	**1.000**

Likewise, we can create the array of ratios of remaining in reserve compared to the open reserve of the prior calendar period and complete the projection of ultimate reserve outcomes. Note that this statistic is the amount remaining on reserve so that the 84 month to ultimate development must be zero. Thus selection of tail factors is not an issue in this reserve projection method.

Remaining in Reserve Ratio by Report Year

Report Year	Age in months: 24	36	48	60	72	84
1994	0.683	0.590	0.508	0.523	0.564	0.600
1995	0.685	0.580	0.535	0.709	0.460	
1996	0.691	0.547	0.605	0.565		
1997	0.701	0.648	0.476			
1998	0.746	0.627				
1999	0.645					
Average	0.692	0.598	0.531	0.599	0.512	0.600
Average Last 3	0.697	0.607	0.539	0.599		
Average Last 4	0.696	0.600	0.531			
Avg Exc Hi & Lo	0.690	0.599	0.521	0.565		

The following development patterns were chosen as the most likely to occur for remaining reserve in reserve amounts.

Report			Age in months:				
Year	24	36	48	60	72	84	Ultimate
1994	0.683	0.590	0.508	0.523	0.564	0.600	**0.000**
1995	0.685	0.580	0.535	0.709	0.460	**0.600**	**0.000**
1996	0.691	0.547	0.605	0.565	**0.500**	**0.600**	**0.000**
1997	0.701	0.648	0.476	**0.600**	**0.500**	**0.600**	**0.000**
1998	0.746	0.627	**0.530**	**0.600**	**0.500**	**0.600**	**0.000**
1999	0.645	**0.635**	**0.530**	**0.600**	**0.500**	**0.600**	**0.000**
2000	**0.690**	**0.635**	**0.530**	**0.600**	**0.500**	**0.600**	**0.000**

The sum of the Paid on Reserve and the Remaining Reserve on Reserve ratios gives a history of the amount developed on open reserves in the prior calendar period.

Incremental Loss Development by Report Year

Report			Age in months				
Year	24	36	48	60	72	84	Ultimate
1994	1.026	1.032	0.945	1.143	1.188	1.006	**1.000**
1995	1.023	1.042	1.044	1.145	0.824	**1.000**	**1.000**
1996	1.100	1.002	1.081	1.061	**1.000**	**1.000**	**1.000**
1997	1.079	1.096	0.993	**1.100**	**1.000**	**1.000**	**1.000**
1998	1.129	1.126	**1.030**	**1.100**	**1.000**	**1.000**	**1.000**
1999	1.064	**1.135**	**1.030**	**1.100**	**1.000**	**1.000**	**1.000**
2000	**1.110**	**1.135**	**1.030**	**1.100**	**1.000**	**1.000**	**1.000**

Once development factor scenarios have been constructed, it is necessary to complete the settlement projections in terms of dollar amounts. An example using the 1998 report year is the most direct illustration of the completion technique.

There are $22.2 million of case reserves outstanding on the 1998 report year as of the end of 2000. The completed development factors indicate that 53% of this reserve will remain in reserve, while 50% of the reserve will be paid out, for a total adverse development of 3% on the 1998 report year cases during 2001.

We can complete the projection of remaining reserves, by using the relationships as above.

Case Loss Reserves by Report Year

Report	Age in months							
Year	12	24	36	48	60	72	84	Ultimate
1994	46,770	31,944	18,832	9,559	4,999	2,821	1,693	**0**
1995	53,422	36,588	21,214	11,345	8,049	3,701	**2,221**	**0**
1996	41,802	28,899	15,798	9,560	5,403	**2,702**	**1,621**	**0**
1997	40,334	28,266	18,312	8,724	**5,234**	**2,617**	**1,570**	**0**
1998	47,500	35,455	22,225	**11,779**	**7,068**	**3,534**	**2,120**	**0**
1999	42,219	27,221	**17,285**	**9,161**	**5,497**	**2,748**	**1,649**	**0**
2000	30,416	**20,987**	**13,327**	**7,063**	**4,238**	**2,119**	**1,271**	**0**

Based on the amounts remaining in reserve, annual paid amounts by report period can be derived simply by using the selected paid on reserve factors. For instance, on the $22,225 of reserves open at the end of 2000 on 1998 reported claims, 50% will be paid out in 2001. This is $11.1 million of paid claims.

Incremental Paid Losses by Report Year

Report	Age in months							
Year	12	24	36	48	60	72	84	Ultimate
1994	30,001	16,021	14,144	8,238	5,923	3,119	1,145	**1,693**
1995	29,421	18,081	16,904	10,811	4,942	2,930	**1,480**	**2,221**
1996	26,601	17,078	13,169	7,522	4,739	**2,702**	**1,081**	**1,621**
1997	24,981	15,251	12,665	9,465	**4,362**	**2,617**	**1,047**	**1,570**
1998	27,595	18,196	17,687	**11,113**	**5,890**	**3,534**	**1,414**	**2,120**
1999	25,886	17,700	**13,611**	**8,643**	**4,581**	**2,748**	**1,099**	**1,649**
2000	15,220	**12,775**	**10,494**	**6,663**	**3,532**	**2,119**	**848**	**1,271**

Thus the incremental paid loss projections accumulate to paid losses by report year as shown below.

Cumulative Paid Losses by Report Year

Report Year	12	24	36	48	60	72	84	Ultimate
1994	30,001	46,022	60,166	68,404	74,327	77,446	78,591	**80,284**
1995	29,421	47,502	64,406	75,217	80,159	83,089	**84,569**	**86,790**
1996	26,601	43,679	56,848	64,370	69,109	**71,811**	**72,891**	**74,512**
1997	24,981	40,232	52,897	62,362	**66,724**	**69,341**	**70,388**	**71,958**
1998	27,595	45,791	63,478	**74,591**	**80,480**	**84,014**	**85,427**	**87,548**
1999	25,886	43,586	**57,197**	**65,839**	**70,420**	**73,168**	**74,267**	**75,917**
2000	15,220	**27,995**	**38,488**	**45,152**	**48,683**	**50,802**	**51,650**	**52,921**

(Age in Months: 12–84)

Report Year	Ultimate	Unpaid	Case Reserve	Case Reserve Development	Case Reserve Adequacy
1994	80,284	1,693	1,693	0	100.0%
1995	86,790	3,701	3,701	0	100.0%
1996	74,512	5,403	5,403	0	100.0%
1997	71,958	9,596	8,724	872	90.9%
1998	87,548	24,070	22,225	1,845	92.3%
1999	75,917	32,331	27,221	5,110	84.2%
2000	52,921	37,701	30,416	7,285	80.7%
	Total Case Reserve Development			15,112	

This analysis indicates a case reserving pattern with case reserves deficient 20% at age 12 months, 15% at age 24 months, and 8–10% at age 36 to 48 months. After 60 months we expect case reserves to be adequate. Note that report year analysis can only evaluate case reserve adequacy. The IBNR liability for this line would require additional analysis.

