SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM C CONSTRUCTION AND EVALUATION OF ACTUARIAL MODELS

EXAM C SAMPLE QUESTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

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1. You are given:

(i) Losses follow a loglogistic distribution with cumulative distribution function:

$$F(x) = \frac{(x/\theta)^{\gamma}}{1+(x/\theta)^{\gamma}}$$

(ii) The sample of losses is:

10	35	80	86	90	120	158	180	200	210	1500

Calculate the estimate of θ by percentile matching, using the 40^{th} and 80^{th} empirically smoothed percentile estimates.

(A) Less than 77

(B) At least 77, but less than 87

(C) At least 87, but less than 97

(D) At least 97, but less than 107

(E) At least 107

2. You are given:

(i) The number of claims has a Poisson distribution.

(ii) Claim sizes have a Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$.

(iii) The number of claims and claim sizes are independent.

(iv) The observed pure premium should be within 2% of the expected pure premium 90% of the time.

Determine the expected number of claims needed for full credibility.

(A) Less than 7,000

(B) At least 7,000, but less than 10,000

(C) At least 10,000, but less than 13,000

(D) At least 13,000, but less than 16,000

(E) At least 16,000

3. You study five lives to estimate the time from the onset of a disease to death. The times to death are:

2 3 3 3 7

Using a triangular kernel with bandwidth 2, estimate the density function at 2.5.

(A) 8/40

(B) 12/40

(C) 14/40

(D) 16/40

(E) 17/40

4. You are given:

(i) Losses follow a Single-parameter Pareto distribution with density function:

$$f(x) = \frac{\alpha}{x^{(\alpha+1)}}, \quad x > 1, \quad 0 < \alpha < \infty$$

(ii) A random sample of size five produced three losses with values 3, 6 and 14, and two losses exceeding 25.

Determine the maximum likelihood estimate of α.

(A) 0.25

(B) 0.30

(C) 0.34

(D) 0.38

(E) 0.42

5. You are given:

(i) The annual number of claims for a policyholder has a binomial distribution with probability function:

$$p(x|q) = \binom{2}{x} q^x (1-q)^{2-x}, \quad x = 0, 1, 2$$

(ii) The prior distribution is:

$$\pi(q) = 4q^3, \ 0 < q < 1$$

This policyholder had one claim in each of Years 1 and 2.

Determine the Bayesian estimate of the number of claims in Year 3.

(A) Less than 1.1

(B) At least 1.1, but less than 1.3

(C) At least 1.3, but less than 1.5

(D) At least 1.5, but less than 1.7

(E) At least 1.7

6. For a sample of dental claims $x_1, x_2, \ldots, x_{10}$, you are given:

(i) $\sum x_i = 3860$ and $\sum x_i^2 = 4{,}574{,}802$

(ii) Claims are assumed to follow a lognormal distribution with parameters μ and σ.

(iii) μ and σ are estimated using the method of moments.

Calculate $E[X \wedge 500]$ for the fitted distribution.

(A) Less than 125

(B) At least 125, but less than 175

(C) At least 175, but less than 225

(D) At least 225, but less than 275

(E) At least 275

7. Two independent samples are combined yielding the following ranks:

Sample I: 1, 2, 3, 4, 7, 9, 13, 19, 20
Sample II: 5, 6, 8, 10, 11, 12, 14, 15, 16, 17, 18

You test the null hypothesis that the two samples are from the same continuous distribution.

The variance of the rank sum statistic is:

$$\frac{nm(n+m+1)}{12}$$

Using the classical approximation for the two-tailed rank sum test, determine the *p*-value.

(A) 0.015

(B) 0.021

(C) 0.105

(D) 0.210

(E) 0.420

8. You are given:

(i) Claim counts follow a Poisson distribution with mean θ.

(ii) Claim sizes follow an exponential distribution with mean 10θ.

(iii) Claim counts and claim sizes are independent, given θ.

(iv) The prior distribution has probability density function:

$$\pi(\theta) = \frac{5}{\theta^6}, \quad \theta > 1$$

Calculate Bühlmann's *k* for aggregate losses.

(A) Less than 1

(B) At least 1, but less than 2

(C) At least 2, but less than 3

(D) At least 3, but less than 4

(E) At least 4

9. You are given:

(i) A survival study uses a Cox proportional hazards model with covariates Z_1 and Z_2, each taking the value 0 or 1.

(ii) The maximum partial likelihood estimate of the coefficient vector is:

$$\left(\hat{\beta}_1, \hat{\beta}_2\right) = (0.71, 0.20)$$

(iii) The baseline survival function at time t_0 is estimated as $\hat{S}(t_0) = 0.65$.

Estimate $S(t_0)$ for a subject with covariate values $Z_1 = Z_2 = 1$.

(A) 0.34

(B) 0.49

(C) 0.65

(D) 0.74

(E) 0.84

10. You are given:

(i) Z_1 and Z_2 are independent N(0,1) random variables.

(ii) *a*, *b*, *c*, *d*, *e*, *f* are constants.

(iii) $Y = a + bZ_1 + cZ_2$ and $X = d + eZ_1 + f\,Z_2$

Determine $\mathrm{E}(Y\,|\,X)$.

(A) a

(B) $a + (b + c)(X - d)$

(C) $a + (be + cf)(X - d)$

(D) $a + \left[(be + cf)/\left(e^2 + f^2\right)\right]X$

(E) $a + \left[(be + cf)/\left(e^2 + f^2\right)\right](X - d)$

11. You are given:

(i) Losses on a company's insurance policies follow a Pareto distribution with probability density function:

$$f(x|\theta) = \frac{\theta}{(x+\theta)^2}, \quad 0 < x < \infty$$

(ii) For half of the company's policies $\theta = 1$, while for the other half $\theta = 3$.

For a randomly selected policy, losses in Year 1 were 5.

Determine the posterior probability that losses for this policy in Year 2 will exceed 8.

(A) 0.11

(B) 0.15

(C) 0.19

(D) 0.21

(E) 0.27

12. You are given total claims for two policyholders:

	Year			
Policyholder	1	2	3	4
X	730	800	650	700
Y	655	650	625	750

Using the nonparametric empirical Bayes method, determine the Bühlmann credibility premium for Policyholder Y.

(A) 655

(B) 670

(C) 687

(D) 703

(E) 719

13. A particular line of business has three types of claims. The historical probability and the number of claims for each type in the current year are:

Type	Historical Probability	Number of Claims in Current Year
A	0.2744	112
B	0.3512	180
C	0.3744	138

You test the null hypothesis that the probability of each type of claim in the current year is the same as the historical probability.

Calculate the chi-square goodness-of-fit test statistic.

(A) Less than 9

(B) At least 9, but less than 10

(C) At least 10, but less than 11

(D) At least 11, but less than 12

(E) At least 12

14. The information associated with the maximum likelihood estimator of a parameter θ is $4n$, where n is the number of observations.

Calculate the asymptotic variance of the maximum likelihood estimator of 2θ.

(A) $1/2n$

(B) $1/n$

(C) $4/n$

(D) $8n$

(E) $16n$

15. You are given:

(i) The probability that an insured will have at least one loss during any year is *p*.

(ii) The prior distribution for *p* is uniform on $[0, 0.5]$.

(iii) An insured is observed for 8 years and has at least one loss every year.

Determine the posterior probability that the insured will have at least one loss during Year 9.

(A) 0.450

(B) 0.475

(C) 0.500

(D) 0.550

(E) 0.625

16-17. *Use the following information for questions* 21 *and* 22.
For a survival study with censored and truncated data, you are given:

Time (*t*)	Number at Risk at Time *t*	Failures at Time *t*
1	30	5
2	27	9
3	32	6
4	25	5
5	20	4

16. The probability of failing at or before Time 4, given survival past Time 1, is ${}_3q_1$.
Calculate Greenwood's approximation of the variance of ${}_3\hat{q}_1$.

(A) 0.0067

(B) 0.0073

(C) 0.0080

(D) 0.0091

(E) 0.0105

17. Calculate the 95% log-transformed confidence interval for $H(3)$, based on the Nelson-Aalen estimate.

(A) (0.30, 0.89)

(B) (0.31, 1.54)

(C) (0.39, 0.99)

(D) (0.44, 1.07)

(E) (0.56, 0.79)

18. You are given:

(i) Two risks have the following severity distributions:

Amount of Claim	Probability of Claim Amount for Risk 1	Probability of Claim Amount for Risk 2
250	0.5	0.7
2,500	0.3	0.2
60,000	0.2	0.1

(ii) Risk 1 is twice as likely to be observed as Risk 2.

A claim of 250 is observed.

Determine the Bühlmann credibility estimate of the second claim amount from the same risk.

(A) Less than 10,200

(B) At least 10,200, but less than 10,400

(C) At least 10,400, but less than 10,600

(D) At least 10,600, but less than 10,800

(E) At least 10,800

19. You are given:

(i) A sample $x_1, x_2, \ldots, x_{10}$ is drawn from a distribution with probability density function:

$$\tfrac{1}{2}\left[\frac{1}{\theta}\exp(-\frac{x}{\theta}) + \frac{1}{\sigma}\exp(-\frac{x}{\sigma})\right], \quad 0 < x < \infty$$

(ii) $\theta > \sigma$

(iii) $\sum x_i = 150$ and $\sum x_i^2 = 5000$

Estimate θ by matching the first two sample moments to the corresponding population quantities.

(A) 9

(B) 10

(C) 15

(D) 20

(E) 21

20. You are given a sample of two values, 5 and 9.

You estimate Var(X) using the estimator $g(X_1, X_2) = \dfrac{1}{2}\sum (X_i - \bar{X})^2$.

