CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam 8

Advanced Ratemaking

October 25, 2011 **3 HOURS**

INSTRUCTIONS TO CANDIDATES

1. This 59 point examination consists of 25 problem and essay questions.

2. For problem and essay questions the number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer the questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.

   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.

   - The answer should be concise and confined to the question as posed. When a specific number of items is requested, do not offer more items than the number requested. For example, if three items are requested, only the first three responses will be graded.

   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

- Verify that you have received the reference materials:

  a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (as of June 30, 2010)
  b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
  c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (as of June 30, 2010)

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2011.

END OF INSTRUCTIONS

1. (3 points)

An insurance company is using a merit rating plan for drivers in two states. State X has the following claims experience:

Group	Number of Accident-Free Years	Earned Premium at Present Group D rates	Number of Claims Incurred
A	3 or more	$500,000	240
B	2	$150,000	125
C	1	$200,000	190
D	None	$300,000	300
Total		$1,150,000	855

State Y has the following relative claim frequencies for accident-free experience:

Number of Accident-Free Years	Relative Claim Frequencies to Total
3 or more	0.70
2 or more	0.77
1 or more	0.84

Assuming that no new risks enter or leave either state, use relative credibility to explain which state has more variation in an individual insured's probability of an accident.

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2. (1.5 points)

A multi-dimensional credibility technique has been developed to predict claim frequencies for major permanent partial claims.

- **Seven years of data were collected.**
- **The technique produced a raw predicted relativity based on the oldest five years.**
- **The most recent two years were used as the holdout sample.**

Major Permanent Partial Claims			
Quintile	Holdout Sample Relativity	Prediction Based on Raw	Prediction Based on Credibility Procedure
1	0.6	0.3	0.4
2	0.8	0.5	0.7
3	1.0	1.1	1.0
4	1.2	1.9	1.5
5	1.4	3.0	1.8

Demonstrate whether the credibility technique produces an improved estimate using the sum of squared errors.

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3. (1.5 points)

An actuary is considering performing a one-way analysis to provide pricing guidance for an insurance company's personal auto book of business.

a. (0.5 point)

Briefly describe two shortcomings associated with one-way analyses.

b. (1 point)

Provide an example of how each shortcoming in part a. above may arise.

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4. (2 points)

In "NCCI's 2007 Hazard Group Mapping," Robertson discusses his use of the *k-means algorithm* to determine clusters.

a. (1.25 points)

Discuss how this algorithm works.

b. (0.75 point)

Describe the desirable optimality properties that result from using *k*-means to determine clusters.

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5. (3 points)

Given the following:

- **An insurance company is exposed to four independent catastrophic events:**

Event	Size
1	$5,000,000
2	$10,000,000
3	$20,000,000
4	$40,000,000

- **The annual probability of occurrence of a catastrophic event is given by** $p(x) = \frac{500{,}000}{x}$**; where x is the amount of loss.**

- **While each event can only happen once, the total number of events per year is not limited to one.**

Plot the exceedance probability curve associated with the insurance company's exposure and label the x and y axes.

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6. (2.5 points)

An insurance company is planning to introduce catastrophe modeling for portfolio management and ratemaking purposes. The company has three portfolios that are exposed to the same five catastrophic events. The details for each event are as follows:

Event (E_i)	Probability (p_i)	Loss for Portfolio 1	Loss for Portfolio 2	Loss for Portfolio 3
1	1.0%	$85,000	$40,000	$90,000
2	1.0%	$40,000	$90,000	$40,000
3	2.0%	$85,000	$50,000	$80,000
4	2.0%	$55,000	$85,000	$55,000
5	1.0%	$45,000	$50,000	$50,000

The standard deviations of loss for the three portfolios, including the catastrophic losses, are as follows:

Portfolio 1: $50,000
Portfolio 2: $125,000
Portfolio 3: $100,000

a. (1.5 points)

The company has decided to minimize risk. Explain which portfolio the insurer should eliminate.

b. (1 point)

Describe two potential problems with using catastrophe models for ratemaking.

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7. (4 points)

An actuary is pricing the $4,000,000 excess of $1,000,000 layer for an excess of loss policy on a property with a total insured value of $10,000,000.

Historical policy information is as follows:

Accident Year	LDF	On-Level Trended Subject Premium
2008	1.00	$6,400,000
2009	1.04	$6,900,000
2010	1.08	$7,400,000

Historical loss information is as follows:

Accident Date	Actual Loss	Trended Loss
March 4, 2008	$2,544,684	$2,976,920
October 12, 2008	$831,360	$972,574
April 2, 2009	$1,969,985	$2,215,965
June 6, 2009	$1,394,744	$1,564,398
August 14, 2010	$2,525,492	$2,731,572

The exposure curve below applies to the insured's risk profile:

% of Insured Value	Exposure Factor
10%	35%
20%	46%
30%	55%
40%	63%
50%	70%
60%	77%

% of Insured Value	Exposure Factor
70%	83%
80%	88%
90%	92%
100%	95%
110%	98%
120%	100%

Use a mixture of experience and exposure loss rating to calculate the policy's loss cost as a percentage of subject premium. Include a discussion on the appropriate layer for experience rating and the appropriate layer for exposure rating.

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8. (2 points)

The table below represents a reinsurer's aggregate loss distribution for a single ceding company's treaty:

Range of Loss Ratios	Average in Range	Probability Loss Ratio is in Range
0% to 60%	47.9%	0.45
60% to 75%	67.8%	0.29
75% to 90%	81.5%	0.17
90% or above	99.6%	0.09

The ceding company will reassume 50% of the losses from a 60% to 75% loss ratio and 80% of the losses from a 75% to 90% loss ratio.

Calculate the reinsurer's expected loss ratio after the application of the loss corridor.

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9. (2 points)

A reinsurer uses the following exposure curve under a non-proportional treaty:

$$G(x) = \frac{\ln(0.1 + 0.01^x) - \ln(1.1)}{\ln(0.11) - \ln(1.1)}.$$

The cedant's maximum retention under the treaty is \$50 million and the maximum possible first-dollar loss is \$100 million.

A function with the form of

$$G(x) = \frac{\ln(a + b^x) - \ln(1 + a)}{\ln(a + b) - \ln(a + 1)}$$

has a derivative of

$$G'(x) = \frac{\frac{\ln(b) \times b^x}{a + b^x}}{\ln(a + b) - \ln(a + 1)}$$

a. (0.5 point)

Calculate the ratio of pure risk premium retained by the cedant.

b. (1.5 points)

Calculate the probability of a total loss.

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10. (3.5 points)

Consider the following set of consistent increased limits factors (ILFs):

Per Occurrence Limit	ILF
$100,000	1.000
$250,000	1.250
$500,000	1.450
$1,000,000	1.750
$2,000,000	2.250

a. (2.5 points)

Assume an ILF for a limit of $750,000 is to be added to the table above.

Determine the range of possible ILFs for a limit of $750,000 that would maintain the consistency of the above set of ILFs.

b. (1 point)

Explain how the consistency test has both a mathematical interpretation and a practical meaning.

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11. (3 points)

Losses follow a uniform distribution between \$0 and \$100.

Assume a 10% trend is applied uniformly to all losses.

Use a Lee diagram to calculate the implied trend for the layer \$50 excess of \$25. Label all relevant features of the diagram.

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12. (1.5 points)

In 1998, the National Council on Compensation Insurance (NCCI) made the following changes to the Workers Compensation Experience Rating Plan:

- Reduced the weight on medical-only losses
- Increased the weight on excess losses
- Made the primary-excess split of actual losses inflation-sensitive

Briefly discuss whether each of these changes supports the following goals of experience rating:

- Safety incentive
- Predictive accuracy

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13. (2 points)

The following information applies to an experience rating plan:

Risk Class	Experience Mod Factor	Standard Premium
A	0.85	$47,600
B	0.95	$77,900
C	1.10	$45,100
D	1.20	$31,200

The current off-balance factor is 0.99.

In the upcoming year, half of the manual premium from risk class D will move to a new risk class, class E. Assume that there are no other changes.

Calculate the experience mod factor for risk class E that yields an off-balance of 1.01.

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14. (1.5 points)

An underwriter is rating an account containing general liability and workers compensation coverage.