Accident Year Approach

While the reserve development method is simplest to interpret on report year data, it may also be used on accident period data on an accident year basis. New claims will enter the claim inventory so that the interpretation of paid on open reserve history is made more difficult. In order to apply the method to accident period data one must be able to assume that IBNR claim activity is related consistently to claims already reported. This assumption is a reasonable one for most lines of business that have the bulk of their claims reported in the first accident period to serve as a stable base for IBNR projections.

Assume we take the accident year paid and incurred triangles that were presented above.

Remaining Case Reserves by Accident Year

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	36,038	34,740	23,022	13,776	9,471	6,330	4,148
1995	41,678	35,542	24,943	13,525	10,076	6,126	
1996	31,613	29,028	18,483	9,277	3,639		
1997	20,846	30,623	24,753	15,600			
1998	35,110	37,398	25,129				
1999	26,400	23,979					
2000	17,454						

Incremental Paid by Accident Year

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	22,603	17,461	14,237	9,813	7,143	4,693	2,274
1995	22,054	21,916	14,767	13,104	6,235	3,211	
1996	20,166	18,981	12,172	9,098	5,985		
1997	19,297	18,058	13,036	11,956			
1998	20,555	22,343	19,934				
1999	17,001	16,567					
2000	11,346						

Consider the following ratios of paid on open reserves.

Paid on Reserve Ratio by Accident Year

Accident Year	Age in months 24	36	48	60	72	84
1994	0.485	0.410	0.426	0.519	0.496	0.359
1995	0.526	0.415	0.525	0.461	0.319	
1996	0.600	0.419	0.492	0.645		
1997	0.866	0.426	0.483			
1998	0.636	0.533				
1999	0.628					

Accident Year	Age in months 24	36	48	60	72	84
Average	0.623	0.441	0.482	0.542	0.407	0.359
Average Last 3	0.710	0.459	0.500	0.542		
Average Last 4	0.683	0.448	0.482			
Avg Exc Hi & Lo	0.598	0.420	0.488	0.519		

Likewise, we can create the array of ratios of remaining in reserve.

Remaining in Reserve Ratio by Accident Year

Accident Year	Age in months 24	36	48	60	72	84
1994	0.964	0.663	0.598	0.688	0.668	0.655
1995	0.853	0.702	0.542	0.745	0.608	
1996	0.918	0.637	0.502	0.392		
1997	1.469	0.808	0.630			
1998	1.065	0.672				
1999	0.908					
Average	1.030	0.696	0.568	0.608	0.638	0.655
Average Last 3	1.147	0.706	0.558	0.608		
Average Last 4	1.090	0.705	0.568			
Avg Exc Hi & Lo	0.964	0.679	0.570	0.688		

Note that the sum of these two ratios, gives a history of the amount developed on reserves, now including IBNR claims.

Accident Year	Age in months: 24	36	48	60	72	84
1994	1.448	1.073	1.025	1.206	1.164	1.015
1995	1.379	1.117	1.068	1.206	0.927	
1996	1.519	1.056	0.994	1.037		
1997	2.335	1.234	1.113			
1998	1.702	1.205				
1999	1.536					

In order to complete the projection of ultimate reserve outcomes both the paid on reserve and the remaining reserve on reserve developments must be projected.

Assume the analyst chose the following development patterns as the most likely to be paid as a percent of the reserve over the next 12-month periods.

Accident	Age in months:						
Year	24	36	48	60	72	84	Ultimate
1994	0.485	0.410	0.426	0.519	0.496	0.359	**1.010**
1995	0.526	0.415	0.525	0.461	0.319	**0.430**	**1.010**
1996	0.600	0.419	0.492	0.645	**0.510**	**0.430**	**1.010**
1997	0.866	0.426	0.483	**0.500**	**0.510**	**0.430**	**1.010**
1998	0.636	0.533	**0.495**	**0.500**	**0.510**	**0.430**	**1.010**
1999	0.628	**0.460**	**0.495**	**0.500**	**0.510**	**0.430**	**1.010**
2000	**0.600**	**0.460**	**0.495**	**0.500**	**0.510**	**0.430**	**1.010**

The same exercise must be carried out for the remaining reserve on reserve ratios on an accident year basis. Assume the analyst chose the following development patterns of reserves as a ratio to prior accident period reserves.

Accident	Age in months:						
Year	24	36	48	60	72	84	Ultimate
1994	0.964	0.663	0.598	0.688	0.668	0.655	**0.000**
1995	0.853	0.702	0.542	0.745	0.608	**0.655**	**0.000**
1996	0.918	0.637	0.502	0.392	**0.650**	**0.655**	**0.000**
1997	1.469	0.808	0.630	**0.600**	**0.650**	**0.655**	**0.000**
1998	1.065	0.672	**0.600**	**0.600**	**0.650**	**0.655**	**0.000**
1999	0.908	**0.750**	**0.600**	**0.600**	**0.650**	**0.655**	**0.000**
2000	**1.000**	**0.750**	**0.600**	**0.600**	**0.650**	**0.655**	**0.000**

We can complete the projection of remaining reserves.

Case Loss Reserves by Accident Year

Accident	Age in months							
Year	12	24	36	48	60	72	84	Ultimate
1994	36,038	34,740	23,022	13,776	9,471	6,330	4,148	**0**
1995	41,678	35,542	24,943	13,525	10,076	6,126	**4,013**	**0**
1996	31,613	29,028	18,483	9,277	3,639	**2,365**	**1,549**	**0**
1997	20,846	30,623	24,753	15,600	**9,360**	**6,084**	**3,985**	**0**
1998	35,110	37,398	25,129	**15,077**	**9,046**	**5,880**	**3,852**	**0**
1999	26,400	23,979	**17,984**	**10,791**	**6,474**	**4,208**	**2,756**	**0**
2000	17,454	**17,454**	**13,091**	**7,854**	**4,713**	**3,063**	**2,006**	**0**

Based on the amounts remaining in reserve, annual paid amounts by accident period can be derived simply by using the selected paid on reserve factors.

Incremental Paid Losses by Accident Year

Accident	Age in months							
Year	12	24	36	48	60	72	84	Ultimate
1994	22,603	17,461	14,237	9,813	7,143	4,693	2,274	**4,189**
1995	22,054	21,916	14,767	13,104	6,235	3,211	**2,634**	**4,053**
1996	20,166	18,981	12,172	9,098	5,985	**1,856**	**1,017**	**1,565**
1997	19,297	18,058	13,036	11,956	**7,800**	**4,774**	**2,616**	**4,025**
1998	20,555	22,343	19,934	**12,439**	**7,539**	**4,614**	**2,528**	**3,890**
1999	17,001	16,567	**11,030**	**8,902**	**5,395**	**3,302**	**1,810**	**2,784**
2000	11,346	**10,472**	**8,029**	**6,480**	**3,927**	**2,403**	**1,317**	**2,026**

Then the incremental paid loss projections can be accumulated to yield ultimate payment estimates by accident year.