Determine the bootstrap approximation to the mean square error of g.

(A) 1

(B) 2

(C) 4

(D) 8

(E) 16

21. You are given:

(i) The number of claims incurred in a month by any insured has a Poisson distribution with mean λ.

(ii) The claim frequencies of different insureds are independent.

(iii) The prior distribution is gamma with probability density function:

$$f(\lambda) = \frac{(100\lambda)^6 e^{-100\lambda}}{120\lambda}$$

(iv)

Month	Number of Insureds	Number of Claims
1	100	6
2	150	8
3	200	11
4	300	?

Determine the Bühlmann-Straub credibility estimate of the number of claims in Month 4.

(A) 16.7

(B) 16.9

(C) 17.3

(D) 17.6

(E) 18.0

22. You fit a Pareto distribution to a sample of 200 claim amounts and use the likelihood ratio test to test the hypothesis that $\alpha = 1.5$ and $\theta = 7.8$.

You are given:

(i) The maximum likelihood estimates are $\hat{\alpha}$ = 1.4 and $\hat{\theta}$ = 7.6.

(ii) The natural logarithm of the likelihood function evaluated at the maximum likelihood estimates is –817.92.

(iii) $\sum \ln(x_i + 7.8) = 607.64$

Determine the result of the test.

(A) Reject at the 0.005 significance level.

(B) Reject at the 0.010 significance level, but not at the 0.005 level.

(C) Reject at the 0.025 significance level, but not at the 0.010 level.

(D) Reject at the 0.050 significance level, but not at the 0.025 level.

(E) Do not reject at the 0.050 significance level.

23. For a sample of 15 losses, you are given:

(i)

Interval	Observed Number of Losses
(0, 2]	5
(2, 5]	5
(5, ∞)	5

(ii) Losses follow the uniform distribution on $(0,\theta)$.

Estimate θ by minimizing the function $\sum_{j=1}^{3}\frac{\left(E_j - O_j\right)^2}{O_j}$, where E_j is the expected number of losses in the *j*th interval and O_j is the observed number of losses in the *j*th interval.

(A) 6.0

(B) 6.4

(C) 6.8

(D) 7.2

(E) 7.6

24. You are given:

(i) The probability that an insured will have exactly one claim is θ.

(ii) The prior distribution of θ has probability density function:

$$\pi(\theta) = \frac{3}{2}\sqrt{\theta},\ 0 < \theta < 1$$

A randomly chosen insured is observed to have exactly one claim.

Determine the posterior probability that θ is greater than 0.60.

(A) 0.54

(B) 0.58

(C) 0.63

(D) 0.67

(E) 0.72

25. The distribution of accidents for 84 randomly selected policies is as follows:

Number of Accidents	Number of Policies
0	32
1	26
2	12
3	7
4	4
5	2
6	1
Total	84

Which of the following models best represents these data?

(A) Negative binomial

(B) Discrete uniform

(C) Poisson

(D) Binomial

(E) Either Poisson or Binomial

26. You are given:

(i) Low-hazard risks have an exponential claim size distribution with mean θ.

(ii) Medium-hazard risks have an exponential claim size distribution with mean 2θ.

(iii) High-hazard risks have an exponential claim size distribution with mean 3θ.

(iv) No claims from low-hazard risks are observed.

(v) Three claims from medium-hazard risks are observed, of sizes 1, 2 and 3.

(vi) One claim from a high-hazard risk is observed, of size 15.

Determine the maximum likelihood estimate of θ.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

27. You are given:

(i) $X_{\text{partial}} =$ pure premium calculated from partially credible data

(ii) $\mu = \mathrm{E}\left[X_{\text{partial}}\right]$

(iii) Fluctuations are limited to $\pm k\,\mu$ of the mean with probability P

(iv) Z = credibility factor

Which of the following is equal to P?

(A) $\Pr\left[\mu - k\mu \le X_{\text{partial}} \le \mu + k\mu\right]$

(B) $\Pr\left[Z\mu - k \le Z\,X_{\text{partial}} \le Z\mu + k\right]$

(C) $\Pr\left[Z\mu - \mu \le Z\,X_{\text{partial}} \le Z\mu + \mu\right]$

(D) $\Pr\left[1 - k \le Z\,X_{\text{partial}} + (1 - Z)\mu \le 1 + k\right]$

(E) $\Pr\left[\mu - k\mu \le Z\,X_{\text{partial}} + (1 - Z)\mu \le \mu + k\mu\right]$

28. You are given:

Claim Size (X)	Number of Claims
$(0, 25]$	25
$(25, 50]$	28
$(50, 100]$	15
$(100, 200]$	6

Assume a uniform distribution of claim sizes within each interval.

Estimate $\mathrm{E}(X^2) - \mathrm{E}\left[(X \wedge 150)^2\right]$.

(A) Less than 200

(B) At least 200, but less than 300

(C) At least 300, but less than 400

(D) At least 400, but less than 500

(E) At least 500

29. You are given:

(i) Each risk has at most one claim each year.

(ii)

Type of Risk	Prior Probability	Annual Claim Probability
I	0.7	0.1
II	0.2	0.2
III	0.1	0.4

One randomly chosen risk has three claims during Years 1-6.

Determine the posterior probability of a claim for this risk in Year 7.

(A) 0.22

(B) 0.28

(C) 0.33

(D) 0.40

(E) 0.46

30. You are given the following about 100 insurance policies in a study of time to policy surrender:

(i) The study was designed in such a way that for every policy that was surrendered, a new policy was added, meaning that the risk set, r_j, is always equal to 100.

(ii) Policies are surrendered only at the end of a policy year.

(iii) The number of policies surrendered at the end of each policy year was observed to be:

1 at the end of the 1st policy year
2 at the end of the 2nd policy year
3 at the end of the 3rd policy year
⋮
n at the end of the n^{th} policy year

(iv) The Nelson-Aalen empirical estimate of the cumulative distribution function at time n, $\hat{F}(n)$, is 0.542.

What is the value of n?

(A) 8

(B) 9

(C) 10

(D) 11

(E) 12

31. You are given the following claim data for automobile policies:

200 255 295 320 360 420 440 490 500 520 1020

Calculate the smoothed empirical estimate of the 45th percentile.

(A) 358

(B) 371

(C) 384

(D) 390

(E) 396

32. You are given:

(i) The number of claims made by an individual insured in a year has a Poisson distribution with mean λ.

(ii) The prior distribution for λ is gamma with parameters $\alpha = 1$ and $\theta = 1.2$.

Three claims are observed in Year 1, and no claims are observed in Year 2.

Using Bühlmann credibility, estimate the number of claims in Year 3.

(A) 1.35

(B) 1.36

(C) 1.40

(D) 1.41

(E) 1.43

33. In a study of claim payment times, you are given:

(i) The data were not truncated or censored.

(ii) At most one claim was paid at any one time.

(iii) The Nelson-Aalen estimate of the cumulative hazard function, $H(t)$, immediately following the second paid claim, was 23/132.

Determine the Nelson-Aalen estimate of the cumulative hazard function, $H(t)$, immediately following the fourth paid claim.

(A) 0.35

(B) 0.37

(C) 0.39

(D) 0.41

(E) 0.43

34. The number of claims follows a negative binomial distribution with parameters β and r, where β is unknown and r is known. You wish to estimate β based on n observations, where $\bar{x}$ is the mean of these observations.

Determine the maximum likelihood estimate of β.

(A) $\dfrac{\bar{x}}{r^2}$

(B) $\dfrac{\bar{x}}{r}$

(C) $\bar{x}$

(D) $r\bar{x}$

(E) $r^2\bar{x}$

35. You are given the following information about a credibility model:

First Observation	Unconditional Probability	Bayesian Estimate of Second Observation
1	1/3	1.50
2	1/3	1.50
3	1/3	3.00

Determine the Bühlmann credibility estimate of the second observation, given that the first observation is 1.

(A) 0.75

(B) 1.00

(C) 1.25

(D) 1.50

(E) 1.75

36. For a survival study, you are given:

(i) The Product-Limit estimator $\hat{S}(t_0)$ is used to construct confidence intervals for $S(t_0)$.

(ii) The 95% log-transformed confidence interval for $S(t_0)$ is $(0.695, 0.843)$.

Determine $\hat{S}(t_0)$.

(A) 0.758

(B) 0.762

(C) 0.765

(D) 0.769

(E) 0.779

37. A random sample of three claims from a dental insurance plan is given below:

225 525 950

Claims are assumed to follow a Pareto distribution with parameters $\theta = 150$ and α.

Determine the maximum likelihood estimate of α.

(A) Less than 0.6

(B) At least 0.6, but less than 0.7

(C) At least 0.7, but less than 0.8

(D) At least 0.8, but less than 0.9

(E) At least 0.9

38. An insurer has data on losses for four policyholders for 7 years. The loss from the i^{th} policyholder for year *j* is X_{ij}.

You are given:

$$\sum_{i=1}^{4}\sum_{j=1}^{7}\left(X_{ij} - \overline{X}_i\right)^2 = 33.60$$

$$\sum_{i=1}^{4}\left(\overline{X}_i - \overline{X}\right)^2 = 3.30$$

Using nonparametric empirical Bayes estimation, calculate the Bühlmann credibility factor for an individual policyholder.