Given the following:

- **The insured has average schedule rating risk characteristics except for equipment, employees and premises.**
- **All of the insured's equipment has been replaced within the last four months and all employees have been trained on the new equipment.**
- **Five months ago, the insured cleaned its premises inside and installed new lighting outside to improve visibility for its customers at night.**

a. (1 point)

Calculate the maximum schedule credit that can be applied to this account according to the ISO Commercial General Liability Experience and Schedule Rating Plan.

b. (0.5 point)

The account has an overall debit mod for workers compensation. Briefly describe two conclusions that can be drawn from this.

15. (1.5 points)

Experience rating plans can be classified as split or no split plans.

Describe the steps of the experience modification calculation under a single split plan. Include a discussion on how a single split plan considers frequency and severity and the relative importance of each.

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16. (3.5 points)

Given the following information:

Risks with Expected Loss Size (Quintile)	Actual Losses	Expected Losses	Plan A Modified Expected Loss
Stratum 1	$187,000	$190,000	$182,000
Stratum 2	$195,000	$195,000	$187,000
Stratum 3	$201,000	$200,000	$195,000
Stratum 4	$227,000	$205,000	$210,000
Stratum 5	$238,000	$210,000	$255,000

a. (1 point)

In "Workers Compensation Experience Rating: What Every Actuary Should Know," Dorweiler's necessary and sufficient conditions for correct credibility are noted. Describe these conditions.

b. (2 points)

Calculate the quintiles test statistic for Plan A.

c. (0.5 point)

Another experience rating plan, Plan B has a quintiles test statistic of 0.50. Explain whether Plan A or Plan B has assigned more appropriate credibility.

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17. (3.5 points)

Let X represent the size of loss of a given claim. Assume the following average severities for varying claim size ranges:

Size of Loss Range	Average Size of Loss
X <= $500	$250
$500 < X <= $1,000	$750
X > $1,000	$2,000

The following information is also available:

Total number of claims in study	500
Overall average claim size	$650
Safety factor	0.80
Tempered loss elimination ratio for a $500 straight deductible	0.40

Calculate the tempered loss elimination ratio for a straight deductible of $1,000.

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18. (3 points)

The following information relates to a large dollar deductible (LDD) workers compensation policy with a per occurrence deductible of $250,000 and an aggregate deductible of $2,000,000:

Standard premium	$1,000,000
Expected loss and ALAE ratio (% of standard premium)	65.0%
Insurance charge at $2,000,000 (% of loss and ALAE)	8.0%
Excess loss pure premium factor at $250,000 (% of loss and ALAE)	3.0%
Loss based assessment (% of loss and ALAE)	5.0%
ULAE load (% of loss and ALAE)	7.0%
General expenses (% of standard premium)	6.5%
Credit risk (% of standard premium)	4.0%
Acquisition expense (% of net LDD premium)	2.0%
Taxes (% of net LDD premium)	4.5%
Profit (% of net LDD premium)	2.5%

a. (1.5 points)

Calculate the LDD premium for this policy.

b. (1.5 points)

Identify two primary differences between LDD and excess insurance and briefly describe how each may directly affect the cost.

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19. (2 points)

An actuary is pricing a large dollar deductible workers compensation policy based on the information below:

Deductible	$200,000
Aggregate limit on deductible	$900,000
Standard premium	$925,000
State hazard group relativity	0.90
Expected unlimited loss ratio	0.68
Excess loss factor (% of standard premium)	0.18

Expected Loss Group	Expected Loss Range
31	620,000 – 710,000
30	710,001 – 820,000
29	820,001 – 930,000
28	930,001 – 1,030,000
27	1,030,001 – 1,200,000

The following table provides insurance charges for select entry ratios and expected loss groups:

	Expected Loss Group				
Entry Ratio	31	30	29	28	27
1.0	0.290	0.276	0.266	0.256	0.245
1.5	0.180	0.169	0.158	0.148	0.137
2.0	0.091	0.080	0.070	0.060	0.049
2.5	0.041	0.033	0.024	0.014	0.003

Calculate the loss cost for this policy using the ICRLL procedure.

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20. (2.5 points)

The following information is known about a balanced retrospectively rated policy:

Losses at minimum premium	$150,000
Losses at maximum premium	$400,000
Loss conversion factor	1.20
Basic premium	$50,000

- There is a 10% chance that this policyholder has no losses.
- There is a 90% chance that the losses are uniformly distributed between $0 and $500,000.
- There are no taxes.

Calculate the guaranteed cost premium for this policy.

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21. (1.5 points)

An Alaska workers compensation insured has expected unlimited loss and ALAE of $2,000,000. The insured is interested in a retrospectively rated policy with a loss and ALAE limit of $50,000.

Assuming that the insured is in Hazard Group F, determine the NCCI Expected Loss Group.

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22. (3.5 points)

The unlimited and limited loss ratios for five identical risks are as follows:

Risk #	Unlimited Loss Ratio	Limited Loss Ratio
1	30%	15%
2	45%	45%
3	45%	45%
4	90%	60%
5	90%	90%

a. (2.5 points)

Calculate Table L charges at loss ratios of 0% to 90% using increments of 15%.

b. (0.5 point)

Describe the impact on the insurance charge when a loss limit is introduced.

c. (0.5 point)

In practice, sample loss ratios may not equal expected loss ratios. When this occurs, briefly describe two approaches used to address this issue.

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23. (2 points)

An actuary calculated the aggregate loss for a policy using the following:

Expected total loss	$250,000
Deductible	$100,000
Percentage of total loss excess of $100,000	40%
Aggregate limit	$300,000

The following table provides insurance charges contemplating the indicated deductibles:

	Deductible			
Entry Ratio	$100,000	$200,000	$300,000	$400,000
1.0	0.430	0.460	0.475	0.487
1.5	0.280	0.330	0.350	0.360
2.0	0.170	0.235	0.260	0.275
2.5	0.105	0.165	0.195	0.205
3.0	0.065	0.115	0.150	0.170

The actuary later learns that the expected total loss should have been $333,000.

Calculate the dollar difference between the correct expected insurance charge and the insurance charge that was used to price the policy.

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24. (1.5 points)

In "Workers Compensation Excess Ratios: An Alternative Method of Estimation," Mahler models loss distributions in a piecewise fashion. Discuss his approach for building a loss distribution model and his rationale for doing so.

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25. (1.5 points)

The following information is available for a retrospectively rated policy:

Standard premium	$20,000
Guaranteed cost premium	$19,760
Provisions for losses and expenses exclusive of taxes	$19,160
Expected losses	$12,000
Loss conversion factor	1.250
Tax multiplier	1.025
Selected maximum loss ratio	95%
Selected minimum loss ratio	20%
Charge for maximum	0.055
Charge for minimum	0.700

Calculate the maximum premium ratio for this policy.

END OF EXAMINATION

Question 1

Sample 1

State X

# of yrs clm free	EP	# clms	Rel. Clm Free (M)	Z = 1-M
3+	500,000	240	$\frac{240/500,000}{A} = 0.6456$	.354
2+	650,000	365	$\frac{365/650,000}{A} = 0.755$	.245
1+	850,000	555	$\frac{555/850,000}{A} = 0.878$	.122
0	300,000	300		
	1,150,000	855		

Let total clm freq for the state = 855/1,150,000 = A

State Y	Mod	Z = 1-M	n yr Z / 1 yr Z
3+	.70	.30	1.875
2+	.77	.23	1.438
1+	.84	.16	1.00

State X	n yr Z / 1 yr Z
3+	2.90
2+	2.00
1+	1.00

State X's n yr Z / 1 yr Z ratio is closer to 3,2,1 for 3+,2+,1+

- ⇨ State X is more stable
- ⇨ State Y has more variation

Sample 2

State X

3+	500K	240	.48	.48/.743 = .646	.354
2+	650K	365	.561	.561/.743=.755	.245
1+	850K	555	.653	0.879	.122
	1,150K	855	.743		

State X

1+	1-.879 = .121
2+	1-.755 = .245 ≈ .121 * 2
3+	1-.646 = .354 ≈ .121 * 3

State y

1+	1-.84 = .16
2+	1-.77 = .23 < .16 * 2
3+	1-.70 = .30 < .16 * 3

Since the credibilities of 2 and 3 years without an accident for State X are approximately 2 & 3 times the 1 year credibility, State X has less variation in insured's probability of an accident, so State Y has more.