Paid Losses by Accident Year

Accident	Age in months									
Year	12	24	36	48	60	72	84	Ultimate	Paid	Reserve
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224	**82,413**	78,224	4,189
1995	22,054	43,970	58,737	71,841	78,076	81,287	**83,921**	**87,974**	81,287	6,687
1996	20,166	39,147	51,319	60,417	66,402	**68,258**	**69,275**	**70,840**	66,402	4,438
1997	19,297	37,355	50,391	62,347	**70,147**	**74,921**	**77,537**	**81,562**	62,347	19,215
1998	20,555	42,898	62,832	**75,271**	**82,810**	**87,423**	**89,952**	**93,842**	62,832	31,010
1999	17,001	33,568	**44,598**	**53,501**	**58,896**	**62,198**	**64,007**	**66,791**	33,568	33,223
2000	11,346	**21,818**	**29,847**	**36,327**	**40,254**	**42,658**	**43,975**	**46,001**	11,346	34,655
								Total Reserve		133,417

This analysis results in a total indicated reserve requirement of $133 million. Note this estimate is closest to the $130 million estimate based on the method that used aggregate incurred historical data.

Bornhuetter–Ferguson (BF) Method

There are many cases where relying solely on paid or incurred loss development methods may be inappropriate. The development methods may produce unreliable results for a new line of business with little historical information or a volatile line of business that is subject to very large occasional losses, such as contract bond surety. Also fitting into this category are cases where losses are reported over a long period of time (10 years or longer) and have very little loss reported in the first two to three years such as excess insurance and reinsurance. In situations such as these, a method that offers a blend of stability and responsiveness is more appropriate.

The Bornhuetter–Ferguson (BF) method estimates ultimate loss by adding together actual reported loss with expected future incurred development. Expected future incurred development relies on expected losses and selected loss development factors. Assume we have the following incurred loss data and selected loss development factors by acident year.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	58,641	74,804	77,323	77,890	80,728	82,280	82,372
1995	63,732	79,512	83,680	85,366	88,152	87,413	
1996	51,779	68,175	69,802	69,694	70,041		
1997	40,143	67,978	75,144	77,947			
1998	55,665	80,296	87,961				
1999	43,401	57,547					
2000	28,800						

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.276	1.034	1.007	1.036	1.019	1.001	**1.010**	**1.010**
1995	1.248	1.052	1.020	1.033	0.992	**1.000**	**1.010**	**1.010**
1996	1.317	1.024	0.998	1.005	**1.000**	**1.000**	**1.010**	**1.010**
1997	1.693	1.105	1.037	**1.020**	**1.000**	**1.000**	**1.010**	**1.030**
1998	1.442	1.095	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.051**
1999	1.326	**1.095**	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.151**
2000	**1.350**	**1.095**	**1.020**	**1.020**	**1.000**	**1.000**	**1.010**	**1.553**

In this example, the 1996 accident year is 60 months old. The development from 60 months to ultimate is given by the factor of 1.010. The age to ultimate factor can be interpreted to mean that the accident year is 99% reported as of 60 months. Likewise, the 2000 accident year as of 12 months is expected to be 64.4% or 1/1.553 reported. This implies that the 2000 accident year is 35.6% unreported as of 12/31/2000.

Clearly, if you knew what the 2000 accident year ultimate losses were going to be, you would set aside 35.6% of the ultimate loss estimate as the appropriate IBNR reserve as of 12/31/2000. This is the amount of future incurred loss that has yet to be reported on the 2000 accident year. Although we won't know what the ultimate value of the 2000 accident year will be for many years, we can calculate the expected value by multiplying earned premium by an expected loss ratio (ELR). The expected loss ratio can be estimated by reviewing the pricing assumptions, historical results, or industry results for the line of business.

If the earned premiums and expected loss ratios for each year are given below, we can apply this method to our incurred losses.

Bornhuetter–Ferguson (BF) Method

Accident Year	(1) Earned Premium	(2) ELR	(3) = (1) × (2) Expected Ultimate	(4) Ultimate Develop't	(5) = 1.0– 1.0/(4) Remaining Develop't	(6) = (5) × (3) Expected Unreported	(7) Reported to Date	(8) = (6) + (7) Total Ultimate	(9) Paid to Date	(10) = (8) – (9) Reserve
1994	101,946	80%	81,557	1.010	1.0%	816	82,372	83,188	78,224	4,964
1995	112,068	80%	89,654	1.010	1.0%	897	87,413	88,310	81,287	7,023
1996	97,796	80%	78,237	1.010	1.0%	782	70,041	70,823	66,402	4,421
1997	101,930	78%	79,505	1.030	2.9%	2,306	77,947	80,253	62,347	17,906
1998	107,357	78%	83,738	1.051	4.9%	4,103	87,961	92,064	62,832	29,232
1999	84,531	78%	65,934	1.151	13.1%	8,637	57,547	66,184	33,568	32,616
2000	57,697	78%	45,004	1.553	35.6%	16,021	28,800	44,821	11,346	33,475
								Total Reserve		129,637

One advantage of the BF method is that it avoids overreacting to unexpected losses. For illustration, assume that actual losses in accident year 2000 are 35,000 rather than 28,800. The incurred loss development method would produce an ultimate loss estimate of 54,355 (= 35,000 × 1.553) well above the expected loss estimate of 45,004. The Bornhuetter–Ferguson method would produce an estimate of 51,025. This estimate is the weighted result of incurred loss development method and the expected loss estimate where the weights are based on the reciprocal of the age-to-ultimate factor.

$$51{,}025 = 54{,}355 \times (1/1.553) + 45{,}004 \times (1 - 1/1.553)$$

If this had been an older accident year, say 1999, the loss development factor would be 1.151. In this case, 86.9% weight would be given to the loss development method rather than 64.4% weight as shown for accident year 2000. Generally, as an accident year matures, the intial expected incurred loss estimate becomes less important and actual reported loss experience becomes more important. In this fashion, the BF method produces a blend of stability and responsiveness in the loss reserve estimate.

Loss Adjustment Expenses

An extremely important part of the loss reserve evaluation process is the evaluation of loss adjustment expense liabilities. Loss adjustment expenses (LAE) can be split into two components, commonly called allocated loss adjustment expenses (ALAE) and unallocated loss adjustment expenses (ULAE). Until 1998, the determining factor in whether or not a loss expense was considered ALAE or ULAE was whether the expense could be assigned to an individual claim. If an expense could be assigned to a claim, it was considered ALAE.

This approach made it difficult to make comparisons between companies. For example, claim adjuster fees are one type of loss adjustment expense. Claim adjusters are typically either company adjusters or independent adjusters. The distinguishing feature is whether or not the claim adjuster is employed by the company. Company adjuster expenses were typically considered ULAE and independent adjuster expenses were typically considered ALAE. So depending on whether the company uses its own employees or uses outside vendors, the same expense from the same type of activity could be classified in two different ways.

The National Association of Insurance Commissioners (NAIC) introduced new reporting requirements for loss adjustment expenses that became effective 1/1/1998. The goal is to have consistent reporting of expenses among companies. Allocated loss adjustment expenses were deemed by the NAIC to mean expenses, whether internal or external to the company, related to defense, litigation and medical cost containment. Unallocated loss adjustment expenses are considered to be all loss expenses not specifically defined as ALAE. All adjuster fees are considered ULAE. Effective with the 1999 Annual Statement, the NAIC changed the titles of these expenses to match the revised definitions. ALAE became "Defense and Cost Containment" expenses and ULAE became "Adjusting and Other" expenses.