(A) Less than 0.74

(B) At least 0.74, but less than 0.77

(C) At least 0.77, but less than 0.80

(D) At least 0.80, but less than 0.83

(E) At least 0.83

39. You are given the following information about a commercial auto liability book of business:

(i) Each insured's claim count has a Poisson distribution with mean λ, where λ has a gamma distribution with $\alpha = 1.5$ and $\theta = 0.2$.

(ii) Individual claim size amounts are independent and exponentially distributed with mean 5000.

(iii) The full credibility standard is for aggregate losses to be within 5% of the expected with probability 0.90.

Using classical credibility, determine the expected number of claims required for full credibility.

(A) 2165

(B) 2381

(C) 3514

(D) 7216

(E) 7938

40. You are given:

(i) A sample of claim payments is:

29 64 90 135 182

(ii) Claim sizes are assumed to follow an exponential distribution.

(iii) The mean of the exponential distribution is estimated using the method of moments.

Calculate the value of the Kolmogorov-Smirnov test statistic.

(A) 0.14

(B) 0.16

(C) 0.19

(D) 0.25

(E) 0.27

41. You are given:

(i) Annual claim frequency for an individual policyholder has mean λ and variance σ^2.

(ii) The prior distribution for λ is uniform on the interval [0.5, 1.5].

(iii) The prior distribution for σ^2 is exponential with mean 1.25.

A policyholder is selected at random and observed to have no claims in Year 1.

Using Bühlmann credibility, estimate the number of claims in Year 2 for the selected policyholder.

(A) 0.56

(B) 0.65

(C) 0.71

(D) 0.83

(E) 0.94

42. You study the time between accidents and reports of claims. The study was terminated at time 3.

You are given:

Time of Accident	Time between Accident and Claim Report	Number of Reported Claims
0	1	18
0	2	13
0	3	9
1	1	14
1	2	10
2	1	11

Use the Product-Limit estimator to estimate the conditional probability that the time between accident and claim report is less than 2, given that it does not exceed 3.

(A) Less than 0.4

(B) At least 0.4, but less than 0.5

(C) At least 0.5, but less than 0.6

(D) At least 0.6, but less than 0.7

(E) At least 0.7

43. You are given:

(i) The prior distribution of the parameter Θ has probability density function:

$$\pi(\theta) = \frac{1}{\theta^2}, \quad 1 < \theta < \infty$$

(ii) Given $\Theta = \theta$, claim sizes follow a Pareto distribution with parameters $\alpha = 2$ and θ.

A claim of 3 is observed.

Calculate the posterior probability that Θ exceeds 2.

(A) 0.33

(B) 0.42

(C) 0.50

(D) 0.58

(E) 0.64

44. You are given:

(i) Losses follow an exponential distribution with mean θ.

(ii) A random sample of 20 losses is distributed as follows:

Loss Range	Frequency
[0, 1000]	7
(1000, 2000]	6
(2000, ∞)	7

Calculate the maximum likelihood estimate of θ.

(A) Less than 1950

(B) At least 1950, but less than 2100

(C) At least 2100, but less than 2250

(D) At least 2250, but less than 2400

(E) At least 2400

45. You are given:

(i) The amount of a claim, X, is uniformly distributed on the interval $[0,\theta]$.

(ii) The prior density of θ is $\pi(\theta)=\dfrac{500}{\theta^2}, \quad \theta>500$.

Two claims, $x_1=400$ and $x_2=600$, are observed. You calculate the posterior distribution as:

$$f(\theta|x_1,x_2)=3\left(\frac{600^3}{\theta^4}\right), \quad \theta>600$$

Calculate the Bayesian premium, $\mathrm{E}(X_3|x_1,x_2)$.

(A) 450

(B) 500

(C) 550

(D) 600

(E) 650

46. The claim payments on a sample of ten policies are:

$$2 \quad 3 \quad 3 \quad 5 \quad 5^{+} \quad 6 \quad 7 \quad 7^{+} \quad 9 \quad 10^{+}$$

+ indicates that the loss exceeded the policy limit

Using the Product-Limit estimator, calculate the probability that the loss on a policy exceeds 8.

(A) 0.20

(B) 0.25

(C) 0.30

(D) 0.36

(E) 0.40

47. You are given the following observed claim frequency data collected over a period of 365 days:

Number of Claims per Day	Observed Number of Days
0	50
1	122
2	101
3	92
4+	0

Fit a Poisson distribution to the above data, using the method of maximum likelihood.

Regroup the data, by number of claims per day, into four groups:

0 1 2 3+

Apply the chi-square goodness-of-fit test to evaluate the null hypothesis that the claims follow a Poisson distribution.

Determine the result of the chi-square test.

(A) Reject at the 0.005 significance level.

(B) Reject at the 0.010 significance level, but not at the 0.005 level.

(C) Reject at the 0.025 significance level, but not at the 0.010 level.

(D) Reject at the 0.050 significance level, but not at the 0.025 level.

(E) Do not reject at the 0.050 significance level.

48. You are given the following joint distribution:

	Θ	
X	0	1
0	0.4	0.1
1	0.1	0.2
2	0.1	0.1

For a given value of Θ and a sample of size 10 for X:

$$\sum_{i=1}^{10} x_i = 10$$

Determine the Bühlmann credibility premium.

(A) 0.75

(B) 0.79

(C) 0.82

(D) 0.86

(E) 0.89

49. You are given:

x	0	1	2	3
$\Pr[X = x]$	0.5	0.3	0.1	0.1

The method of moments is used to estimate the population mean, μ, and variance, σ^2, by $\bar{X}$ and $S_n^2 = \dfrac{\sum\left(X_i - \bar{X}\right)^2}{n}$, respectively.

Calculate the bias of S_n^2, when $n = 4$.

(A) –0.72

(B) –0.49

(C) –0.24

(D) –0.08

(E) 0.00

50. You are given four classes of insureds, each of whom may have zero or one claim, with the following probabilities:

Class	Number of Claims	
	0	1
I	0.9	0.1
II	0.8	0.2
III	0.5	0.5
IV	0.1	0.9

A class is selected at random (with probability ¼), and four insureds are selected at random from the class. The total number of claims is two.

If five insureds are selected at random from the same class, estimate the total number of claims using Bühlmann-Straub credibility.

(A) 2.0

(B) 2.2

(C) 2.4

(D) 2.6

(E) 2.8

51. The following results were obtained from a survival study, using the Product-Limit estimator:

t	$\hat{S}(t)$	$\sqrt{\hat{V}[\hat{S}(t)]}$
17	0.957	0.0149
25	0.888	0.0236
32	0.814	0.0298
36	0.777	0.0321
39	0.729	0.0348
42	0.680	0.0370
44	0.659	0.0378
47	0.558	0.0418
50	0.360	0.0470
54	0.293	0.0456
56	0.244	0.0440
57	0.187	0.0420
59	0.156	0.0404
62	0.052	0.0444

Determine the lower limit of the 95% linear confidence interval for $x_{0.75}$, the 75$^{\text{th}}$ percentile of the survival distribution.

(A) 32

(B) 36

(C) 50

(D) 54

(E) 56

52. With the bootstrapping technique, the underlying distribution function is estimated by which of the following?

(A) The empirical distribution function

(B) A normal distribution function

(C) A parametric distribution function selected by the modeler

(D) Any of (A), (B) or (C)

(E) None of (A), (B) or (C)

53. You are given:

Number of Claims	Probability	Claim Size	Probability
0	$\frac{1}{5}$		
1	$\frac{3}{5}$	25	$\frac{1}{3}$
		150	$\frac{2}{3}$
2	$\frac{1}{5}$	50	$\frac{2}{3}$
		200	$\frac{1}{3}$

Claim sizes are independent.

Determine the variance of the aggregate loss.

(A) 4,050

(B) 8,100

(C) 10,500

(D) 12,510

(E) 15,612

54. You are given:

(i) Losses follow an exponential distribution with mean θ.

(ii) A random sample of losses is distributed as follows:

Loss Range	Number of Losses
(0 – 100]	32
(100 – 200]	21
(200 – 400]	27
(400 – 750]	16
(750 – 1000]	2
(1000 – 1500]	2
Total	100

Estimate θ by matching at the 80^{th} percentile.

(A) 249

(B) 253

(C) 257

(D) 260

(E) 263

55. You are given:

Class	Number of Insureds	Claim Count Probabilities				
		0	1	2	3	4
1	3000	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
2	2000	0	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0
3	1000	0	0	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

A randomly selected insured has one claim in Year 1.

Determine the expected number of claims in Year 2 for that insured.

(A) 1.00

(B) 1.25

(C) 1.33

(D) 1.67

(E) 1.75

56. You are given the following information about a group of policies:

Claim Payment	Policy Limit
5	50
15	50
60	100
100	100
500	500
500	1000

Determine the likelihood function.

(A) $f(50)\,f(50)\,f(100)\,f(100)\,f(500)\,f(1000)$

(B) $f(50)\,f(50)\,f(100)\,f(100)\,f(500)\,f(1000)\,/\,[1-F(1000)]$

(C) $f(5)\,f(15)\,f(60)\,f(100)\,f(500)\,f(500)$

(D) $f(5)\,f(15)\,f(60)\,f(100)\,f(500)\,f(500)\,/\,[1-F(1000)]$

(E) $f(5)\,f(15)\,f(60)\,[1-F(100)]\,[1-F(500)]\,f(500)$

57. You are given:

Claim Size	Number of Claims
0-25	30
25-50	32
50-100	20
100-200	8

Assume a uniform distribution of claim sizes within each interval.

Estimate the second raw moment of the claim size distribution.

