Sample Questions Exam S

1.

Each member of a population independently produces a random number of offspring with the following probabilities:

Number of offspring	0	1	2	3+
Probability	0.375	0.125	0.500	0.000

In the current generation, there are six people, each of whom has the ability to produce offspring.

Calculate the probability that this population will eventually go extinct.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

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1. Solution
Key: A

Solution:

The expected number of offspring, $\mu = \sum_{j=0}^{\infty} jP_j = .125 + 2 * .5 = 1.125 > 1$. This implies that $\pi_0 < 1$, so that the family of each current member of the population will go extinct with probability:

$$\pi_0 = \sum_{j=0}^{\infty} \pi_0^j P_j = .375 + .125\pi_0 + .5\pi_0^2$$

The solutions to this quadratic equation are $\{.75, 1\}$. According to Ross, π_0 is the smallest positive value satisfying the above equation, so the extinction probability of each member is:

$$\pi_0 = 0.75.$$

Since there are currently six members, each with independent extinction probabilities, the population extinction probability is:

$$\pi_0^6 = 0.75^6 = 0.178$$

Classification: A.6.g – Branching Processes
Text Reference: *Ross*, Chapter 4.7 (p 234)
Item Notes: Similar to *Ross* Example 4.33 & 4.34 (p 236-7), but with different values.

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2.

You are given the following information:

- A fair coin is flipped by a gambler with 20 chips at time 0.
- The gambler wins 1 chip if the coin turns “head”.
- The gambler loses 1 chip if the coin turns “tail”.
- The gambler will stop playing when he either has lost all of his chips or he reaches 60 chips.
- Of the first 10 flips, there are 7 “heads” and 3 “tails”.

Calculate the probability that he will lose all of his chips, given the results of the first 10 flips.

A. Less than 0.50
B. At least 0.50, but less than 0.65
C. At least 0.65, but less than 0.80
D. At least 0.80, but less than 0.95
E. At least 0.95

2. Solution
Key: B

Solution:
Since we are told that the coin is fair, the probability of winning on any given toss is 1/2. After the first 10 flips, he had 24 chips (20+7-3).

The probability of going down 24 chips before going up another 36 chips is:

$$\frac{36}{36+24} = 0.60$$

Classification: A.6.f – Gamblers ruin problem
Text Reference: Ross 4.5
Item Notes:

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3.

You are given:

- A response variable Y is in the exponential family,

$$f(y) = c(y, \phi) * \exp[\frac{y\theta - a(\theta)}{\phi}]$$

- $a(\theta) = e^{\theta}$
- $\mu = E(Y)$
- Dispersion parameter, $\phi = 1$

Determine an expression for $Var(Y)$ as a function of μ.

A. μ^2
B. μ
C. $1/\mu$
D. 1
E. e^{μ}

3. Solution

Solution: B

$Var(y) = \ddot{a}(\theta) = e^{\theta}$
$\mu = \dot{a}(\theta) = e^{\theta}$

➔ $Var(y) = \mu$

Alternatively, recognize (e.g. Table 3.1 p 36 *de Jong*) from the given information that Y is Poisson(μ) from which it follows that variance is also μ.

Classification: C.1.a – Understand the relationship between mean and variance by model family member
Text Reference: *de Jong*, Chapter 3.1
Item Notes:

4.

An actuary wants to estimate the probability of a home insurance policy having a claim by using a logistic regression model. He has the following pieces of information from 1,000 historical policies:

- Cost of the home, in $000s
- Age of the home, in years
- Whether or not there was a claim on the policy

The actuary is considering a number of different model specifications. Below are the models he is considering along with the calculated deviance of each model:

Model #	Included Variables	Deviance
1	Intercept + Cost	1085.0
2	Intercept + Cost + Age	1084.8
3	Intercept + Cost + (Cost * Age)	1083.0
4	Intercept + Cost + Cost^2 + Cost^3	1081.9
5	Intercept + Cost + Cost^2 + Cost^3 + Cost^4	1081.6

Determine the optimal model using the Bayesian Information Criterion.

A. 1
B. 2
C. 3
D. 4
E. 5

4. Solution

Key: A

Solution:

The BIC chooses the model which minimizes:

*Deviance + p*ln(n)*

With n = 1000, and p is the number of parameters (including intercept):

Model #	Included Parameters	Deviance	p	p ln(n)	BIC*
1	Intercept + Cost	1085.0	2	13.8	1098.8
2	Intercept + Cost + Age	1084.8	3	20.7	1105.5
3	Intercept + Cost + (Cost * Age)	1083.0	3	20.7	1103.7
4	Intercept + Cost + $Cost^2$ + $Cost^3$	1081.9	4	27.6	1109.5
5	Intercept + Cost + $Cost^2$ + $Cost^3$ + $Cost^4$	1081.6	5	34.5	1116.1

This quantity is minimized with Model #1.

Classification: C.2.f – Akaike's/Bayesian Information Criterion (AIC/BIC)
Text Reference: *de Jong*, Chapter 4.19 (p 62)
Item Notes:

Question is based off of *de Jong* Table 7.4 (p 103), but with different values.

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5.

You are given the three autoregressive time series models

$$x_t = 0.5\, x_{t-1} + w_t$$
$$y_t = \left(\frac{4}{3}\right) y_{t-1} - \left(\frac{1}{3}\right) y_{t-2} + w_t$$
$$z_t = \left(\frac{5}{2}\right) z_{t-1} - z_{t-2} + w_t$$

Determine which of the following statements is true.

A. $\{x_t\}$ is stationary, $\{y_t\}$ and $\{z_t\}$ are non-stationary

B. $\{y_t\}$ is stationary, $\{x_t\}$ and $\{z_t\}$ are non-stationary

C. $\{z_t\}$ is stationary, $\{x_t\}$ and $\{y_t\}$ are non-stationary

D. $\{x_t\}$ and $\{y_t\}$ are stationary, $\{z_t\}$ is non-stationary

E. $\{x_t\}$ and $\{z_t\}$ are stationary, $\{y_t\}$ is non-stationary

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5. Solution

Key: A

Solution:

$(1 - 0.5B)x_t = w_t$ The characteristic equation $(1 - 0.5B) = 0$ has solution B = 2 > 1. Thus $\{x_t\}$ is stationary.

$\left(1 - \left(\frac{4}{3}\right)B + \left(\frac{1}{3}\right)B^2\right)y_t = w_t$ The characteristic equation $(3 - 4B + B^2) = 0$ has solution B = 3 > 1 and B = 1. Thus $\{y_t\}$ is non-stationary.

$\left(1 - \left(\frac{5}{2}\right)B + B^2\right)z = w_t$ The characteristic equation $(1 - 2B)(2 - B) = 0$ has solution B = 2 > 1 and B = 0.5 < 1. Thus $\{z_t\}$ is non-stationary.

Classification: <u>D.2</u> – Time Series
Text Reference: Pages 79, 80 PSC, AVM