The key in grouping expenses for reserving purposes is still whether or not the expenses are assigned to an individual claim. Significantly more analysis can be completed for those expenses that are assigned to an individual claim (allocated expenses) because more data exists. For instance, the accident date of the claim that generated the expense is known for an allocated expense, but unknown for an unallocated expense.

In evaluating any loss adjustment expense liabilities, the loss reserve analyst needs to know how the data is being defined and, if the definition has changed, how the data is impacted. Once the answers to these questions are known, any necessary adjustments can be made to the liability estimates.

Allocated Loss Adjustment Expenses

One approach to estimating allocated loss adjustment expense liabilities is to combine ALAE with losses and estimate the total liability. Generally, this approach is not desirable because the two development patterns may be quite different. Combining ALAE with losses is often similar to combining two non-homogeneous lines of business. Separate analyses of loss and allocated loss adjustment expense are also necessary to allow for monitoring of actual experience versus projected experience for each component.

The allocated loss adjustment expenses can often be further split into subcategories. The most important subcategory is attorneys' fees and court costs. It will often be conducive to obtaining better estimates of loss adjustment expense to develop legal expense separate from all other allocated expense items.

Case reserve estimates sometimes are not established for allocated loss adjustment expenses. This means that the actuary only has paid allocated loss adjustment expenses to work with. The allocated expense reserve is established on a bulk basis by actuarial estimates, or may be spread to cases by some formula approach. In either case, allocated paid amounts are the only meaningful history available for the analysis.

A common analysis procedure is to compare the allocated expenses paid to the paid losses on the same claims, and follow the development of the relationship of paid allocated expense to paid loss over time.

Assume the same paid loss history from our previous example.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224
1995	22,054	43,970	58,737	71,841	78,076	81,287	
1996	20,166	39,147	51,319	60,417	66,402		
1997	19,297	37,355	50,391	62,347			
1998	20,555	42,898	62,832				
1999	17,001	33,568					
2000	11,346						

We also have available the history of paid allocated loss expense by accident year.

Accident Year	Age in months: 12	24	36	48	60	72	84
1994	554	1,110	2,118	3,231	4,211	4,170	5,429
1995	485	1,244	2,256	3,578	4,567	5,202	
1996	446	1,104	1,981	2,973	3,785		
1997	405	953	1,809	2,905			
1998	388	1,025	2,161				
1999	357	843					
2000	216						

The relationship of paid allocated loss expense to paid loss is then derived as follows for this history.

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	2.45%	2.77%	3.90%	5.04%	5.91%	5.49%	6.94%
1995	2.20%	2.83%	3.84%	4.98%	5.85%	6.40%	
1996	2.21%	2.82%	3.86%	4.92%	5.70%		
1997	2.10%	2.55%	3.59%	4.66%			
1998	1.89%	2.39%	3.44%				
1999	2.10%	2.51%					
2000	1.90%						

Age-to-age development factors applying to the ratios of paid allocated loss adjustment expense are given below. We are trying to estimate the ultimate ratio of ALAE to loss. Once the ultimate ratio is chosen it can be applied to the estimate of ultimate losses to obtain an estimate of ultimate allocated loss adjustment expense. It often helps to think of these ratios of paid allocated expense to paid loss as the cost to settle $100 of loss. For example, in the 1994 accident year at 12/31/2000, $6.94 of allocated expenses has been paid for every $100 of paid loss.

The development triangle and selected averages of the development factors for these expense amounts are given below.

Accident	Age in months:						
Year	12–24	24–36	36–48	48–60	60–72	72–84	84–Ult
1994	1.130	1.408	1.292	1.173	0.929	1.264	
1995	1.286	1.358	1.297	1.174	1.094		
1996	1.275	1.369	1.275	1.158			
1997	1.216	1.407	1.298				
1998	1.266	1.439					
1999	1.196						
Average	1.228	1.396	1.291	1.168	1.012	1.264	
Average Last 3	1.226	1.405	1.290	1.168			
Average Last 4	1.238	1.393	1.291				
Avg Exc Hi & Lo	1.238	1.395	1.295	1.173			
Weighted Average	1.226	1.394	1.290	1.169	1.011	1.264	
Harmonic Mean	1.227	1.396	1.290	1.168	1.008	1.264	

Assume the analyst chose the following development patterns along the right diagonal as the most likely in the future.

Accident Year	Age in months: 12–24	24–36	36–48	48–60	60–72	72–84	84–Ult	Dev to Ult
1994	1.130	1.408	1.292	1.173	0.929	1.264	**1.010**	**1.010**
1995	1.286	1.358	1.297	1.174	1.094	**1.010**	**1.010**	**1.020**
1996	1.275	1.369	1.275	1.158	**1.025**	**1.010**	**1.010**	**1.046**
1997	1.216	1.407	1.298	**1.160**	**1.025**	**1.010**	**1.010**	**1.213**
1998	1.266	1.439	**1.295**	**1.160**	**1.025**	**1.010**	**1.010**	**1.571**
1999	1.196	**1.400**	**1.295**	**1.160**	**1.025**	**1.010**	**1.010**	**2.199**
2000	**1.300**	**1.400**	**1.295**	**1.160**	**1.025**	**1.010**	**1.010**	**2.859**

Based on development factors chosen above, paid allocated loss adjustment expense per $100 of loss by accident year may be projected to an ultimate basis.

Accident Year	Age in months: 12	24	36	48	60	72	84	Ultimate per $100
1994	2.45	2.77	3.90	5.04	5.91	5.49	6.94	**7.01**
1995	2.20	2.83	3.84	4.98	5.85	6.40	**6.46**	**6.53**
1996	2.21	2.82	3.86	4.92	5.70	**5.84**	**5.90**	**5.96**
1997	2.10	2.55	3.59	4.66	**5.40**	**5.54**	**5.60**	**5.65**
1998	1.89	2.39	3.44	**4.45**	**5.17**	**5.30**	**5.35**	**5.40**
1999	2.10	2.51	**3.52**	**4.55**	**5.28**	**5.41**	**5.47**	**5.52**
2000	1.90	**2.47**	**3.46**	**4.49**	**5.20**	**5.33**	**5.39**	**5.44**

Accident Year	Ultimate per $100	Ultimate Loss	Ultimate ALAE	Paid ALAE	ALAE Reserve
1994	7.01	83,196	5,832	5,429	403
1995	6.53	88,287	5,764	5,202	562
1996	5.96	70,741	4,216	3,785	431
1997	5.65	80,301	4,538	2,905	1,633
1998	5.40	92,430	4,993	2,161	2,832
1999	5.52	66,215	3,657	843	2,814
2000	5.44	44,821	2,439	216	2,223
				Total Reserve	10,898

This analysis indicates a reserve need of $10.9 million for allocated loss adjustment expenses.