(A) Less than 3300

(B) At least 3300, but less than 3500

(C) At least 3500, but less than 3700

(D) At least 3700, but less than 3900

(E) At least 3900

58. You are given:

(i) The number of claims per auto insured follows a Poisson distribution with mean λ.

(ii) The prior distribution for λ has the following probability density function:

$$f(\lambda)=\frac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)}$$

(iii) A company observes the following claims experience:

	Year 1	Year 2
Number of claims	75	210
Number of autos insured	600	900

The company expects to insure 1100 autos in Year 3.
Determine the expected number of claims in Year 3.

(A) 178

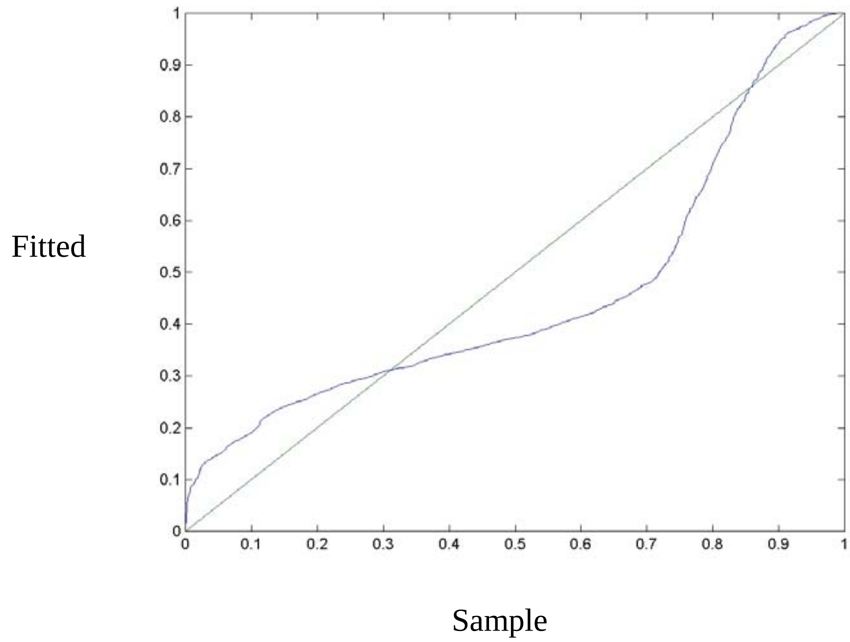
(B) 184

(C) 193

(D) 209

(E) 224

59. The graph below shows a *q-q* plot of a fitted distribution compared to a sample.



Which of the following is true?

(A) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has less probability around the median than the sample.

(B) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has more probability around the median than the sample.

(C) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has less probability around the median than the sample.

(D) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has more probability around the median than the sample.

(E) The tail of the fitted distribution is too thick on the left, too thin on the right, and the fitted distribution has less probability around the median than the sample.

60. You are given the following information about six coins:

Coin	Probability of Heads
1 – 4	0.50
5	0.25
6	0.75

A coin is selected at random and then flipped repeatedly. X_i denotes the outcome of the *i*th flip, where "1" indicates heads and "0" indicates tails. The following sequence is obtained:

$$S = \{X_1, X_2, X_3, X_4\} = \{1,1,0,1\}$$

Determine $E(X_5 | S)$ using Bayesian analysis.

(A) 0.52

(B) 0.54

(C) 0.56

(D) 0.59

(E) 0.63

61. You observe the following five ground-up claims from a data set that is truncated from below at 100:

125 150 165 175 250

You fit a ground-up exponential distribution using maximum likelihood estimation.

Determine the mean of the fitted distribution.

(A) 73

(B) 100

(C) 125

(D) 156

(F) 173

62. An insurer writes a large book of home warranty policies. You are given the following information regarding claims filed by insureds against these policies:

(i) A maximum of one claim may be filed per year.

(ii) The probability of a claim varies by insured, and the claims experience for each insured is independent of every other insured.

(iii) The probability of a claim for each insured remains constant over time.

(iv) The overall probability of a claim being filed by a randomly selected insured in a year is 0.10.

(v) The variance of the individual insured claim probabilities is 0.01.

An insured selected at random is found to have filed 0 claims over the past 10 years.

Determine the Bühlmann credibility estimate for the expected number of claims the selected insured will file over the next 5 years.

(A) 0.04

(B) 0.08

(C) 0.17

(D) 0.22

(E) 0.25

63. A study of the time to first claim includes only policies issued during 1996 through 1998 on which claims occurred by the end of 1999.

The table below summarizes the information about the 50 policies included in the study:

Number of Policies			
Year of Issue	Time to First Claim		
	1 year	2 years	3 years
1996	5	9	13
1997	6	10	
1998	7		

Use the Product-Limit estimator to estimate the conditional probability that the first claim on a policy occurs less than 2 years after issue given that the claim occurs no later than 3 years after issue.

(A) Less than 0.20

(B) At least 0.20, but less than 0.25

(C) At least 0.25, but less than 0.30

(D) At least 0.30, but less than 0.35

(E) At least 0.35

64. For a group of insureds, you are given:

(i) The amount of a claim is uniformly distributed but will not exceed a certain unknown limit θ.

(ii) The prior distribution of θ is $\pi(\theta) = \frac{500}{\theta^2}$, $\theta > 500$.

(iii) Two independent claims of 400 and 600 are observed.

Determine the probability that the next claim will exceed 550.

(A) 0.19

(B) 0.22

(C) 0.25

(D) 0.28

(E) 0.31

65. You are given the following information about a general liability book of business comprised of 2500 insureds:

(i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the i^{th} insured.

(ii) $N_1, N_2, \ldots, N_{2500}$ are independent and identically distributed random variables following a negative binomial distribution with parameters $r = 2$ and $\beta = 0.2$.

(iii) $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent and identically distributed random variables following a Pareto distribution with $\alpha = 3.0$ and $\theta = 1000$.

(iv) The full credibility standard is to be within 5% of the expected aggregate losses 90% of the time.

Using classical credibility theory, determine the partial credibility of the annual loss experience for this book of business.

(A) 0.34

(B) 0.42

(C) 0.47

(D) 0.50

(E) 0.53

66. To estimate $E[X]$, you have simulated X_1, X_2, X_3, X_4 and X_5 with the following results:

1	2	3	4	5

You want the standard deviation of the estimator of $E[X]$ to be less than 0.05.

Estimate the total number of simulations needed.

(A) Less than 150

(B) At least 150, but less than 400

(C) At least 400, but less than 650

(D) At least 650, but less than 900

(E) At least 900

67. You are given the following information about a book of business comprised of 100 insureds:

(i) $X_i = \sum_{j=1}^{N_i} Y_{ij}$ is a random variable representing the annual loss of the i^{th} insured.

(ii) $N_1, N_2, \ldots, N_{100}$ are independent random variables distributed according to a negative binomial distribution with parameters r (unknown) and $\beta = 0.2$.

(iii) Unknown parameter r has an exponential distribution with mean 2.

(iv) $Y_{i1}, Y_{i2}, \ldots, Y_{iN_i}$ are independent random variables distributed according to a Pareto distribution with $\alpha = 3.0$ and $\theta = 1000$.

Determine the Bühlmann credibility factor, Z, for the book of business.

(A) 0.000

(B) 0.045

(C) 0.500

(D) 0.826

(E) 0.905

68. For a mortality study of insurance applicants in two countries, you are given:

(i)

	Country A			Country B		
t_i	d_i	Y_i	θ_i	d_i	Y_i	θ_i
1	20	200	0.05	15	100	0.10
2	54	180	0.10	20	85	0.10
3	14	126	0.15	20	65	0.10
4	22	112	0.20	10	45	0.10

(ii) Y_i is the number at risk over the period (t_{i-1}, t_i). Deaths during the period (t_{i-1}, t_i) are assumed to occur at t_i.

(iii) θ_i is the reference hazard rate over the period (t_{i-1}, t_i). Within a country, θ_i is the same for all study participants.

(iv) $S^{\mathrm{T}}(t)$ is the Product-Limit estimate of $S(t)$ based on the data for all study participants.

(v) $S^{\mathrm{B}}(t)$ is the Product-Limit estimate of $S(t)$ based on the data for study participants in Country B.

Determine $\left|S^{\mathrm{T}}(4)-S^{\mathrm{B}}(4)\right|$.

(A) 0.06

(B) 0.07

(C) 0.08

(D) 0.09

(E) 0.10

69. You fit an exponential distribution to the following data:

1000	1400	5300	7400	7600

Determine the coefficient of variation of the maximum likelihood estimate of the mean, θ.

(A) 0.33

(B) 0.45

(C) 0.70

(D) 1.00

(E) 1.21

70. You are given the following information on claim frequency of automobile accidents for individual drivers:

	Business Use		Pleasure Use	
	Expected Claims	Claim Variance	Expected Claims	Claim Variance
Rural	1.0	0.5	1.5	0.8
Urban	2.0	1.0	2.5	1.0
Total	1.8	1.06	2.3	1.12

You are also given:

(i) Each driver's claims experience is independent of every other driver's.

(ii) There are an equal number of business and pleasure use drivers.

Determine the Bühlmann credibility factor for a single driver.

(A) 0.05

(B) 0.09

(C) 0.17

(D) 0.19

(E) 0.27

71. You are investigating insurance fraud that manifests itself through claimants who file claims with respect to auto accidents with which they were not involved. Your evidence consists of a distribution of the observed number of claimants per accident and a standard distribution for accidents on which fraud is known to be absent. The two distributions are summarized below:

Number of Claimants per Accident	Standard Probability	Observed Number of Accidents
1	0.25	235
2	.35	335
3	.24	250
4	.11	111
5	.04	47
6+	.01	22
Total	1.00	1000

Determine the result of a chi-square test of the null hypothesis that there is no fraud in the observed accidents.