Question 2

Sample 1

Couret and Venter look at sum of squared error for the holdout relativity next to three things:

1) Total hazard group relativity
2) Prediction based on raw data
3) Prediction based on credibility procedure

1) Sum of squared error = $(1 – 0.6)^2 + (1 – 0.8)^2 + (1 – 1)^2 + (1 – 1.2)^2 + (1 – 1.4)^2 = 0.4$
2) Sum of squared error = $(0.3 – 0.6)^2 + (0.5 – 0.8)^2 + (1.1 – 1)^2 + (1.9 – 1.2)^2 + (3 – 1.4)^2 = 3.24$
3) Sum of squared error = $(0.4 – 0.6)^2 + (0.7 – 0.8)^2 + (1 – 1)^2 + (1.5 – 1.2)^2 + (1.8 – 1.4)^2 = 0.3$

Since #3 produces the lowest sum of squared errors, the credibility procedure is an improvement over #1 (hazard group membership) and #2 (using actual data).

Sample 2

SSE for Raw = (raw - holdout)^2
= $(0.3 – 0.6)^2 + (0.5 – 0.8)^2 + (1.1 – 1)^2 + (1.9 – 1.2)^2 + (3 – 1.4)^2 = 3.24$
SSE for Cred Proc = (cred – holdout)^2
= $(0.4 – 0.6)^2 + (0.7 – 0.8)^2 + (1 – 1)^2 + (1.5 – 1.2)^2 + (1.8 – 1.4)^2 = 0.3$
Yes, the credibility technique produces a lower SSE when compared to holdout (0.3) than the raw data (SSE of 3.24).

Question 3

Sample 1

a., b.
One-way analysis doesn't consider:

1. correlations between rating variables. e.g. Young people drive older cars more often. Worse loss ratio in older cars can be partially driven by youthful drivers.
2. interdependencies among rating variables. e.g. The rate differentials between male and female drivers vary by age.

Sample 2

a.

1. It fails to recognize the interdependencies among variables
2. It fails to consider correlation

b.

1. Young drivers who have expensive cars may be poor risks while old drivers who have expensive cars may be good risks
2. Age may be correlated with territory if older people live in certain parts of a state or county and younger people live in certain other parts of a state or country

Question 4

Sample 1

a)

Select some initial assignment of classes to clusters (hazard groups), then perform these two steps iteratively:

1. Calculate the centroid of each cluster.
2. Assign each class to the cluster to which it is closest.

If any of the classes changes clusters in step 2, then repeat.

b)

1. Maximizes the proportion of total variance explained by the hazard groups:

$$1-\frac{\sum_{i=1}^{k}\sum_{c \in HG_i}\|R_c-\bar{R}_i\|_2^2}{\sum_c\|R_c-\bar{R}\|_2^2}$$

2. Minimizes the within hazard group variance and maximizes the between hazard group variance.

Sample 2

a)

1. Assign classes to k arbitrary groups
2. Calculate centroid of excess ratios of each group (essentially weighted excess ratios)
3. Compare excess ratios of each class to those of all centroids
4. Move each class to group with closest centroid
5. If any classes move, go back to step 2 and continue

b)

k-means clustering is equivalent to maximizing R-squared statistic in linear regression. It maximizes the variance between groups while minimizing the variance within each group.

Question 5

Sample 1

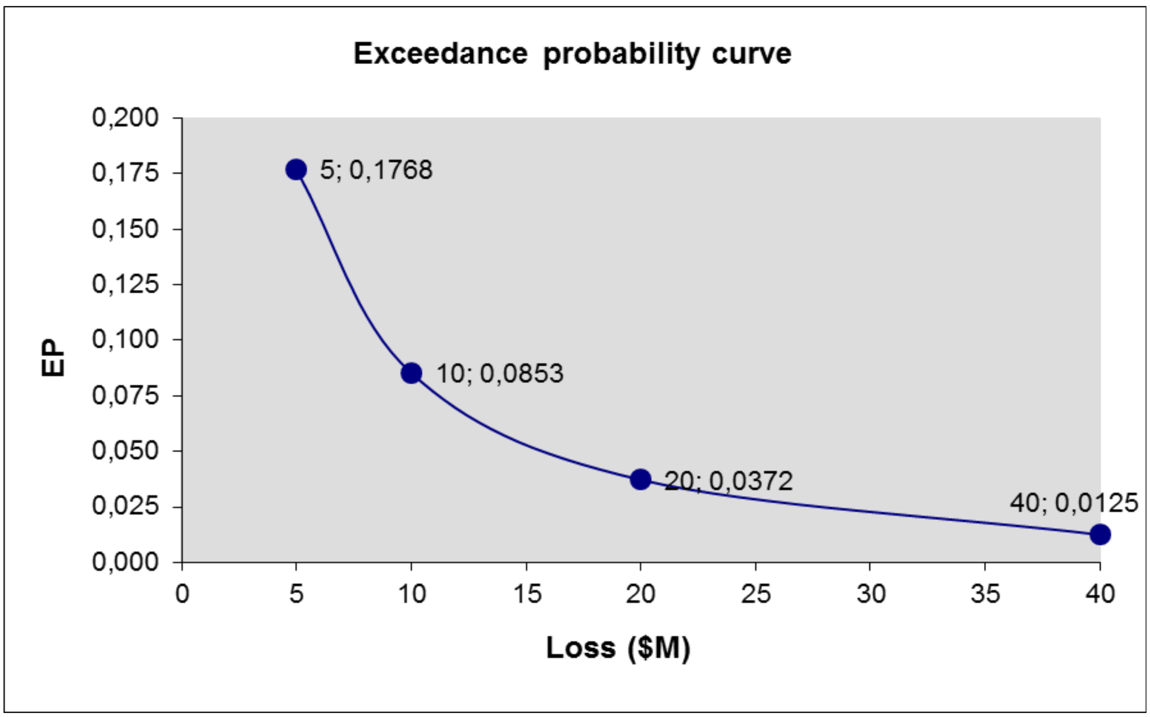
Event	Size of loss (x)	Annualprobability = 500k / x	Exceedanceprobability
4	40,000,000	0.0125	0.0125
3	20,000,000	0.025	0.0372
2	10,000,000	0.05	0.0853
1	5,000,000	0.1	0,1768

$EP = 1 - \pi\ (1 - p_i)$

$1 - (1 - 0.0125) * (1 - 0.025) = 0.0372$

$1 - (1 - 0.0125) * (1 - 0.025) * (1 - 0.05) = 0.0853$

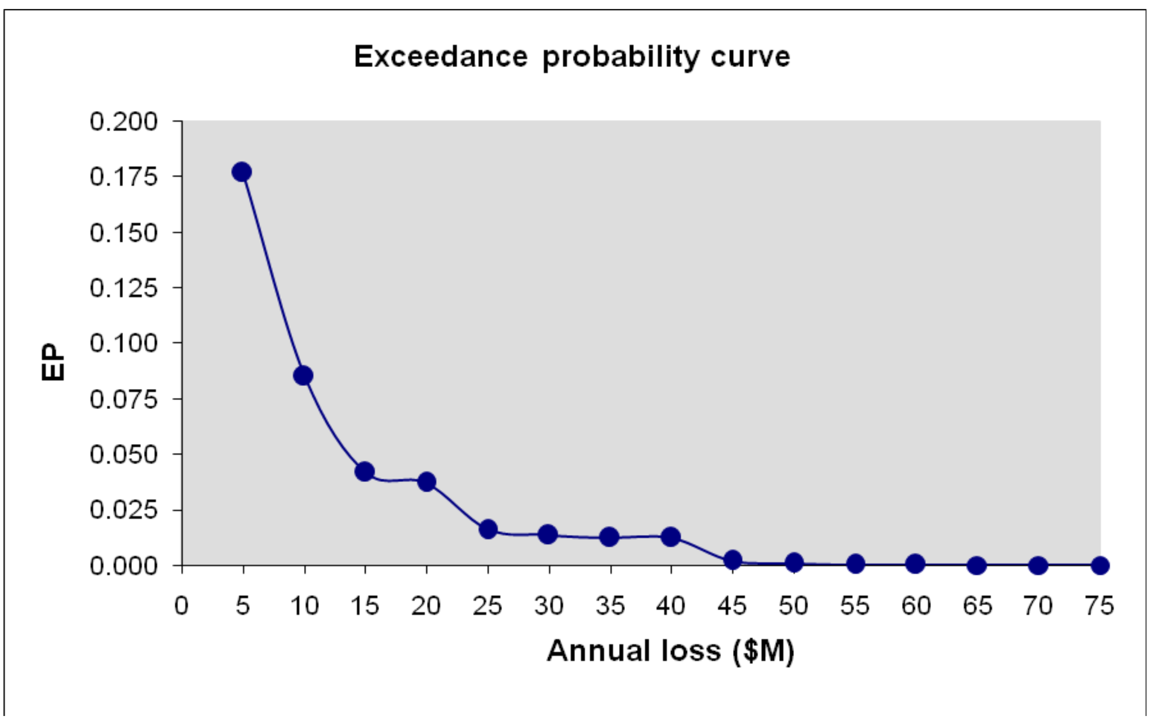
$1 - (1 - 0.0125) * (1 - 0.025) * (1 - 0.05) * (1 - 0.1) = 0.1768$