The premise of the above analysis is that the relationship of paid allocated expense to paid loss dollars is usually fairly stable. The premise must be validated by the analyst in discussion with the insurer's management. For example, changes in defense strategies may distort the ratios. Sometimes significant ALAE dollars are spent on claims that will eventually close with no loss payment. One advantage of this method is that it does recognize the relationship of paid loss expense to loss. However, any errors made in estimating ultimate losses will be carried forward into the estimate of allocated loss adjustment expense.

A variation on the above method includes developing the additive increments to the allocated expense to loss ratios in place of multiplicative development of these ratios. If the ratios are very small at early maturities, the additive approach seems to be more stable.

Another common method of estimating the ultimate value of ALAE is to simply develop the paid allocated loss expense history. The drawback of this approach is that the estimate is not related to the ultimate level of losses, hence it could produce widely varying results in terms of allocated expense paid per $100 of claims paid. Clearly, the methods chosen will depend heavily on a review of the data and its characteristics, as well as an understanding of the insurer's operating characteristics with regard to handling of defense and other allocated expenses.

Unallocated Loss Adjustment Expenses

In addition to estimating the liability for allocated loss adjustment expenses, we must also estimate the liability for unallocated loss adjustment expenses. Most ULAE is the expense of operating a claims department and includes such items as claim adjuster fees, office rent, and utilities.

Unallocated loss adjustment expense tends not to be recorded in the same level of detail as allocated loss adjustment expense. Usually some type of internal allocation procedure is used to distribute calendar year ULAE expenses to line of business. This internal allocation procedure should reflect the claim activity that gave rise to the expense such as the number of claims incurred during the year, claims closed during the year, number of claims remaining open during the year, number of days claims are open, or number of payment or reserve transactions during the year. Ideally, standard costs are assigned to each transaction and total claim department costs are allocated in accordance to the distribution of standard costs. The analyst needs to understand how expenses are allocated to line of business and any changes that have occurred in the allocation procedure before meaningful numerical analysis of ULAE can be done.

Once unallocated loss expense payments have been assigned to lines of business, we can begin to estimate the reserve. The most common procedure is to estimate the amount of unallocated loss adjustment expense that is needed per $100 of claims payments.

Suppose the following unallocated loss expense payment history, by calendar year of payment.

Calendar Year	Paid ULAE	Paid Loss	Paid ULAE/ $100 Loss
1994	12,345	91,955	13.43
1995	13,826	100,576	13.75
1996	15,486	111,530	13.89
1997	17,344	130,708	13.27
1998	19,425	145,889	13.31
1999	21,756	164,051	13.26
2000	24,367	171,397	14.22
Total/Avg	124,549	916,106	13.60

These amounts average $13.60 in unallocated loss adjustment expense per $100 of paid loss. It is important to remember that the paid amounts in any calendar year come from a mixture of new claims and claims that were already open.

ULAE expenses are generated throughout the lifetime of a claim. A common simple assumption is that 50% of the total unallocated loss adjustment expense is paid when a new claim is opened and the other 50% over the life of the claim. However, it is preferable to review this assumption with the claim department. Assume that this review indicates that 40% of unallocated loss expense is paid to set up an initial claim. Then the estimated liability for unallocated loss adjustment expense is given by

$$.1360 \times \text{IBNR Reserve} + .1360 \times (1 - .40) \times \text{Case Reserve}$$

If the IBNR reserve required is \$90 million and the case reserves total \$320 million, then the total ultimate amount of allocated claim expense liability is

$$.1360 \times \$90,000,000 + .1360 \times .60 \times \$320,000,000$$

which totals \$38,352,000. Note that the reserve amount needed is higher than the amount paid in any one calendar year. This is because we are estimating the total ULAE liability, not simply how much ULAE will be paid in any given calendar year.

One adjustment that can be made to this method is to split the "pure" IBNR reserve from the remaining broad definition of IBNR. The "pure" IBNR claims would be multiplied by the .1360 factor since no loss adjustment work has yet been done on these claims. The remaining IBNR reserve would be multiplied by $.1360 \times (1 - .4)$ to recognize that some ULAE had already been paid on these claims.

PHASE 3: EVALUATION OF ULTIMATE LOSS ESTIMATES

The application of any particular reserve method to a given body of data will yield a set of estimated ultimate losses. However, each method applied will result in a different set of ultimate losses and an associated reserve estimate. The actuary must still decide on either a best estimate reserve, or a range of possible reserve estimates, or both. Of course, for financial statement purposes, a point estimate of loss reserve requirements must be supplied for the balance sheet.

While a substantial amount of judgment has been an element of the selection and application of each reserving method, the selection of a final reserve estimate is most often a subject of the actuary's experience and judgment. In this section we will present a number of practical tests that will allow one to test a set of estimated ultimate losses for reasonability.

It is important to evaluate the results of each reserving method by attempting to diagnose the reasons the methods vary. The explanation must be the result of the actuary's analysis and experience. The attempt to reconcile a number of different estimates is extremely difficult, but often yields important new insights.

The analysis conducted on the data presented above yielded six different estimates of ultimate losses by accident year. These estimates are the result of different methods that are sensitive to different aspects of the reserve development process. This is to be expected, since each method uses only a limited amount of data about the loss development process. After reviewing the results of the various methods, the analyst makes an initial selection of ultimate loss as follows:

Estimated Ultimate Losses by Accident Year and Method

Accident Year	Paid Devel't	Incurred Devel't	Average Paid	Average Incurred	BF Method	Reserve Devel't	Average	Selected
1994	82,370	83,196	79,092	82,284	83,188	82,413	82,090	83,196
1995	88,163	88,287	84,023	87,505	88,310	87,974	87,377	88,287
1996	76,340	70,741	71,866	69,982	70,823	70,840	71,765	70,741
1997	78,846	80,301	74,204	79,487	80,253	81,562	79,109	80,301
1998	102,293	92,430	91,107	91,399	92,064	93,842	93,856	92,430
1999	69,344	66,215	71,520	65,521	66,184	66,791	67,596	66,215
2000	45,939	44,737	60,461	53,757	44,821	46,001	49,286	44,821

Required Reserve in $1,000's by Method

	Paid Devel't	Incurred Devel't	Average Paid	Average Incurred	BF Method	Reserve Devel't	Average	Selected
All Years	147,289	129,901	136,267	133,929	129,637	133,417	135,073	129,986

The analyst has selected the results of the incurred development method for all years except the most recent. For the most recent year, the BF method has been used because the incurred loss data is still immature. The initial selection of these ultimate loss estimates should be tested for reasonability by comparing them to various loss development history displays.

Comparing these selected ultimate losses to the paid and incurred history yields the following displays of paid and incurred as a percent of ultimate.