(A) Reject at the 0.005 significance level.

(B) Reject at the 0.010 significance level, but not at the 0.005 level.

(C) Reject at the 0.025 significance level, but not at the 0.010 level.

(D) Reject at the 0.050 significance level, but not at the 0.025 level.

(E) Do not reject at the 0.050 significance level.

72. You are given the following data on large business policyholders:

(i) Losses for each employee of a given policyholder are independent and have a common mean and variance.

(ii) The overall average loss per employee for all policyholders is 20.

(iii) The variance of the hypothetical means is 40.

(iv) The expected value of the process variance is 8000.

(v) The following experience is observed for a randomly selected policyholder:

Year	Average Loss per Employee	Number of Employees
1	15	800
2	10	600
3	5	400

Determine the Bühlmann-Straub credibility premium per employee for this policyholder.

(A) Less than 10.5

(B) At least 10.5, but less than 11.5

(C) At least 11.5, but less than 12.5

(D) At least 12.5, but less than 13.5

(E) At least 13.5

73. You are given the following information about a group of 10 claims:

Claim Size Interval	Number of Claims in Interval	Number of Claims Censored in Interval
(0-15,000]	1	2
(15,000-30,000]	1	2
(30,000-45,000]	4	0

Assume that claim sizes and censorship points are uniformly distributed within each interval.

Estimate, using the life table methodology, the probability that a claim exceeds 30,000.

(A) 0.67

(B) 0.70

(C) 0.74

(D) 0.77

(E) 0.80

74. You are making credibility estimates for regional rating factors. You observe that the Bühlmann-Straub nonparametric empirical Bayes method can be applied, with rating factor playing the role of pure premium.

X_{ij} denotes the rating factor for region *i* and year *j*, where $i = 1,2,3$ and $j = 1,2,3,4$. Corresponding to each rating factor is the number of reported claims, m_{ij}, measuring exposure.

You are given:

i	$m_i = \sum_{j=1}^{4} m_{ij}$	$\bar{X}_i = \frac{1}{m_i}\sum_{j=1}^{4} m_{ij} X_{ij}$	$\hat{v}_i = \frac{1}{3}\sum_{j=1}^{4} m_{ij}\left(X_{ij} - \bar{X}_i\right)^2$	$m_i\left(\bar{X}_i - \bar{X}\right)^2$
1	50	1.406	0.536	0.887
2	300	1.298	0.125	0.191
3	150	1.178	0.172	1.348

Determine the credibility estimate of the rating factor for region 1 using the method that preserves $\sum_{i=1}^{3} m_i \bar{X}_i$.

(A) 1.31

(B) 1.33

(C) 1.35

(D) 1.37

(E) 1.39

75. You are given:

(i) Claim amounts follow a shifted exponential distribution with probability density function:

$$f(x) = \frac{1}{\theta} e^{-(x-\delta)/\theta}, \quad \delta < x < \infty$$

(ii) A random sample of claim amounts $X_1, X_2, ..., X_{10}$:

5 5 5 6 8 9 11 12 16 23

(iii) $\sum X_i = 100$ and $\sum X_i^2 = 1306$

Estimate δ using the method of moments.

(A) 3.0

(B) 3.5

(C) 4.0

(D) 4.5

(E) 5.0

76. You are given:

(i) The annual number of claims for each policyholder follows a Poisson distribution with mean θ.

(ii) The distribution of θ across all policyholders has probability density function:

$$f(\theta) = \theta e^{-\theta}, \theta > 0$$

(iii) $\int_0^\infty \theta e^{-n\theta} d\theta = \frac{1}{n^2}$

A randomly selected policyholder is known to have had at least one claim last year.

Determine the posterior probability that this same policyholder will have at least one claim this year.

(A) 0.70

(B) 0.75

(C) 0.78

(D) 0.81

(E) 0.86

77. A survival study gave (1.63, 2.55) as the 95% linear confidence interval for the cumulative hazard function $H(t_0)$.

Calculate the 95% log-transformed confidence interval for $H(t_0)$.

(A) (0.49, 0.94)

(B) (0.84, 3.34)

(C) (1.58, 2.60)

(D) (1.68, 2.50)

(E) (1.68, 2.60)

78. You are given:

(i) Claim size, X, has mean μ and variance 500.

(ii) The random variable μ has a mean of 1000 and variance of 50.

(iii) The following three claims were observed: 750, 1075, 2000

Calculate the expected size of the next claim using Bühlmann credibility.

(A) 1025

(B) 1063

(C) 1115

(D) 1181

(E) 1266

79. Losses come from a mixture of an exponential distribution with mean 100 with probability p and an exponential distribution with mean 10,000 with probability $1-p$.

Losses of 100 and 2000 are observed.

Determine the likelihood function of p.

(A) $$\left(\frac{pe^{-1}}{100}\cdot\frac{(1-p)e^{-0.01}}{10{,}000}\right)\cdot\left(\frac{pe^{-20}}{100}\cdot\frac{(1-p)e^{-0.2}}{10{,}000}\right)$$

(B) $$\left(\frac{pe^{-1}}{100}\cdot\frac{(1-p)e^{-0.01}}{10{,}000}\right)+\left(\frac{pe^{-20}}{100}\cdot\frac{(1-p)e^{-0.2}}{10{,}000}\right)$$

(C) $$\left(\frac{pe^{-1}}{100}+\frac{(1-p)e^{-0.01}}{10{,}000}\right)\cdot\left(\frac{pe^{-20}}{100}+\frac{(1-p)e^{-0.2}}{10{,}000}\right)$$

(D) $$\left(\frac{pe^{-1}}{100}+\frac{(1-p)e^{-0.01}}{10{,}000}\right)+\left(\frac{pe^{-20}}{100}+\frac{(1-p)e^{-0.2}}{10{,}000}\right)$$

(E) $$p.\left(\frac{e^{-1}}{100}+\frac{e^{-0.01}}{10{,}000}\right)+(1-p)\cdot\left(\frac{e^{-20}}{100}+\frac{e^{-0.2}}{10{,}000}\right)$$

80. A fund is established by collecting an amount P from each of 100 independent lives age 70. The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or
- P, payable at age 72, to those who survive.

You are given:

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.08$

For this question only, you are also given:

The number of claims in the first year is simulated from the binomial distribution using the inverse transform method (where smaller random numbers correspond to fewer deaths). The random number for the first trial, generated using the uniform distribution on [0, 1], is 0.18.

Calculate the simulated claim amount.

(A) 0

(B) 10

(C) 20

(D) 30

(E) 40

81. You wish to simulate a value, Y, from a two point mixture.

With probability 0.3, Y is exponentially distributed with mean 0.5. With probability 0.7, Y is uniformly distributed on $[-3, 3]$. You simulate the mixing variable where low values correspond to the exponential distribution. Then you simulate the value of Y, where low random numbers correspond to low values of Y. Your uniform random numbers from $[0, 1]$ are 0.25 and 0.69 in that order.

Calculate the simulated value of Y.

(A) 0.19

(B) 0.38

(C) 0.59

(D) 0.77

(E) 0.95

82. N is the random variable for the number of accidents in a single year. N follows the distribution:

$$\Pr(N = n) = 0.9(0.1)^{n-1}, \qquad n = 1, 2, \ldots$$

X_i is the random variable for the claim amount of the ith accident. X_i follows the distribution:

$$g(x_i) = 0.01\,e^{-0.01x_i}, \quad x_i > 0, \quad i = 1, 2, \ldots$$

Let U and $V_1, V_2, \ldots$ be independent random variables following the uniform distribution on (0, 1). You use the inverse transformation method with U to simulate N and V_i to simulate X_i with small values of random numbers corresponding to small values of N and X_i.
You are given the following random numbers for the first simulation:

u	v_1	v_2	v_3	v_4
0.05	0.30	0.22	0.52	0.46

Calculate the total amount of claims during the year for the first simulation.

(A) 0

(B) 36

(C) 72

(D) 108

(E) 144

83. You are the consulting actuary to a group of venture capitalists financing a search for pirate gold.

It's a risky undertaking: with probability 0.80, no treasure will be found, and thus the outcome is 0.

The rewards are high: with probability 0.20 treasure will be found. The outcome, if treasure is found, is uniformly distributed on [1000, 5000].

You use the inverse transformation method to simulate the outcome, where large random numbers from the uniform distribution on [0, 1] correspond to large outcomes.

Your random numbers for the first two trials are 0.75 and 0.85.

Calculate the average of the outcomes of these first two trials.

(A) 0

(B) 1000

(C) 2000

(D) 3000

(E) 4000

SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM C CONSTRUCTION AND EVALUATION OF ACTUARIAL MODELS

EXAM C SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

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Question #1
Key: E

The 40^{th} percentile is the .4(12) = 4.8^{th} smallest observation. By interpolation it is .2(86) +.8(90) = 89.2. The 80^{th} percentile is the .8(12) = 9.6^{th} smallest observation. By interpolation it is .4(200) +.6(210) = 206.

The equations to solve are

$.4 = \dfrac{(89.2/\theta)^\gamma}{1+(89.2/\theta)^\gamma}$ and $.8 = \dfrac{(206/\theta)^\gamma}{1+(206/\theta)^\gamma}$.

Solving each for the parenthetical expression gives $\dfrac{2}{3} = (89.2/\theta)^\gamma$ and $4 = (206/\theta)^\gamma$.