Sample 2

Size of loss (x)	Annualprobability = 500k / x
40,000,000	0.0125
20,000,000	0.025
10,000,000	0.05
5,000,000	0.1

AnnualLoss	Probability	Exceedanceprobability
5	0.1*0.95*0.975*0.9875 = 0.0915	0.1768
10	0.9*0.05*0.975*0.9875 = 0.0433	0.0853
15	0.1*0.05*0.975*0.9875 = 0.0048	0.0420
20	0.9*0.95*0.025*0.9875 = 0.0211	0.0372
25	0.1*0.95*0.025*0.9875 = 0.0024	0.0161
30	0.9*0.05*0.025*0.9875 = 0.0011	0.0137
35	0.1*0.05*0.025*0.9875 = 0.0001	0.0126
40	0.9*0.95*0.975*0.0125 = 0.0104	0.0125
45	0.1*0.95*0.975*0.0125 = 0.0012	0.0021
50	0.9*0.05*0.975*0.0125 = 0.0005	0.0009
55	0.1*0.05*0.975*0.0125 = 0.0001	0.0004
60	0.9*0.95*0.025*0.0125 = 0.0003	0.0003
65	0.1*0.95*0.025*0.0125 = 0.0000	0.0000
70	0.9*0.05*0.025*0.0125 = 0.0000	0.0000
75	0.1*0.05*0.025*0.0125 = 0.0000	0.0000



Question 6

Sample 1

A) EL_1 = \$4.5k; EL_2 = \$4.5k; EL_3 = \$4.5k
The insurer eliminates portfolio 3 since portfolios 1 and 2 are negatively correlated. Eliminating portfolio 2 would leave portfolios 1 and 3, which are highly correlated. Even though portfolio 2 has the highest standard deviation, it should remain because it is negatively correlated with portfolio 1 and the standard deviation is only 25k greater than that of portfolio 3.

B) 1. Model to model variance – the same input could produce very different results if another model is used.
2. Public acceptance – the public has been slow to accept the models since they generally result in higher rates.

Sample 2

A) AAL: Portfolio 1 = Portfolio 2 = Portfolio 3 = 4500
All three portfolios have the same expected loss, but the variation of loss varies. Since portfolios 1 and 3 are highly correlated, as evidenced by the similar expected loss amounts, the insurer could eliminate one of these to reduce overall risk. Since an increase in variation of expected loss increases the risk, portfolio 3 could be considered riskier. Therefore, I would recommend eliminating portfolio 3.

B) 1. Historically, regulators have not widely accepted the use of models. It requires expertise to evaluate, modeling firms don't release all proprietary aspects of model; and they often result in rate increases.
2. They lie outside most actuaries' expertise. Therefore, actuaries must rely on outside experts. However, they should still have a basic understanding of the model, validate results and inputs, and determine if it is appropriate for its intended purpose.

Question 7

Sample 1

4M x 1M policies covers losses up to $5M ground up.
The largest loss in the experience is 2.977M.
Since no loss is observed b/w 3M to 5M, experience rating in this layer is inappropriate.
In order to leverage the experience, though, the best method is to exposure rate both layers (free cover + exposed cover), and then apply their ratio to the experience rate.

Exposure Rating

4M x 1M on 10 M property corresponds to of exposure factor	70% - 35% = 35%
Exposed layer: 2M x 1M corresponds to of exposure factor	55% - 35% = 20%
Free cover: 2M x 3M corresponds to of exposure factor	70% - 55% = 15%

Experience Rating
Assume LDF's are excess loss LDF factors!

Losses: take reinsured layer, then apply LDF.

			LDF	
2,976,920	→	1,976,920	→	1,976,920
		X		
		1,215,965	→	1,264,604
		564,398	→	586,974
		1,731,572	→	1,870,098
				5,698,596

∑ Subj. Prem = 20,700,000
LR = 0.2753

Using Exposure Rate Ratio
=> 0.2753 * (35%/20%)

= 0.4818

Sample 2

Find policy's loss cost as % of subject premium

DOL	Trended Loss	Losses in Layer 4Mxs1M
3/08	2,976,920	1,976,920
10/08	972,574	0
4/09	2,215,965	1,215,965
6/09	1,564,398	564,398
8/10	2,731,572	1,731,572

AY	Subj Prem	Trended Loss in Layer	LDF	Developed Trended Loss	LR
2008	6,400,000	1,976,920	1.00	1,976,920	.309
2009	6,900,000	1,780,363	1.04	1,851,578	.268
2010	7,400,000	1,731,572	1.08	1,870,098	.253
	20,700,000			5,698,596	.275

No losses trend into the 3M-5M portion of the layer

Exposure Factor 1M-3M portion = EF(3M/10M)-EF(1M/10M)
=.55-.35 = .20
Exposure Factor 3M-5M portion = EF(5M/10M)-EF(3M/10M)
= .70-.55 = .15
Exposure Factor 1M-5M = .70-.35 = .35

Layer Portion	Experience Rating	Exposure Rating
1M-3M	.275	.20
3M-5M	.275*(.15/.20)=.206	.15
Total	.481	.35

Loss cost as a % of subject premium is 48.1%.

Since experience losses do not trend into upper portion of layer use experience rating for lower portion and exposure rating to use as relativities for determining rate for upper portion. This avoids free cover.

Sample 3

Assuming given LDF is a LDF on total ground-up losses.

AY	Trended Losses	LDF	Trended and Ultimate	Losses in Layer	On-level Trended Subject Premium
2008	2,976,920+972,574=3,949,494	1.00	3,949,494	2,949,494	6,400,000
2009	2,215,965+1,564,398=3,780,363	1.04	3,931,578	2,931,578	6,900,000
2010	2,731,572	1.08	2,950,098	1,950,098	7,400,000
				7,831,169	20,700,000

The trended and ultimate ground-up losses from actual experience is maximum at 40% of insured value.

The experience is in sufficient amount and reliable, consequently the excess layer from 3,000,000 excess of 1,000,000 can be rated based on experience.

No historical data penetrate the upper portion of the excess layer, so the remaining upper portion shall be rated based on exposure using relativity to avoid free cover.

The experience rate for the lower portion of the XL layer

=7,831,169/20,700,000 = .378317

Relativity from exposure rate = (70%-63%)/(63%-35%) = .25

Consequently, the final rate for the entire XL = .378317*(1+0.25) = 0.472896

Question 8

Sample 1

Range	Avg %	Reassumed by cedent	net to reins
0-60	47.9	0	47.9
60-70	67.8	=50%(67.8%-60%)=3.9%	63.9=67.8-3.9
75-90	81.5	=50%(15%)+80%(81.5%-75%)=12.7%	68.8=81.5-12.7
90+	99.6	=50%(15%)+80%(15%)=19.5%	80.1=99.6-19.5

Σ(net to reins)(prob in range)
=47.9(0.45)+63.9(0.29)+68.8(0.17)+80.1(0.09)
=58.99%

Sample 2

Range	Avg %	Pr(Range)	Reins LR
0-60	47.9	0.45	47.9
60-70	67.8	0.29	0.60+(.678-.60)*0.5=0.639
75-90	81.5	0.17	0.60+(.75-.60)*0.5+(0.815-0.75)*0.2=0.688
90+	99.6	0.09	0.60+(.75-.60)*0.5+(0.9-0.75)*0.2+(0.996-0.9)=0.801

E[LR]=0.479(0.45)+0.639(0.29)+0.688(0.17)+0.801(0.09) = .58991

Sample 3

Loss Ratio	Avg	Probability	Avg * Prob	Corridor	
0-60	47.9	0.45	0.21555	-	0
60-70	67.8	0.29	0.19662	=0.078*0.5	0.039
75-90	81.5	0.17	0.13885	=0.15*0.5+0.065*0.8	0.127
90+	99.6	0.09	0.08964	=0.15*0.5+0.15*0.8	0.195
			0.64036		