Paid Losses as % of Ultimate Losses

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	27.2%	48.2%	65.3%	77.1%	85.6%	91.3%	94.0%
1995	25.0%	49.8%	66.5%	81.4%	88.4%	92.1%	
1996	28.5%	55.3%	72.5%	85.4%	93.9%		
1997	24.0%	46.5%	62.8%	77.6%			
1998	22.2%	46.4%	68.0%				
1999	25.7%	50.7%					
2000	25.3%						

Incurred Losses as % of Ultimate Losses

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	70.5%	89.9%	92.9%	93.6%	97.0%	98.9%	99.0%
1995	72.2%	90.1%	94.8%	96.7%	99.8%	99.0%	
1996	73.2%	96.4%	98.7%	98.5%	99.0%		
1997	50.0%	84.7%	93.6%	97.1%			
1998	60.2%	86.9%	95.2%				
1999	65.5%	86.9%					
2000	64.2%						

Review of these statistics indicates that the ratios of paid loss to expected ultimate loss in the 1999 and 2000 accident years are somewhat high, at 50.7% and 25.3%, respectively. However the comparison to 1997 and 1998 is difficult because of the anomalous behavior of these two years.

A similar review of the ultimate amounts with respect to carried case reserves is also useful. This display of course is merely the difference between the previous two displays.

Case Reserves as % of Ultimate Losses

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	43.3%	41.8%	27.7%	16.6%	11.4%	7.6%	5.0%
1995	47.2%	40.3%	28.3%	15.3%	11.4%	6.9%	
1996	44.7%	41.0%	26.1%	13.1%	5.1%		
1997	26.0%	38.1%	30.8%	19.4%			
1998	38.0%	40.5%	27.2%				
1999	39.9%	36.2%					
2000	38.9%						

Based on the selections, the required reserve is the difference between the paid losses and the ultimate. This required reserve is also a "hindsight" test of the selected ultimate.

Required Reserves as % of Ultimate Losses

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	72.8%	51.8%	34.7%	22.9%	14.4%	8.7%	6.0%
1995	75.0%	50.2%	33.5%	18.6%	11.6%	7.9%	
1996	71.5%	44.7%	27.5%	14.6%	6.1%		
1997	76.0%	53.5%	37.2%	22.4%			
1998	77.8%	53.6%	32.0%				
1999	74.3%	49.3%					
2000	74.7%						

It is also useful in some cases to review the ratio of the required reserve to the carried case reserve, as this ratio can be very stable for some lines.

Required Reserves as % of Case Reserves

Accident	Age in months:						
Year	12	24	36	48	60	72	84
1994	168.1%	124.2%	125.5%	138.5%	126.1%	114.5%	119.9%
1995	158.9%	124.7%	118.5%	121.6%	101.3%	114.3%	
1996	160.0%	108.8%	105.1%	111.3%	119.2%		
1997	292.6%	140.2%	120.8%	115.1%			
1998	204.7%	132.4%	117.8%				
1999	186.4%	136.1%					
2000	191.9%						

It is also necessary to compare the ultimate losses to other benchmarks, such as earned premium or exposures. Below the selected ultimate loss is compared to earned premium. The ultimate loss ratio is also compared to the expected loss ratio that was used in the pricing assumptions.

Accident Year	Selected Ultimate	Earned Premium	Loss Ratio	Expected LR
1994	83,196	101,946	81.6%	80.0%
1995	88,287	112,068	78.8%	80.0%
1996	70,741	97,796	72.3%	80.0%
1997	80,301	101,930	78.8%	78.0%
1998	92,430	107,357	86.1%	78.0%
1999	66,215	84,531	78.3%	78.0%
2000	44,832	57,697	77.7%	78.0%

Immediately we see that the loss ratio for the 1998 accident year is higher than the other years. This may be a result of the more hazardous mix of business that was written in that year. The reserving actuary should communicate with the Underwriting Department that results for the 1998 accident year are worse than other years and are significantly higher than expected. In this case, the more hazardous business is no longer written, but the Underwriting Department must get feedback on results from the reserving actuary so appropriate corrective underwriting actions can be taken if they haven't already.

Another benchmark is average ultimate claim size, or severity. Absent any changes in the book of business, we would expect severity to trend with inflation. Below we have compared the selected ultimate losses to the projected number of claims net of claims closed with no payment.

Accident Year	Selected Ultimate	Ultimate Claims	Average Claim	Change in Avg.
1994	83,196	34,328	2,424	
1995	88,287	32,542	2,713	11.9%
1996	70,741	26,916	2,628	−3.1%
1997	80,301	24,571	3,268	24.3%
1998	92,430	26,531	3,484	6.6%
1999	66,215	18,015	3,676	5.5%
2000	44,832	13,599	3,297	−10.3%

History shows that the average claim for this book of business is volatile. However, the 10.3% decrease in severity in the 2000 accident year appears unusual. The actuary should consider whether the ultimate loss selection or the ultimate claim count selection should be modified, or if this change in severity is reasonable.

Claim frequency, which is the number of claims relative to some measure of exposure, can be used as a reasonableness check for the projected number of claims. In this case, we will use earned premium as a proxy for exposure.

Accident Year	Ultimate Claims	Earned Premium	Frequency	Change in Freq.
1994	34,328	101,946	337	
1995	32,542	112,068	290	−13.8%
1996	26,916	97,796	275	−5.2%
1997	24,571	101,930	241	−12.4%
1998	26,531	107,357	247	2.5%
1999	18,015	84,531	213	−13.8%
2000	13,599	57,697	236	10.6%

This exhibit shows that for every $1 million in premium, there were 337 claims in accident year 1994. The frequency for this business is also volatile, and the increase of 10.6% in accident year 2000 is very high.

After discussions with underwriting and claims management, the actuary determines that there has been an increase in the number of reported claims under $1,000. There has been a shift in the portfolio toward high-frequency, low-severity risks. Based on this information, the actuary is comfortable that the 2000 selected ultimate loss is correct. In future reserve reviews, the actuary should watch for possible shifts in loss and allocated loss adjustment expense development patterns that may result from this shifting mix of business.

PHASE 4: MONITORING RESULTS

Once ultimate losses have been selected, it is extremely important for the analyst to be able to derive projections of expected development in the upcoming period. These predictions can be monitored over the next period—month, quarter, or year. If actual loss statistics, such as paid losses, case reserves, IBNR counts, and the number of claims closed with payment actually come in close to the forecast development amount, the analyst can have more confidence in the analysis and understanding of the reserve situation. If results are not as forecast, additional work is required to determine if the differences were random occurrences, or if it is an indication that the projections should be revised.

For example, based on the selected ultimate loss of $44,832,000 for the 2000 accident year, we should expect to see emergence of incurred losses of $10,082,000 during 2001 on the 2000 accident year. The expected loss emergence forecast comes directly from the incurred loss development factors previously selected by the analyst. The selected development factor from age 12 to ultimate of 1.553 implies that 64.4% (= 1/1.553) of losses are unreported at 12 months. The factor of 1.151 for 24 to ultimate implies that 86.9% of the losses are unreported at

24 months. Thus, we can calculate that expected reported losses from 12 to 24 months are 22.5% of ultimate, and 22.5% of the selected ultimate is 10,082,000.

The forecast incurred loss emergence expected in calendar year 2001 is (in $1,000's):

Accident Year	Emergence
1994	0
1995	0
1996	0
1997	1,559
1998	1,759
1999	5,467
2000	10,082
Total	18,867

This indicates a total of $18,867,000 of incurred loss should emerge during 2001 based on the selected loss development factors. The benefit of monitoring the near term forecast is clear. The accuracy of the ultimate estimate on accident year 2000 will take several more years to ascertain, however the accuracy of the development projection for the next calendar year can be measured in only one year. Similar projections can be made for paid loss or loss adjustment expenses.