Taking the ratio of the second equation to the first gives $6 = (206/89.2)^\gamma$ which leads to $\gamma = \ln(6)/\ln(206/89.2) = 2.1407$. Then $4^{1/2.1407} = 206/\theta$ for $\theta = 107.8$.

Question #2
Key: E

The standard for full credibility is $\left(\dfrac{1.645}{.02}\right)^2\left(1+\dfrac{Var(X)}{E(X)^2}\right)$ where X is the claim size variable. For the Pareto variable, $E(X) = .5/5 = .1$ and $Var(X) = \dfrac{2(.5)^2}{5(4)} - (.1)^2 = .015$.

Then the standard is $\left(\dfrac{1.645}{.02}\right)^2\left(1+\dfrac{.015}{.1^2}\right) = 16{,}913$ claims.

Question #3
Key: B

The kernel is a triangle with a base of 4 and a height at the middle of 0.5 (so the area is 1). The length of the base is twice the bandwidth. Any observation within 2 of 2.5 will contribute to the estimate. For the observation at 2, when the triangle is centered at 2, the height of the triangle at 2.5 is .375 (it is one-quarter the way from 2 to the end of the triangle at 4 and so the height is one-quarter the way from 0.5 to 0). Similarly the points at 3 are also 0.5 away and so the height of the associated triangle is also .375. Each triangle height is weighted by the empirical probability at the associated point. So the estimate at 2.5 is (1/5)(3/8) + (3/5)(3/8) + (1/5)(0) = 12/40.

Question #4
Key: A

The distribution function is $F(x)=\int_1^x \alpha t^{-\alpha-1}dt = -t^{-\alpha}\Big|_1^x = 1-x^{-\alpha}$. The likelihood function is

$$\begin{aligned} L &= f(3)f(6)f(14)[1-F(25)]^2 \\ &= \alpha 3^{-\alpha-1}\alpha 6^{-\alpha-1}\alpha 14^{-\alpha-1}(25^{-\alpha})^2 \\ &\propto \alpha^3[3(6)(14)(625)]^{-\alpha}. \end{aligned}$$

Taking logs, differentiating, setting equal to zero, and solving:

$\ln L = 3\ln\alpha - \alpha\ln 157{,}500$ plus a constant

$(\ln L)' = 3\alpha^{-1} - \ln 157{,}500 = 0$

$\hat{\alpha} = 3/\ln 157{,}500 = .2507.$

Question #5
Key: C

$\pi(q\,|\,1,1) \propto p(1\,|\,q)p(1\,|\,q)\pi(q) = 2q(1-q)2q(1-q)4q^3 \propto q^5(1-q)^2$

$\int_0^1 q^5(1-q)^2\,dq = 1/168, \quad \pi(q\,|\,1,1) = 168q^5(1-q)^2.$

The expected number of claims in a year is $E(X\,|\,q) = 2q$ and so the Bayesian estimate is

$E(2q\,|\,1,1) = \int_0^1 2q(168)q^5(1-q)^2\,dq = 4/3.$

The answer can be obtained without integrals by recognizing that the posterior distribution of *q* is beta with *a* = 6 and *b* = 3. The posterior mean is $E(q\,|\,1,1) = a/(a+b) = 6/9 = 2/3$. The posterior mean of 2*q* is then 4/3.

Question #6
Key: D

For the method of moments estimate,

$386 = e^{\mu+.5\sigma^2}, \quad 457{,}480.2 = e^{2\mu+2\sigma^2}$

$5.9558 = \mu + .5\sigma^2, \quad 13.0335 = 2\mu + 2\sigma^2$

$\hat{\mu} = 5.3949, \quad \hat{\sigma}^2 = 1.1218.$

Then

$$E(X \wedge 500) = e^{5.3949+.5(1.1218)}\Phi\left(\frac{\ln 500 - 5.3949 - 1.1218}{\sqrt{1.1218}}\right) + 500\left[1 - \Phi\left(\frac{\ln 500 - 5.3949}{\sqrt{1.1218}}\right)\right]$$
$$= 386\Phi(-.2853) + 500[1 - \Phi(.7739)]$$
$$= 386(.3877) + 500(.2195) = 259.$$

Note-these calculations use exact normal probabilities. Rounding and using the normal table that accompanies the exam will produce a different numerical answer but the same letter answer.

Question #7
Key: D

Because the values are already ranked, the test statistic is immediately calculated as the sum of the given values for Sample I: *R* = 1 + 2 + 3 + 4 + 7 + 9 + 13 + 19 + 20 = 78. The other needed values are *n* = 9 and *m* = 11, the two sample sizes. The mean is *n*(*n*+*m*+1)/2 = 94.5 and the variance is *nm*(*n*+*m*+1)/12 = 173.25. The test statistic is $Z = (78 - 94.5)/\sqrt{173.25} = -1.25$. The *p*-value is twice (because it is a two-tailed test) the probability of being more extreme than the test statistic, $p = 2\Pr(Z < -1.25) = .210$.

Question #8
Key: C

Let *N* be the Poisson claim count variable, let *X* be the claim size variable, and let *S* be the aggregate loss variable.

$$\mu(\theta) = E(S \mid \theta) = E(N \mid \theta)E(X \mid \theta) = \theta 10\theta = 10\theta^2$$
$$v(\theta) = Var(S \mid \theta) = E(N \mid \theta)E(X^2 \mid \theta) = \theta 200\theta^2 = 200\theta^3$$
$$\mu = E(10\theta^2) = \int_1^\infty 10\theta^2(5\theta^{-6})d\theta = 50/3$$
$$EPV = E(200\theta^3) = \int_1^\infty 200\theta^3(5\theta^{-6})d\theta = 500$$
$$VHM = Var(10\theta^2) = \int_1^\infty (10\theta^2)^2(5\theta^{-6})d\theta - (50/3)^2 = 222.22$$
$$k = 500/222.22 = 2.25.$$

Question #9
Key: A

$c = \exp(.71(1) + .20(1)) = 2.4843$. Then $\hat{S}(t_0; \mathbf{z}) = \hat{S}_0(t_0)^c = .65^{2.4843} = .343$.

Question #10
Key: E

Y and *X* are linear combinations of the same two normal random variables, so they are bivariate normal. Thus $E(Y|X) = E(Y) + [\mathrm{Cov}(Y,X)/\mathrm{Var}(X)][X - E(X)]$. From the definitions of *Y* and *X*, $E(Y) = a$, $E(X) = d$, $\mathrm{Var}(X) = e^2 + f^2$, and $\mathrm{Cov}(Y,X) = be + cf$.

Question #11
Key: D

$$\Pr(\theta=1 \mid X=5) = \frac{f(5\mid\theta=1)\Pr(\theta=1)}{f(5\mid\theta=1)\Pr(\theta=1)+f(5\mid\theta=3)\Pr(\theta=3)}$$

$$= \frac{(1/36)(1/2)}{(1/36)(1/2)+(3/64)(1/2)} = 16/43$$

$$\Pr(X_2>8 \mid X_1=5) = \Pr(X_2>8\mid\theta=1)\Pr(\theta=1\mid X_1=5)+\Pr(X_2>8\mid\theta=3)\Pr(\theta=3\mid X_1=5)$$

$$=(1/9)(16/43)+(3/11)(27/43)=.2126.$$

For the last line, $\Pr(X>8\mid\theta)=\int_8^\infty \theta(x+\theta)^{-2}\,dx=\theta(8+\theta)^{-1}$ is used.

Question #12
Key: C

The sample mean for *X* is 720 and for *Y* is 670. The mean of all 8 observations is 695.

$$\hat{v}=\frac{\begin{array}{c}(730-720)^2+(800-720)^2+(650-720)^2+(700-720)^2\\ +(655-670)^2+(650-670)^2+(625-670)^2+(750-670)^2\end{array}}{2(4-1)}=3475$$

$$\hat{a}=\frac{(720-695)^2+(670-695)^2}{2-1}-\frac{3475}{4}=381.25$$

$$\hat{k}=3475/381.25=9.1148$$

$$\hat{Z}=\frac{4}{4+9.1148}=.305$$

$$P_c=.305(670)+.695(695)=687.4.$$

Question #13
Key: B

There are 430 observations. The expected counts are 430(.2744) = 117.99, 430(.3512) =151.02, 430(.3744) = 160.99. The test statistic is

$$\frac{(112-117.99)^2}{117.99}+\frac{(180-151.02)^2}{151.02}+\frac{(138-160.99)^2}{160.99}=9.15.$$

Question #14
Key: B

From the information, the asymptotic variance of $\hat{\theta}$ is 1/4*n*. Then
$Var(2\hat{\theta})=4Var(\hat{\theta})=4(1/4n)=1/n.$
Note that the delta method is not needed for this problem, although using it leads to the same answer.