Corridor*Prob= 0.05045

Reinsurer's expected LR = 0.64036-0.05045 = 0.58991

Question 9

Sample 1

a. D = 50M   M = 100M so d = 50/100 = 0.5
   $G(0.5) = [\ln(0.1 + 0.01^{0.5}) - \ln(1.1)] / [\ln(0.11) - \ln(1.1)] = 0.7404$

b. p = G'(1) / G'(0)
   $G'(x) = [(\ln(0.01) * (0.01)^{x}) / (0.1 + 0.01^{x})] / [\ln(0.01 + 0.1) - \ln(1.1)]$
   $G'(1) = [(\ln(0.01) * (0.01)) / (0.1 + 0.01)] / [\ln(0.01 + 0.1) - \ln(1.1)]$
   = 0.1818
   $G'(0) = [(\ln(0.01) * 1)) / (0.1 + 1)] / [\ln(0.01 + 0.1) - \ln(1.1)]$
   = 1.8182
   p = 0.1818 / 1.8182 = 0.1

Sample 2

a. Using exposure curves → D = 50M   M = Total Loss = 100M d = D/M = 0.5
   G(0.5) = 0.74

b. a = 0.1 b = 0.01
   a = [(g – 1)*b] / (1 – gb)
   0.1 = [(g – 1)*0.01] / (1 – 0.01g)
   10 – 0.1g = g – 1
   11 = 1.1g
   g = 10
   p = 1/g = 1/10 = 0.1 = probability of total loss

Question 10

Sample 1

a)

Limit	ILF	ΔILF / ΔLimit (must decrease)
100	1.000	
250	1.250	.001667
500	1.450	.000800
750	x	(x – 1.45) / 250
1000	1.750	(1.75 – x) / 250
2000	2.250	.000500

must satisfy both 1.45 < x < 1.75 and

$$.0005 < \frac{1.75-x}{250} < \frac{x-1.45}{250} < .0008$$

x < 1.625

1.6 < x
x > 1.6

x < 1.65

To satisfy all conditions, **1.6 < x < 1.625**

b) Consistency test means mathematically that the ILF curve is increasing at a decreasing rate (concave down)

Practically, it means you pay more for more coverage, but each additional unit of coverage is cheaper, because a higher layer of equal width can never have more expected losses than a lower layer of equal width.

Sample 2

a) Range of possible ILFs for a 750K limit

let x = ILF for 750K

$$\frac{x-1.45}{750-500} > \frac{1.75-x}{1000-750}$$

$$\frac{\cancel{250}}{\cancel{250}}(x-1.45) > 1.750 - x$$

$2x > 3.2$

$x > 1.6$ $\qquad$ $x > 1.6$

and $\frac{x-1.45}{250} < \frac{1.45-1.25}{250}$

$x < 1.650$ $\qquad$ $x < 1.650$

$$\frac{2.250-1.750}{1000} < \frac{1.75-x}{250}$$

$.125 < 1.750 - x$ $\qquad$ $x < 1.625$

$x < 1.625$

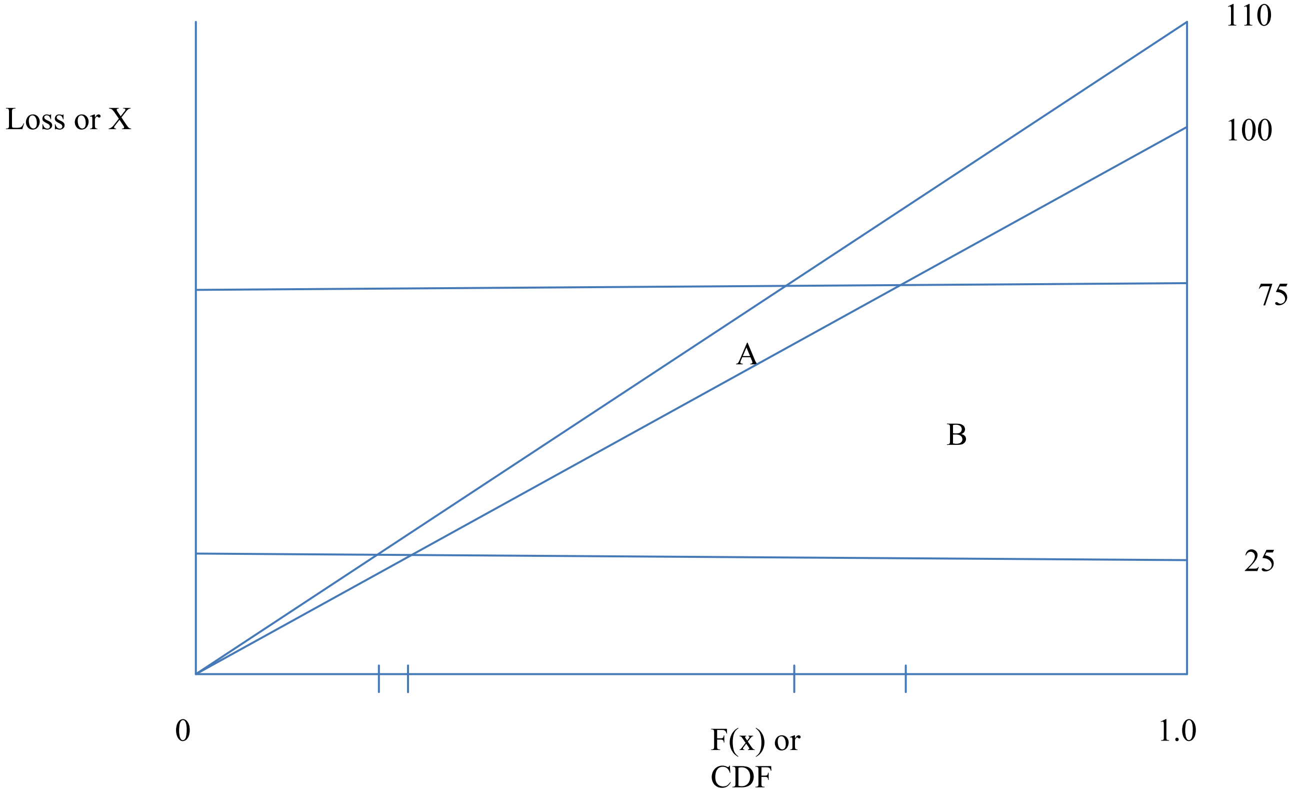
$x > 1.6$ and $x < 1.625$

b) Mathematic interpretation is that the increased limit function increases at a decreasing rate because the first derivative is positive and the 2nd derivative is negative.

Practical interpretation is that as limit increase, there are less losses expected at higher limits, so rates should not increase more for higher limits than for lower limits