MISCELLANEOUS TOPICS

Reserve Discounting

In establishing the liabilities for losses and loss adjustment expenses, it is often necessary to recognize the time value of money. We have not taken interest into account in any of the loss reserve estimation procedures reviewed above.

A payout schedule of the liability amount is required to discount the loss reserve liability for the time value of money. If the liability estimate is given by the paid loss development estimate as below, we have an undiscounted liability of $147,289,000. The payment pattern can be deduced from the completed triangle established by the selection of paid loss development factors.

For example, if the paid loss projection is given as follows:

Accident			Age in months:						Paid	
Year	12	24	36	48	60	72	84	Ultimate	to Date	Reserve
1994	22,603	40,064	54,301	64,114	71,257	75,950	78,224	**82,370**	78,224	4,146
1995	22,054	43,970	58,737	71,841	78,076	81,287	**83,726**	**88,163**	81,287	6,876
1996	20,166	39,147	51,319	60,417	66,402	**70,386**	**72,498**	**76,340**	66,402	9,938
1997	19,297	37,355	50,391	62,347	**68,582**	**72,697**	**74,878**	**78,846**	62,347	16,499
1998	20,555	42,898	62,832	**79,231**	**88,977**	**94,315**	**97,145**	**102,293**	62,832	39,461
1999	17,001	33,568	**45,317**	**54,833**	**60,317**	**63,936**	**65,854**	**69,344**	33,568	35,776
2000	11,346	**22,238**	**30,022**	**36,326**	**39,959**	**42,356**	**43,627**	**45,939**	11,346	34,593
										147,289

If we assume that all losses are paid within eight years, we can calculate the forecast of paid amounts by calendar year and accident year. Discount factors are then applied to the payments within each calendar year. Here we have assumed that payment will be made at the mid-point of the calendar year and that interest is earned at an annual rate of 5%.

Accident				Calendar Year				
Year	2001	2002	2003	2004	2005	2006	2007	Total
1994	4,146	0	0	0	0	0	0	4,146
1995	2,439	4,437	0	0	0	0	0	6,876
1996	3,984	2,112	3,842	0	0	0	0	9,938
1997	6,235	4,115	2,181	3,969	0	0	0	16,499
1998	16,399	9,745	5,339	2,829	5,149	0	0	39,461
1999	11,749	9,517	5,483	3,619	1,918	3,490	0	35,776
2000	10,892	7,783	6,305	3,633	2,398	1,271	2,312	34,593
Total	55,843	37,709	23,150	14,050	9,464	4,761	2,312	147,289
Discount Factor	0.976	0.929	0.885	0.843	0.803	0.765	0.728	
Discounted Reserve	54,498	35,048	20,491	11,844	7,599	3,640	1,684	134,804
						Discount Amount		12,485

The discount of \$12.5 million is about 8.5% of the undiscounted amount. The calculation of discount is very sensitive to the selected interest rate. The analyst should research the type of investments held and the expected return on those investments to determine the appropriate rate of discount.

Reserve Estimate Ranges

Throughout our analyses, we have focused on obtaining point estimates of the loss reserve liability. However, we have also found that it is extremely difficult to obtain one single estimate of the loss reserve liability. Each method results in a different answer. Further, to the extent that we are dealing with the estimation of the mean of a stochastic process, the actual result will almost always differ from the estimate.

Clearly, a range of results and a statement of our confidence that the observed reserve liability at final development will be within the stated range are preferable for this sort of process. However, the insurer's balance sheets will continue to require the analyst to supply a point estimate of the reserve liability.

Working with risk theoretical concepts, it is possible to develop a model of the reserve inventory, in terms of frequency and severity. This model can be used to develop confidence intervals for the development of the reserve. The development of such a risk theory model is outside the scope of this chapter. However, work along these lines has been done (see Hayne [1988]).

CONCLUSION

Accurate loss and loss adjustment reserves are critical to insurance company performance. Although the reserve is a balance sheet item, a change to a liability account on the balance sheet has a direct impact on income. In addition, management must have an accurate picture of results so that appropriate strategic and operational decisions can be made.

A strict formula approach to projecting loss and loss adjustment expense reserves will not work. A number of different projection methods must be used, and the experience and judgment of the analyst is critical. The analyst must be in constant communication with claims and underwriting management so that appropriate methods are used and proper adjustments are made. Of equal importance is communication of the results to management that might lead to improvement in claims processing procedures or development of underwriting initiatives. The reserving actuary should also communicate results to the pricing actuary so that pricing procedures can be modified if required.

In addition, because there is frequently a range of estimates that can be considered reasonable, the analyst should be aware of the context in which the reserve estimates will be used. Different projection techniques may be appropriate if results are to be used to determine the profitability of a specific line of business or if results will be used for statutory reporting of reserves.

Reasonability checks should always be performed after ultimate losses or loss expenses are selected. Revisions to ultimate selections should be considered if results differ significantly from expectations or industry benchmarks. Finally, projections should be frequently monitored and adjustments to ultimate selections should be made when indicated.

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APPENDIX A

THE DEFINITION OF LIABILITY

An obligation satisfies the accounting definition of a liability if it possesses three essential characteristics (Davidson et al., 386–387):

1. The obligation involves a probable future sacrifice of resources at a specified or determinable date;
2. The firm has little or no discretion to avoid the transfer, and
3. The transaction or event giving rise to the obligation has already occurred.

A claim liability of a property and casualty insurer satisfies the second and third characteristics above. The first requirement is not generally satisfied in property and casualty claim situations. For instance, in a workers compensation claim, payments must be made periodically at specified times, often weekly. However, in a third-party liability situation it is not possible to specify the date on which settlement will be made.

A loss reserve is a contingent liability in the sense that each specific claim under adjustment depends on some future contingent event to determine the extent of the insurer's liability. Two tests are proposed by the accounting literature to determine if a contingent liability should be recognized on the company's balance sheet. These are:

1. information at the time of preparation of the financial reports indicates that it is likely that a liability has been incurred, and
2. the amount of the liability can be reasonably estimated.

Clearly, an insurer's loss reserves satisfy both these conditions.

APPENDIX B

AN ACTUARIAL MODEL OF LOSS DEVELOPMENT[1]

Most approaches to loss reserving and financial reporting deal with aggregates over a time period. For example, we may consider all claims that occur, or all claims that are reported, or all amounts that are paid, during one calendar year, quarter, or month. Such approaches usually involve assumptions about how the claims process behaves *within* each year, quarter or month—assumptions that may not be correct. Instead we can create actuarial models that analyze claims continuously, without dividing time into discrete periods. Such models give us more flexibility in dealing with unusual or quickly changing situations.

The basic mathematical form of an actuarial loss development model is outlined below. This model can serve as a conceptual starting point for reserve analysis.

THE LOSS FUNCTION

We begin with a loss function $v(x)$, which represents the rate at which losses are occurring at time x. The ultimate loss for the time period (a,b) is then

$$\int_a^b v(x)\,dx.$$

In practice we cannot observe $v(x)$ directly; we can only observe aggregate loss over a period of time.