Question #15
Key: A

The posterior probability density is
$\pi(p\,|\,1,1,1,1,1,1,1,1)\propto \Pr(1,1,1,1,1,1,1,1\,|\,p)\pi(p)\propto p^8(2)\propto p^8.$

$$\pi(p\,|\,1,1,1,1,1,1,1,1)=\frac{p^8}{\int_0^{.5}p^8dp}=\frac{p^8}{(.5^9)/9}=9(.5^{-9})p^8.$$

$$\Pr(X_9=1\,|\,1,1,1,1,1,1,1,1)=\int_0^{.5}\Pr(X_9=1\,|\,p)\pi(p\,|\,1,1,1,1,1,1,1,1)dp$$

$$=\int_0^{.5}p9(.5^{-9})p^8dp=9(.5^{-9})(.5^{10})/10=.45.$$

Question #16
Key: A

${}_3\hat{p}_1=\frac{18}{27}\frac{26}{32}\frac{20}{25}=\frac{13}{30}.$ Greenwood's approximation is

$$\left(\frac{13}{30}\right)^2\left(\frac{9}{18(27)}+\frac{6}{26(32)}+\frac{5}{20(25)}\right)=.0067.$$

Question #17
Key: D

$\hat{H}(3)$ = 5/30 + 9/27 + 6/32 = 0.6875
$V\hat{a}r(\hat{H}(3))$ = 5/(30)^2 + 9/(27)^2 + 6/(32)^2 = 0.02376

The 95% log-transformed confidence interval is:
$\hat{H}(3)U$, where $U = \exp\left(\pm\frac{1.96\sqrt{.02376}}{.6875}\right) = \exp(\pm 0.43945)$
The confidence interval is:
[0.6875 exp(–0.43945), 0.6875 exp(0.43945)] = [0.443, 1.067].

Question #18
Key: D

The means are .5(250) + .3(2,500) + .2(60,000) = 12,875 and .7(250) + .2(2,500) + .1(60,000) = 6,675 for risks 1 and 2 respectively.
The variances are .5(250)^2 + .3(2,500)^2 + .2(60,000)^2 – 12,875^2 = 556,140,625 and .7(250)^2 + .2(2,500)^2 + .1(60,000)^2 – 6,675^2 = 316,738,125 respectively.

The overall mean is (2/3)(12,875) + (1/3)(6,675) = 10,808.33 and so
EPV = (2/3)(556,140,625) + (1/3)(316,738,125) = 476,339,792 and
VHM = (2/3)(12,875)^2 + (1/3)(6,675)^2 – 10,808.33^2 = 8,542,222. Then,
k = 476,339,792/8,542,222 = 55.763 and Z = 1/(1 + 55.763) = .017617.
The credibility estimate is .017617(250) + .982383(10,808.33) = 10,622.

Question #19
Key: D

The first two sample moments are 15 and 500, and the first two population moments are
$E(X) = .5(\theta + \sigma)$ and $E(X^2) = .5(2\theta^2 + 2\sigma^2) = \theta^2 + \sigma^2$. These can be obtained either through integration or by recognizing the density function as a two-point mixture of exponential densities. The equations to solve are $30 = \theta + \sigma$ and $500 = \theta^2 + \sigma^2$. From the first equation, $\sigma = 30 - \theta$ and substituting into the second equation gives $500 = \theta^2 + (30-\theta)^2 = 2\theta^2 - 60\theta + 900$. The quadratic equation has two solutions, 10 and 20. Because $\theta > \sigma$ the answer is 20.

Question #20
Key: D

There are four possible samples, (5,5), (5,9), (9,5), and (9,9). For each, the estimator *g* must be calculated. The values are 0, 4, 4, and 0 respectively. Assuming a population in which the values 5 and 9 each occur with probability .5, the population variance is $.5(5-7)^2 + .5(9-7)^2 = 4$. The mean square error is approximated as $.25[(0-4)^2 + (4-4)^2 + (4-4)^2 + (0-4)^2] = 8$.

Question #21
Key: B

From the Poisson distribution, $\mu(\lambda) = \lambda$ and $v(\lambda) = \lambda$. Then, $\mu = E(\lambda) = 6/100 = .06, \quad EPV = E(\lambda) = .06, \quad VHM = Var(\lambda) = 6/100^2 = .0006$ where the various moments are evaluated from the gamma distribution. Then, $k = .06/.0006 = 100$ and $Z = 450/(450+100) = 9/11$ where the 450 is the total number of insureds contributing experience. The credibility estimate of the expected number of claims for one insured in month 4 is (9/11)(25/450) + (2/11)(.06) = .056364. For 300 insureds the expected number of claims is 300(.056364) = 16.9.

Question #22
Key: C

The likelihood function is $L(\alpha,\theta) = \prod_{j=1}^{200} \frac{\alpha\theta^\alpha}{(x_j+\theta)^{\alpha+1}}$ and its logarithm is $l(\alpha,\theta) = 200\ln(\alpha) + 200\alpha\ln(\theta) - (\alpha+1)\sum_{i=1}^{200}\ln(x_i+\theta)$. When evaluated at the hypothesized values of 1.5 and 7.8, the loglikelhood is −821.77. The test statistic is 2(821.77−817.92) = 7.7. With two degrees of freedom (0 free parameters in the null hypothesis versus 2 in the alternative), the test statistic falls between the 97.5th percentile (7.38) and the 99th percentile (9.21).

Question #23
Key: E

Assume that $\theta > 5$. Then the expected counts for the three intervals are $15(2/\theta) = 30/\theta, 15(3/\theta) = 45/\theta,$ and $15(\theta - 5)/\theta = 15 - 75/\theta$ respectively. The quantity to minimize is

$$\frac{1}{5}\left[(30\theta^{-1} - 5)^2 + (45\theta^{-1} - 5)^2 + (15 - 75\theta^{-1} - 5)^2\right].$$

Differentiating (and ignoring the coefficient of 1/5) gives the equation
$-2(30\theta^{-1} - 5)30\theta^{-2} - 2(45\theta^{-1} - 5)45\theta^{-2} + 2(10 - 75\theta^{-1})75\theta^{-2} = 0$. Multiplying through by θ^3 and dividing by 2 reduces the equation to
$-(30 - 5\theta)30 - (45 - 5\theta)45 + (10\theta - 75)75 = -8550 + 1125\theta = 0$ for a solution of
$\hat{\theta} = 8550/1125 = 7.6$.

Question #24
Key: E

$\pi(\theta\,|\,1) \propto \theta(1.5\theta^{.5}) \propto \theta^{1.5}$. The required constant is the reciprocal of $\int_0^1 \theta^{1.5} d\theta = \theta^{2.5}/2.5\Big|_0^1 = .4$ and so $\pi(\theta\,|\,1) = 2.5\theta^{1.5}$. The requested probability is
$\Pr(\theta > .6\,|\,1) = \int_{.6}^1 2.5\theta^{1.5} d\theta = \theta^{2.5}\Big|_{.6}^1 = 1 - .6^{2.5} = .721.$

Question #25
Key: A

k	kn_k / n_{k-1}
0	
1	0.81
2	0.92
3	1.75
4	2.29
5	2.50
6	3.00

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #26
Key: B

The likelihood function is $\frac{e^{-1/(2\theta)}}{2\theta} \cdot \frac{e^{-2/(2\theta)}}{2\theta} \cdot \frac{e^{-3/(2\theta)}}{2\theta} \cdot \frac{e^{-15/(3\theta)}}{3\theta} = \frac{e^{-8/\theta}}{24\theta^4}$. The loglikelihood function is $-\ln 24 - 4\ln(\theta) - 8/\theta$. Differentiating with respect to θ and setting the result equal to 0 yields $-\frac{4}{\theta} + \frac{8}{\theta^2} = 0$ which produces $\hat{\theta} = 2$.

Question #27
Key: E

The absolute difference of the credibility estimate from its expected value is to be less than or equal to $k\mu$ (with probability *P*). That is,

$$\left|[ZX_{partial} + (1-Z)M] - [Z\mu + (1-Z)M]\right| \le k\mu$$

$$-k\mu \le ZX_{partial} - Z\mu \le k\mu.$$

Adding μ to all three sides produces answer choice (E).

Question #28
Key: C

In general,

$$E(X^2) - E[(X \wedge 150)^2] = \int_0^{200} x^2 f(x)dx - \int_0^{150} x^2 f(x)dx - 150^2 \int_{150}^{200} f(x)dx = \int_{150}^{200} (x^2 - 150^2) f(x)dx.$$

Assuming a uniform distribution, the density function over the interval from 100 to 200 is 6/7400 (the probability of 6/74 assigned to the interval divided by the width of the interval). The answer is

$$\int_{150}^{200} (x^2 - 150^2) \frac{6}{7400} dx = \left(\frac{x^3}{3} - 150^2 x \right) \frac{6}{7400} \Bigg|_{150}^{200} = 337.84.$$

Question #29
Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

$$\Pr(3 \mid \text{I}) = \binom{6}{3}(.1)^3(.9)^3 = .01458, \Pr(3 \mid \text{II}) = \binom{6}{3}(.2)^3(.8)^3 = .08192,$$

$$\Pr(3 \mid \text{III}) = \binom{6}{3}(.4)^3(.6)^3 = .27648$$

The probability of observing three successes is .7(.01458) + .2(.08192) + .1(.27648) = .054238. The three posterior probabilities are:

$$\Pr(\mathrm{I}\,|\,3)=\frac{.7(.01458)}{.054238}=.18817,\ \Pr(\mathrm{II}\,|\,3)=\frac{.2(.08192)}{.054238}=.30208,\ \Pr(\mathrm{III}\,|\,3)=\frac{.1(.27648)}{.054238}=.50975.$$

The posterior probability of a claim is then
.1(.18817) + .2(.30208) + .4(.50975) = .28313.

Question #30
Key: E

$.542=\hat{F}(n)=1-e^{-\hat{H}(n)},\quad \hat{H}(n)=.78.$ The Nelson-Aalen estimate is the sum of successive *s*/*r* values. From the problem statement, *r* = 100 at all surrender times while the *s*-values follow the pattern 1, 2, 3, …. Then,

$.78=\frac{1}{100}+\frac{2}{100}+\cdots+\frac{n}{100}=\frac{n(n+1)}{200}$ and the solution is *n* = 12.