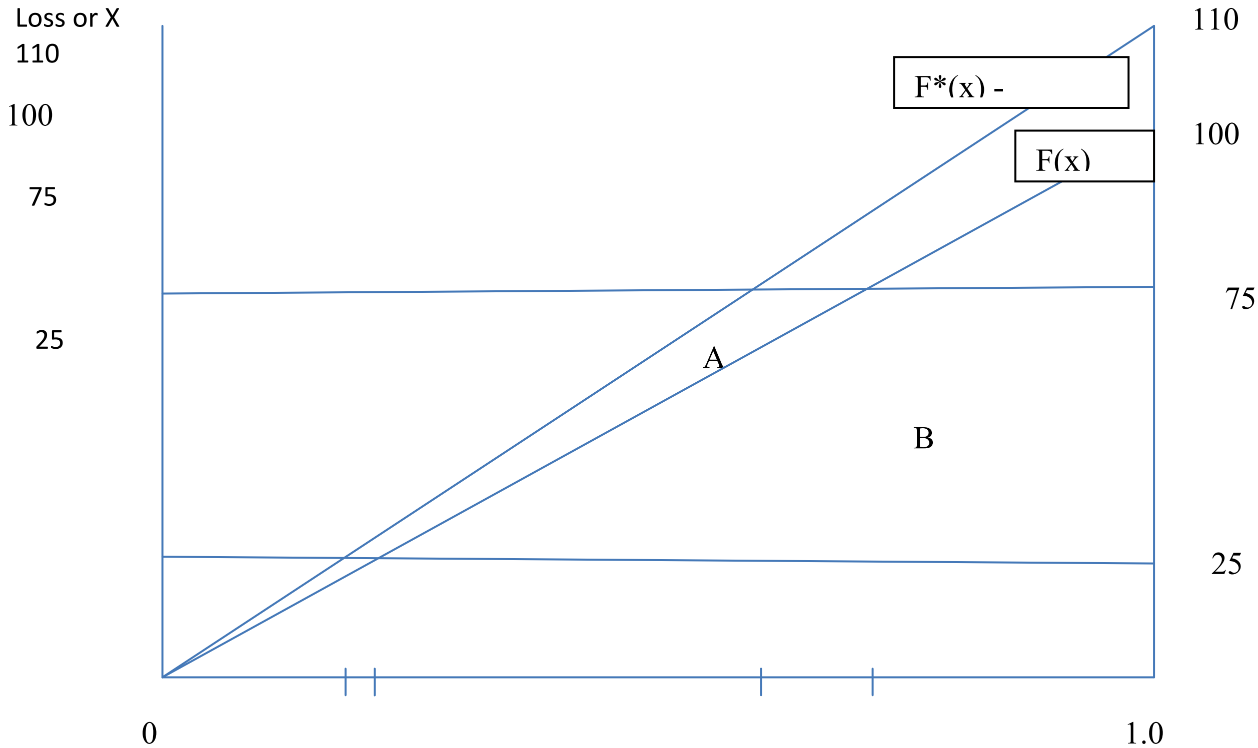
Question 11

Sample 1



E[x;25,75] = 50 [(1-25/100)+(1-75/100)]*1/2 = 25
E[x';25,75] = 50 [(1-25/110)+(1-75/110)]*1/2 = 27.27
Implied Trend = 27.27/25 -1 = 9.09%

Sample 2



Trend in ıayer = <u>a(E[x;75/1.1]-E[x;25/1.1]</u> -1
E[x;.75]-E[x,25]

E[x, 75] = $\int_{0}^{75} \frac{x}{100} dx + \int_{75}^{100} \frac{75}{100} dx$ = 46.875

E[x, 25] = $\int_{0}^{25} \frac{x}{100} dx + \int_{25}^{100} \frac{25}{100} dx$ = 21.875

E[x, 75/1.1] = $\int_{0}^{68.8} \frac{x}{100} dx + \int_{68.8}^{100} \frac{68.8}{100} dx$ = 44.944

E[x, 25/1.1] = $\int_{0}^{22.7} \frac{x}{100} dx + \int_{22.7}^{100} \frac{22.7}{100} dx$ = 20.124

Trend in layer = 1.1 (44.944-20.124)/(46.875-21.875)-1 = .092

Question 12

Sample 1

1) ↓ <u>Weight on med-only losses:</u> Now that the weight has been reduced to 30% previous non-reporters may now report their med-only losses. If they must report them, they have an incentive to keep claims down, so that their mod is as low as possible.
Also improves predictive accuracy because the insurer gets a more complete, true assessment of the insured's loss potential (by receiving all info)

2) ↑ <u>Weight on xs losses:</u> Definitely promotes safety: fewer xs losses (which now have more weight) → lower mod → lower premium.
This change improved plan performance, so necessarily predictive accuracy improved.

3) <u>Split point inflation-sensitive:</u>

- No foreseeable effect on safety incentive.

- Improves predictive accuracy because it retains an appropriate portion of primary losses, which represent frequency (XS losses represent severity). This also improves plan performance.

Sample 2

	safety	predictive accuracy
reduced weight on med-only loss	employers will report more of their med-only losses which will increase their safety	does not affect
increased weight on excess losses	increases the safety incentive as more of the excess losses will be included in the mod calculation	improves predictive accuracy; excess credibility was shown to be too low
primary/excess split inflation sensitive	no support	with inflation it becomes important to the accuracy of the plan to increase the split. D-ratios and W values can be distorted if no movement is made.

Question 13

Sample 1

OB = .99 = STD / MAN
STD = 201,800

MAN: A = 56,000
82,000
41,000
26,000
205,000

Where X = Non Experience Rated Premium

$$.99 = \frac{STD + X}{MAN + X}$$

.99 MAN + .99X = STD + X
.99 MAN – STD = .01 X
X = 115,000

	Mod	MAN Prem	STD Prem
A	.85	56,000	47,600
B	.95	82,000	77,900
C	1.1	41,000	45,100
D	1.2	13,000	15,600
E	M	13,000	13000M
			186,200 + 13,000 M

$$\mathbf{1.01} = \frac{186{,}200 + 13{,}000\ M + 115{,}000}{205{,}000 + 115{,}000} \rightarrow \quad \mathbf{M = 1.692}$$

Sample 2

Manual Prem = Standard Prem / Mod

OB = .99 → X = 115000; Mod = 1.00

Claim	Man Prem
A	56000
B	82000
C	41000
D	26000
X	115000

2nd Year

$$\frac{56*0.85+82*0.95+41*1.1+13*1.2+13*Y+115}{56+82+41+13+13+115} \longrightarrow Y = 1.69$$

Claim	Man Prem	Mod
A	56000	0.85
B	82000	0.95
C	41000	1.1
D	13000	1.2
E	13000	Y
X	115000	1.0

Question 14

Sample 1

A.)
Max Credit for Equipment: 10%
Max Credit for Premises: 10%
Max Credit for Employees: 6%

Total of above = 25%,

However credit is limited to 25%, so Total Credit is 25%

b.)
The account has worse than average experience for its class
The account may be a bad fit for the class.

Sample 2

Max Credit for Equipment: 10%
Max Credit for Premises: 10%
Max Credit for Employees: 6%

Total of above = 25%,

However credit is limited to 25%, so Total Credit is 25%

b.)
The account has worse than average experience for its class
Manual Rates for the class are inadequate

Sample 3

Equipment - replaced within last 4 months (so this would not be in reflected in experience period) Credit of 10%

Employees – all employees have been trained on the new equipment and this is also not reflected in the experience: 6% Credit

Location – Five months ago, so not yet reflected in experience
- Improvement in exposure inside premises: 5% Credit
- Improvement in exposure outside premises: 5% Credit

Total credit = 10% + 6% + 5% + 5% = 26% credit, but subject to a max credit of 25% so Max Credit = 25%

b.)
- The insured is not a good fit within its class
- The insured has worse than average loss experience in its class

Question 15

Sample 1

1) We use the general formula for experience modification

$$M = \frac{A_p + WA_e + (1 - W)E_e + B}{E + B}$$

2) A_p and A_e are actual primary and excess losses. They are calculated as

$$A = \begin{cases} 5{,}000 \text{ if losses} > 5{,}000 \\ \text{losses if losses} \leq 5{,}000 \end{cases}$$

applied by claim. A_e is calculated as total losses – A_p.

Also, losses are limited by various limits found in the NCCI Experience Rating manual.

3) E is calculated as

$$\sum \frac{payroll}{3} \times \text{ELR}$$

for the three years used in the experience period.

E_e is calculated as

$$\text{E} \times \text{D-ratio}$$

ELR and D-ratios are found using classification codes in the NCCI tables.

4) The frequency is reflected in the primary losses that are given more weight than excess losses which represent severity.

Sample 2

The formula used to calculate mod is:

$$M = \frac{A_p + WA_e + (1 - W)E_e + B}{E + B}$$

5) Look at employer's class code and search in NCCI experience rating manual for the respective ELR and D-ratio.

6) Apply ELR to employee's payroll to get expected total loss, E. Then:

$$E_p = E \times \text{D-ratio}$$

$$E_e = E - E_p$$

7) Use the total expected loss to search in NCCI manual for corresponding weighting value W and ballast value B.

8) Look at the actual losses during the experience period, and apply a per occurrence limit of SAL for each loss (single risk). Limit of 2×SAL for multiple risks per occurrence. Actual primary loss A_p is determined by applying a single split of 5,000 to each risk, multiple risks per occurrence has primary loss capped at 10,000. Med-only claims are only taking account 30%. Then:

$$A_e = A - A_p$$

9) Apply the formula above and round to two decimal places.

A_p reflects frequency and A_e reflects severity. Frequency is more important as A_p is taken as a whole, while A_e is applied a factor of W and weighted with E_e.

Question 16

Sample 1

Part a)

The necessary condition is that credit and debit risks equally reproduce the permissible loss ratio (PLR) and any random subgroup of credit and debit risks also reproduce the PLR, provided there is sufficient volume.
The sufficient condition is that there is no way to select a subgroup of credit or debit risks on any experience basis that will produce a different standard loss ratio in the prospective period.