Example 1: Losses are occurring at a constant rate of \$1,000,000 per year. Find the amount of loss from 1/1/88 to 1/1/91.

Answer: Let $x = 0$ correspond to 1/1/88. and $x = 3$ to 1/1/91. Since $v(x) = \$1{,}000{,}000$, we see that

$$\text{Loss} = \int_0^3 \$1{,}000{,}000\,dx = \$3{,}000{,}000.$$

[1]The author thanks Eric Brosius for his efforts in re-drafting the first edition version of this section.

We could have answered this question easily by ordinary multiplication. In most insurance situations, however, $v(x)$ changes over time in response to inflation and other factors.

Example 2: In the previous example, what if the rate of loss were growing exponentially as described by the function $v(x) = \$1{,}000{,}000\, e^{0.1x}$?

Answer: This time we compute

$$\text{Loss} = \int_0^3 \$1{,}000{,}000 e^{0.1x} dx = \$3{,}498{,}588.$$

DEVELOPMENT

The function $v(x)$ tells us how losses *occur*. It says nothing about how those losses are reported and paid. To study this question we introduce a development function D. If we are studying loss payments, $D(t)$ will represent the percentage of losses that are paid within t years after occurrence. If we are studying loss reporting, $D(t)$ will represent the percentage of losses that are reported within that period of time. A little thought about the claims process shows us that $D(t) = 0$ for $t < 0$, and that $\lim_{t\to\infty} D(t) = 1$. We sometimes strengthen the second fact by assuming that there is a fixed time T such that $D(t) = 1$ whenever $t > T$. This implies that all development is complete within T years after occurrence.

Example 3: Is it always true that $0 \le D(t) \le 1$?

Answer: This is often true, but not always. For example, if we look at loss payments net of salvage and subrogation, development may exceed 100% for a while before late recoveries bring net paid losses back down to their ultimate value.

By combining the loss function $v(x)$ with the development function $D(t)$ we can describe development as we see it at a particular point in time. The aggregate losses from period (a,b) as developed to time $x = c$ are given by

$$\int_a^b v(x) D(c - x)\, dx.$$

We can understand this integral as follows. Losses that occurred at time $x = a$ have had $c - a$ years to develop, so we multiply these losses by $D(c - a)$. Losses that occurred at time $x = b$ have had only $c - b$ years to develop, and are multiplied by the (usually smaller) value $D(c - b)$.

Example 4: Suppose $v(x) = \$1{,}000{,}000$ and $D(t) = 1 - e^{-t}$ for $t \geq 0$. Find the losses from 1/1/1996 to 1/1/1999 as developed to 1/1/2000

Answer: We obtain loss developed to 1/1/2000 as

$$\int_0^3 \$1{,}000{,}000(1 - e^{-(4-x)})\,dx \approx \$2{,}650{,}436.$$

At 1/1/2000 losses that occurred on 1/1/1996 are 4 years old. They have developed to 98% of ultimate, since $D(4) \approx 0.98$. Losses that occurred on 1/1/1999 are 1 year old, and have developed to only 63% of ultimate ($D(1) \approx 0.63$). What we observe, however, is the entire block of claims that occurred in the period $(0, 3)$. This block has developed to 88% of ultimate ($\$2{,}650{,}436/\$3{,}000{,}000 \approx 0.88$).

FITTING A MODEL

So far we have used the functions $v(x)$ and $D(t)$ to compute aggregate loss at some stage of development. In practice v and D are unknown to us—we must work backward from aggregate loss observations to determine the underlying functions. As is usual in modeling problems, this process requires some assumptions about what kind of functions v and D might be.

Example 5: Assume that $v(x) = r + sx$ and $D(t) = (1 + t/T)/2$ for $0 \leq t \leq T$. Given the following data,

Reported Loss ($000,000)

(a, b)	$c = 1$	$c = 2$
$(0, 1)$	4.5	5.8
$(1, 2)$	—	6.6

find the ultimate loss for time period $(1, 2)$.

Answer: Reported loss as of time $x = c$ for the period (a,b) is given by

$$\int_a^b (r + sx)D(c - x)\,dx,$$

Our data provides us with three equations:

$$4.5 = \int_0^1 (r + sx)D(1 - x)\,dx,$$

$$5.8 = \int_0^1 (r + sx)D(2 - x)\,dx, \qquad \text{and}$$

$$6.6 = \int_1^2 (r + sx)D(2 - x)\,dx.$$

With some effort we can solve for the model parameters r, s, and T:

$$r = 6.0, \qquad s = 3.6, \qquad T = 3.0.$$

The ultimate loss for time period $(1,2)$ is then

$$\int_1^2 6.0 + 3.6x\,dx = 11.4,$$

or in actual dollars, \$11,400,000.[2]

PRACTICAL CHALLENGES

Real examples are more complicated. Loss and development functions are usually non-linear; and the resulting equations are much harder to solve than those given above are. Although more data is available, no single development function can describe it all. For one thing, development is subject to random fluctuation; this makes it necessary to estimate model parameters by least-squares type methods.

In addition, the development process itself, and hence the function $D(t)$, can change with time.

[2]A fuller treatment of these issues is given in McClenahan, C. L., "Adjusting Loss Development Patterns for Growth," *Proceedings of the Casualty Actuarial Society*, 1987, 74:101–114, and in the discussion by D. P. Gogol on pp. 115–118.

Example 6: A company installs an automated claim handling system. How will this affect the $D(c-x)$ term in the integral?

Answer: If claim reporting speeds up, larger values of $D(c-x)$ will result (and hence larger observed losses.)

Example 7: How does increasing claim litigation affect paid loss development?

Answer: Claims that go to court take longer to settle. As more claims fall into this category, loss payments slow down and the paid loss development function $D(t)$ stretches out.

As these examples show, it may be better to use a family of development functions $Dx(t)$. This allows for changes in loss development patterns, but it adds further complications. It is not usually necessary to solve the resulting equations exactly; solutions can be sought by numerical methods. In particular, the integrals used here can be approximated by finite sums. Even so, few actuaries find these techniques sufficiently tractable for everyday use.

CONCLUSION

One cannot afford to ignore the effects of growth (i.e., changes in $v(x)$) upon loss development. Changes in loss volume can make quite a difference. Actuarial models such as the one described here allow us to correct for these "growth effects" and measure the underlying development patterns.[3]

If these models are to be used regularly, more research into the true form of the loss development functions $D(t)$ will be needed. Such research has only recently begun to take shape.[4]

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[3]cf. Simon, L. J., "Distortion in IBNR Factors," *Proceedings of the Casualty Actuarial Society*, 1970, 57:64–68, where this problem is studied using graphical methods.

[4]Two recent applications of these ideas may be found in McClenahan, C. L., "Liabilities for Extended Reporting, Endorsement Guarantees Under Claims-Made Policies," *CAS Discussion Paper Program*, 1988, 345–363 and Philbrick, S. W., "Reserve Review of a Reinsurance Company," *CAS Discussion Paper Program*, 1986, 147–162.