Part b)
A -> Actual
E -> Expected
Stat = Var (A/Mod E) / Var(A/E)

Group	A/Mod E	A/E	
1	1.027	0.984	
2	1.043	1.000	
3	1.031	1.005	
4	1.081	1.107	
5	0.933	1.133	
Var	0.0024	0.0038	<- population var

Test Stat = 0.0024/0.0038 = 0.632

Part c)
Plan B has assigned more appropriate credibility because of the test statistic is smaller. It eliminated much of the variance after the application of the experience mod.

Sample 2

Part a)

The necessary condition is that credit and debit risks equally reproduce the permissible loss ratio (PLR) and any random subgroup of credit and debit risks also reproduce the PLR, provided there is sufficient volume.
The sufficient condition is that there is no way to select a subgroup of credit or debit risks on any experience basis that will produce a different standard loss ratio in the prospective period.

Part b)

Actual/Expected	Actual/Modified Expected
0.9842	1.0275
1.0000	1.0428
1.0050	1.0308
1.1073	1.0810
1.1333	0.9333
E = 1.0460	E = 1.0231
Var = 0.019/(5-1)	Var = 0.0119/(5-1)
= 0.00475	= 0.00297

Test Statistic = 0.00297/0.00475 = 0.6254

Part c)
Plan B has assigned more appropriate credibility because of the test statistic is smaller. It eliminated much of the variance after the application of the experience mod.

Question 17

Sample 1

Range	Average	#
<= 500	250	X = 350
500-1000	750	Y = 50
>1000	2000	500 – X – Y = 100

f * LER(500) = 0.8 * [250(X) + 500(500-X)] / (650 * 500) = 0.40
500*500 – 250X = 162,500
X = 350

250X + 750Y + 2000*(500-X-Y) = 500*650
87,500 + 750Y + 300,000 – 2000Y = 500*650
Y = 50

F*LER(1000)
= 0.8 * (250*350+750*50+100*1000)/(500*650)
= 0.55

Sample 2

tempered LER @500 => 0.4 = 0.8k => k = 0.5
Total losses=650 x 500 = 325,000
Losses eliminated by a $500 deductible = 325,000 x 0.5 = 162,000
162,500 = (750-500)Y + (2000-500)*Z
where Y = Number of losses from 500 to 1000
Z = Number of losses above 1000

(1) X + Y + Z = 500

(2) 250X + 750Y + 2000Z = 325,000

(3) 250Y + 1500Z = 162,000

Solve for Z

(4) (2) – 250*(1) => 500Y + 1750Z = 200,000

(5) 2*(3) – (4) => 1250Z = 125,000

Z = 100

Loss remaining from $1000 deductible is = (2000-1000)Z = 100,000
k@$1,000 = 1 – 100,000/325,000 = 0.6923
fk_{1000} = 0.5538 = tempered LER @ $1000

Sample 3

X <= 500	250	a%
500 < X <= 1000	750	b%
X > 1000	2000	(1 - a% - b%)

250 a% + 750 b% + 2000 (1 – a% - b%) = 650
250 a + 750 b +2000 – 200a – 200b = 650
1350 = 1750 a + 1250 b
0.4 / 0.8 = [250 a% + 500 (1-a%)] / 650
a = 0.7
b = 0.1

tempered loss elimination ratio for 1000
= {[250 x 0.7 + 750 x 0.1 + 1000 x 0.2] / 650 } x 0.8
= .554

Question 18

Sample 1

a. LDD Premium = $\frac{EL(XL + ULAE + LBA) + SP(GO + CR)}{1 - A - T - P}$

$= \frac{1m(.65)[.11 + .07 + .05] + 1m[.065 + .04]}{1 - .02 - .045 - .025}$

$= \frac{149{,}500 + 105{,}000}{.91}$

=279,670.33

$XL = InsCR + XLPP$

=.08+.03=.11

b. 1. Credit risk – LDD insurers have the risk that the insured might not reimburse them for deductible losses, and so this needs to be charged for in the premium calculation. Excess insurance does not have credit risk because they reimburse the insured for losses above the retention, so there is no credit risk charge fro excess insurance.

ULAE – LDD insurers do all the servicing on all losses, so the ULAE cost is calculated on the entire expected loss. Since excess insurers only step in above the retention, ULAE is only charged on excess losses.

Sample 2

a. Prem = $\frac{650{,}000 \bullet (0.08 + 0.03 + 0.05 + 0.07] + 1{,}000{,}000(0.065 + 0.04)}{1 - 0.045 - 0.02 - 0.025}$

=279,670

b. 1. Taxes & Assessment: LDD is WC, excess is not. Excess pays no assessments & pays G.L. tax rates making it cheaper.

2. Competition & Profit load: Excess competes on price alone, & so profit loads are competitive & low, (LDD also offer claim handling services). Lowers excess price.

Question 19

Sample 1

E = 0.68 * 925,000 = 629,000
XS Loss = 0.18 * 925,000 = 166,500
E(L) = (0.68 – 0.18) * 925,000 = 462,500

LER = 1 – (0.18 / 0.68) = 1 – 0.7353 = .2647

Adjusted E = E * Hazard * (1+ 0.8 * LER)/(1 – LER) = 932,933

ELG = 28

R = 900,000 / 462,500 = Agg. Ded. / E(L) = 1.9459 (round to 2)

Insurance Charge = 0.06 * E(L) = 27,750

Loss Cost = XS Loss + Charge = 166,500 + 27,750 = 194,250

Sample 2

Expected Unlimited Loss = 925,000 * 0.68 = 629,000
LER = 0.18 / 0.68 = 0.265
Loss Group Adjustment Factor = 1 + (.8 * .265) / (1 - .265) = 1.65

Adjusted Loss = 629,000 * 1.65 * .90 = 934,065

Loss Group = 28

Expected Limited Loss = .50 * 925,000 = 462,5000
XS Loss = 166,500
Entry Ratio = 900,000 / 462,500 = 1.95

Insurance Charge = [(1.95 – 1.5) / (2.0 – 1.5) * (.06 - .148)] + .148 = .0688

Total Loss Cost = .0688 * 462,500 + 166,500 = 198,320

Question 20

Sample 1

Losses @ Min= $150,000
Losses @ Max= $400,000

E[Ground Up Loss] = 0.1 * 0 + 0.9 * 250,000 = 225,000

E[L] = Expected losses entering retro formula
= 0.1 * 150,000 + .9*[.3*150,000 + 0.2*400,000 + 0.5 * [(150,000 + 400,000)/2]
= 251,250

Expected Retro Premium = (b + c*E[L])T
= (50,000 + 1.2 * 251,250) = 351,500

Guaranteed cost premium = expected retro premium in a balanced plan.

Sample 2

E[A] = 0.9 * 250,000 = 225,000

Insurance Charge

Above Max =(500k – 400k)/2 * 0.2 * 0.9 = 9,000

Below Min =[(150k-0)/2 * 0.3 * 0.9] + 150k *0.1 = 35,250

Converted Insurance Charge = 1.2 * (9,000 – 35,250) = -31,500

b = e - (c-1)E + cI

e+E = (b + cE - cI)*T

= 50,000 + 1.2 * 225,000 – (-31,500)

= 351,500

The plan is balanced, so GCP = e+E = 351,500.

Question 21

Sample 1

Excess Loss Factor = .69
Hazard Group Differential (F) = .70

LUGS = 2,000,000 (.70) (1+.8 * .69)/(1-.69)
=7,009,032

Using 2008 table ELG = 24

Sample 2

Assume 50,000 limit is per occurrence limit so you are looking to use LUGS as an adjustment for ICRLL

E(L)
LUGS = SP * ELR * m(s/h) * (1+.8LER) / (1 – LER)

m(s/h) = hazard group rel.

=2M (.7)(1+0.8 * .69)/(1-.69) = 7,009,032.26

.7 = lookup in table

LER = .69 <-lookup of excess L+ALAE pure prem. Factor for Alaska HGF (last page of retro manual)

ELG = 24 based on most recent tables [‘08]

Question 22

Part a)

Sample 1

Avg unlim LR = .6
Avg limited LR = .51

$$LER = \frac{E[LR]_{lim}}{E[LR]_{unlim}} = 1 - 0.51/0.6 = 1 - 0.85 = 0.15$$

(1) LR	(2) # risks	(3) # risk >	(4) # losses over	(5) $X_{part\ LR}$	(6) $X_{Full\ LR}$
0.00	0	5	17	1.0	1.0
0.15	1	4	12	0.7059	0.75
0.30	0	4	8	0.4706	0.55
0.45	2	2	4	0.2353	0.35
0.60	1	1	2	0.1176	0.25
0.75	0	1	1	0.0588	0.20
0.90	1	0	0	0	0.15
		17			

(3) = upward sum of (2)
(4) = upward sum of (3)
(5) = (4) / (3 total)
(6) = LER + (1-LER)(5)

Sample 2

E = .6
Ê = .51

$$LER = \frac{.6 - .51}{0.6} \qquad 0.15$$

LR	r	# risks	# above	% above	partial charge	charge
0.00	0	0	5	1	1	1
0.15	0.294	1	4	0.8	0.706	0.75
0.30	0.588	0	4	0.8	0.471	0.55
0.45	0.882	2	2	0.4	0.235	0.35
0.60	1.176	1	1	0.2	0.1176	0.25
0.75	1.471	0	1	0.2	0.0588	0.20
0.90	1.765	1	0	0	0	0.15

charge = LER + (1-LER) partial charge

Sample 3

$$\text{Ave Unlimited LR} = \frac{.3+.45+.45+.9+.9}{5} = .6$$

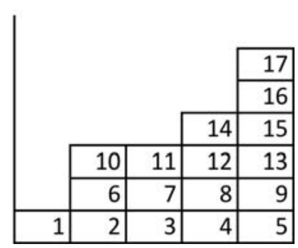
$$\text{Ave Limited LR} = \frac{.15+.45+.45+.6+.9}{5} = 0.51$$

=> 1 - 51/60 = .15 of loss eliminated by loss limitation

Use vertical sum method 1

Table L

LR	L Charge
0	1
15%	.15 + 12/17*.85 = .75
30%	.15 + 8/17*.85 = .55
45%	.15 + 4/17*.85 = .35
60%	.15 + 2/17*.85 = .25
75%	.15 + 1/17*.85 = .2
90%	0.15



Part b)

Sample 1

When an occurrence limit is introduced, some losses are eliminated above that limit and never get considered on an aggregate limit. A charge needs to be included for the occurrence limitation. However, careful not to overlap occurrence charge and aggregate charge if using an unlimited charge table. Table L fixes this by already reflecting the occurrence limitation and adding a charge for it.

Sample 2

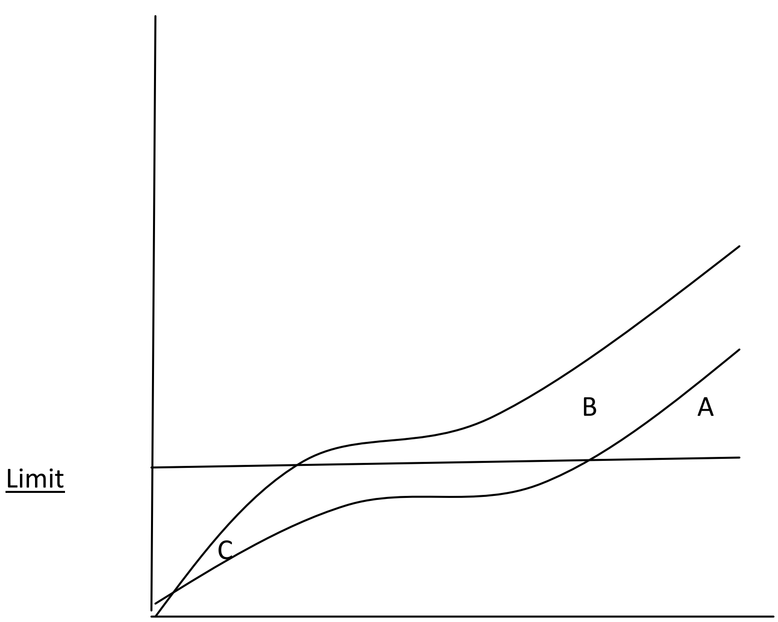
Insurance charge decreases when loss limit is introduced due to overlap between excess loss pure prem factor and insurance charge (assuming insurance charge here is not Table L insurance charge, which includes the LER, and is therefore increased.)

Sample 3

Insurance charge is increased, since both pre occurrence limit and aggregate limit decrease the ratable loss.

Sample 4

When loss limit is introduced, the charge is A + B + C.
Without loss limit, the charge is A + B



Part c)

Sample 1

1) Calculate r using sample LR instead of expected LR.
2) Use expected LR, but at end divide both r and $\phi(r)$ by $\phi(o)$.

Sample 2

When the sample loss ratios are not the same as the expected loss ratios, one of the approach is directly using sample loss ratio to make sure the charge at zero entry ratio is always equal to one. Another approach is to divide all the entry ratios and the charges to the calculated charge at entry ratio zero to obtain the new table of insurance charges.

Question 23

Sample 1

Limited Losses = 250,000 x (1 - .4) = 150,000
R = 300 / 150 = 2, Lookup charge = .170
Total charge = .170 (150,000) = $25,500

If 333,000, but ELPPF still 40%
Limited Loss = 333,000 x (1 - .4) = 199,800
R = 300,000 / 199,800 = 1.50, charge = .280
Charge = .280 (199,800) = $55,944

Difference = 55,944 – 25,500 = $30,444

Sample 2

Exp lim loss = 250K x (1 - .4) = 150K
R = 300 / 150 = 2
Lookup 100, 2
First way = .170 x 150K = 25500

Exp lim loss = 333K x (1 - .4) = 200K
R = 300 / 200 = 1.5
Lookup
Second way = .28 x 200K = 56000

Diff = 56000 – 25500 = 30500

Question 24

Sample 1

Mahler uses empirical data at less than 100k, then he blends exponential and Pareto distributions at higher limits after truncating and shifting the data.
Mahler wanted to rely on actual experience to the extent possible at the lower limits while fitting a curve to make up for sparser data at higher limits. The Pareto has a thick tail which makes it ideal at very high limits while the exponential works well just above the experience cut off point.

Sample 2

In this method, Mahler wanted to be able to calculate the excess ratio based on experience and the exponential/pareto distribution
$R(L) – R(L_{truncpt}) * R_{fit}(L-L_{truncpt})$
Under the trucation point, the excess ratio is calculated directly to rely on the actual data. Above the truncation point, the excess ratio is derived from a combination of the short tailed exponential distribution and the long-tailed pareto distribution. The actual data above truncation point is used to curve fir these two distributions. By using both distributions mid-sized claims will get the exponential shape while large losses with get the fatter tail of the pareto in order to make sure the excess ratios don't drop quickly. In giving with this approach, small sized claims will avoid being curve fitted and the maximum reliance of actual data is done in the lower layer where there is more data and the higher portion one will use curve fitting to predict the data.

Question 25

Sample 1

$X_H - X_G = (e+E-H/T)/cE$
$0.7 - 0.055 = (19{,}160/20{,}000 - H/1.025)/1.25*0.6$
$H = 0.4861$
$r_G = 0.95/0.6 = 1.583$
$r_H = 0.2/0.6 = 0.333$
$r_G - r_H = (G-H)/cET$
$1.583 - 0.333 = (G - 0.4861)/1.25*1.025*0.6$
$G = 1.447$

Sample 2

$\Psi(r_H) = 0.7 + 0.2/0.6 - 1 = 0.033$
$GCP = T(e+E)$
$19{,}760/20{,}000 = 0.988 = 1.025(e+0.6)$
$e = 0.3639$
$E = 12/20 = 0.6$
$I = 0.6(0.055 - 0.033) = 0.013$
$b = e - (c-1)E + cI = 0.3639 - 0.25(0.6) + 1.25(0.013) = 0.2302$
$G = (b+cr_GE)T$
$r_G = 0.95/0.6 = 1.583$
$G = (0.2302 + 1.25*1.583*0.6)*1.025 = 1.4531$

Sample 3

$GCP = 19{,}760 = (e+E)T$
$e+E = 19{,}160$
$E = 12{,}000$
$e = 7{,}160$
$T = 19{,}760/19{,}160 = 1.0313$
$B = e - (c-1)E + cI$
$cI = c(X_G - S_H)E$
$b = 7{,}160 - 0.25*12{,}000+1.25(0.055 - 0.0333)*12{,}000 = 4{,}485$
$X_G = 0.055$
$X_H = 0.700$
$S_H = X_H+r_H-1 = 0.7+1/3-1 = 0.03333$
$r_H = L_H/L = 0.2*SP/12{,}000 = 4{,}000/12{,}000 = 1/3$
$G = (b+L_Gc)T = (4485+19{,}000*1.25)*1.0313 = 29{,}119$
$L_G = 0.95*SP = 19{,}000$
$G \text{ ratio} = 29{,}119/SP = 1.456$