Question # 31
Answer: C

$$g=[12(.45)]=[5.4]=5;\quad h=5.4-5=0.4.$$

$$\hat{\pi}_{.45}=.6x_{(5)}+.4x_{(6)}=.6(360)+.4(420)=384.$$

Question # 32
Answer: D

N is distributed *Poisson*(λ)

$\mu=E(\lambda)=\alpha\theta=1(1.2)=1.2.$

$v=E(\lambda)=1.2;\quad a=Var(\lambda)=\alpha\theta^2=1(1.2)^2=1.44.$

$k=\frac{1.2}{1.44}=\frac{5}{6};\quad Z=\frac{2}{2+5/6}=\frac{12}{17}.$

Thus, the estimate for Year 3 is

$\frac{12}{17}(1.5)+\frac{5}{17}(1.2)=1.41.$

Note that a Bayesian approach produces the same answer.

Question # 33
Answer: C

At the time of the second failure,

$$\hat{H}(t) = \frac{1}{n} + \frac{1}{n-1} = \frac{23}{132} \Rightarrow n = 12.$$

At the time of the fourth failure,

$$\hat{H}(t) = \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} = .3854.$$

Question # 34
Answer: B

The likelihood is:

$$L = \prod_{j=1}^{n} \frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}} \propto \prod_{j=1}^{n} \beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l = \sum_{j=1}^{n} \left[x_j \ln \beta - (r+x_j)\ln(1+\beta) \right]$$

$$l' = \sum_{j=1}^{n} \left[\frac{x_j}{\beta} - \frac{r+x_j}{1+\beta} \right] = 0$$

$$0 = \sum_{j=1}^{n} \left[x_j(1+\beta) - (r+x_j)\beta \right] = \sum_{j=1}^{n} x_j - rn\beta$$

$$0 = n\bar{x} - rn\beta; \quad \hat{\beta} = \bar{x}/r.$$

Question # 35
Answer: C

The Bühlmann credibility estimate is $Zx + (1-Z)\mu$ where *x* is the first observation. The Bühlmann estimate is the least squares approximation to the Bayesian estimate. Therefore, *Z* and μ must be selected to minimize

$$\frac{1}{3}[Z + (1-Z)\mu - 1.5]^2 + \frac{1}{3}[2Z + (1-Z)\mu - 1.5]^2 + \frac{1}{3}[3Z + (1-Z)\mu - 3]^2.$$

Setting partial derivatives equal to zero will give the values. However, it should be clear that μ is the average of the Bayesian estimates, that is,

$$\mu = \frac{1}{3}(1.5+1.5+3) = 2.$$

The derivative with respect to *Z* is (deleting the coefficients of 1/3):

$$2(-Z+.5)(-1)+2(.5)(0)+2(Z-1)(1)=0$$
$$Z=.75.$$

The answer is

$$.75(1)+.25(2)=1.25.$$

Question # 36
Answer: E

The confidence interval is $(\hat{S}(t_0)^{1/\theta}, \hat{S}(t_0)^{\theta})$.
Taking logarithms of both endpoints gives the two equations

$$\ln .695 = -.36384 = \frac{1}{\theta}\ln \hat{S}(t_0)$$
$$\ln .843 = -.17079 = \theta \ln \hat{S}(t_0).$$

Multiplying the two equations gives

$$.06214 = [\ln \hat{S}(t_0)]^2$$
$$\ln \hat{S}(t_0) = -.24928$$
$$\hat{S}(t_0) = .77936.$$

The negative square root is required in order to make the answer fall in the interval (0,1).

Question # 37
Answer: B

The likelihood is:

$$L = \frac{\alpha 150^{\alpha}}{(150+225)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+525)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+950)^{\alpha+1}}$$
$$= \frac{\alpha^3 150^{3\alpha}}{(375 \bullet 675 \bullet 1100)^{\alpha+1}}.$$

The loglikelihood is:

$$l = 3\ln\alpha + 3\alpha\ln 150 - (\alpha+1)\ln(375 \bullet 675 \bullet 1100)$$
$$l' = \frac{3}{\alpha} + 3\ln 150 - \ln(375 \bullet 675 \bullet 1100) = \frac{3}{\alpha} - 4.4128$$
$$\hat{\alpha} = 3/4.4128 = .6798.$$

Question # 38
Answer: D

For this problem, r = 4 and n = 7. Then,

$$\hat{v} = \frac{33.60}{4(7-1)} = 1.4 \text{ and } \hat{a} = \frac{3.3}{4-1} - \frac{1.4}{7} = .9.$$

Then,

$$k = \frac{1.4}{.9} = \frac{14}{9}; \quad Z = \frac{7}{7+(14/9)} = \frac{63}{77} = .82.$$

Question # 39
Answer: B

X is the random sum $Y_1 + Y_2 + ... + Y_N$.
N has a negative binomial distribution with $r = \alpha = 1.5$ and $\beta = \theta = 0.2$.

$$E(N) = r\beta = 0.3$$
$$Var(N) = r\beta(1+\beta) = 0.36$$

$$E(Y) = 5000$$
$$Var(Y) = 25{,}000{,}000$$

$$E(X) = 0.3 \times 5000 = 1500$$
$$Var(X) = 0.3 \times 25{,}000{,}000 + 0.36 \times 25{,}000{,}000 = 16{,}500{,}000$$

Number of exposures (insureds) required for full credibility

$n_{FULL} = (1.645/0.05)^2 \times 16,500,000/(1500)^2 = 7937.67$.

Number of expected claims required for full credibility
$E(N)\times n_{FULL} = 0.3\times 7937.67 = 2381$.

Question # 40
Answer: E

X	$F_n(x)$	$F_n(x^-)$	$F_0(x)$	$\lvert F_n(x)-F_0(x)\rvert$	$\lvert F_n(x^-)-F_0(x)\rvert$
29	0.2	0	0.252	0.052	0.252
64	0.4	0.2	0.473	0.073	0.273
90	0.6	0.4	0.593	0.007	0.193
135	0.8	0.6	0.741	0.059	0.141
182	1.00	0.8	0.838	0.162	0.038

where:
$\hat{\theta} = \bar{x} = 100$ and $F_0(x) = 1 - e^{-x/100}$.
The maximum value from the last two columns is 0.273.

Question # 41
Answer: E

$$\mu = E(\lambda) = 1; \quad v = E(\sigma^2) = 1.25; \quad a = Var(\lambda) = 1/12.$$

$$k = v/a = 15; \quad Z = \frac{1}{1+15} = \frac{1}{16}.$$

Thus, the estimate for Year 2 is
$$\frac{1}{16}(0) + \frac{15}{16}(1) = .9375.$$

Question # 42
Answer: B

Time must be reversed, so let *T* be the time between accident and claim report and let *R* = 3 – *T*. The desired probability is

$$\Pr(T < 2 \mid T \le 3) = \Pr(3-R < 2 \mid 3-R \le 3) = \Pr(R > 1 \mid R \ge 0) = \Pr(R > 1).$$

The product-limit calculation is:

R	*Y*	*d*
0	40	9
1	55	23
2	43	43

The estimate of surviving past (reversed) time 1 is (31/40)(32/55) = .4509.

Question # 43
Answer: E

The posterior density, given an observation of 3 is:

$$\pi(\theta\,|\,3)=\frac{f(3\,|\,\theta)\pi(\theta)}{\int_1^\infty f(3\,|\,\theta)\pi(\theta)d\theta}=\frac{\frac{2\theta^2}{(3+\theta)^3}\frac{1}{\theta^2}}{\int_1^\infty 2(3+\theta)^{-3}d\theta}$$
$$=\frac{2(3+\theta)^{-3}}{-(3+\theta)^{-2}\Big|_1^\infty}=32(3+\theta)^{-3},\quad \theta>1.$$

Then,

$$\Pr(\Theta>2)=\int_2^\infty 32(3+\theta)^{-3}d\theta=-16(3+\theta)^{-2}\Big|_2^\infty=\frac{16}{25}=.64.$$

Question # 44
Answer: B

$$L=F(1000)^7[F(2000)-F(1000)]^6[1-F(2000)]^7$$
$$=(1-e^{-1000/\theta})^7(e^{-1000/\theta}-e^{-2000/\theta})^6(e^{-2000/\theta})^7$$
$$=(1-p)^7(p-p^2)^6(p^2)^7$$
$$=p^{20}(1-p)^{13}$$

where $p=e^{-1000/\theta}$. The maximum occurs at *p* = 20/33 and so $\hat{\theta}=-1000/\ln(20/33)=1996.90.$

Question # 45
Answer: A

$E(X\,|\,\theta)=\theta/2.$

$$E(X_3 \mid 400,600)=\int_{600}^{\infty} E(X \mid \theta) f(\theta \mid 400,600) d\theta=\int_{600}^{\infty} \frac{\theta}{2} 3 \frac{600^3}{\theta^4} d\theta=\frac{3(600^3)}{2} \frac{\theta^{-2}}{-2}\bigg|_{600}^{\infty}$$

$$=\frac{3(600^3)(600^{-2})}{4}=450.$$

Question # 46
Answer: D

The data may be organized as follows:

t	*Y*	*d*	$\hat{S}(t)$
2	10	1	(9/10) = .9
3	9	2	.9(7/9) = .7
5	7	1	.7(6/7) = .6
6	5	1	.6(4/5) = .48
7	4	1	.48(3/4) = .36
9	2	1	.36(1/2) = .18

Because the product-limit estimate is constant between observations, the value of $\hat{S}(8)$ is found from $\hat{S}(7)=.36$.

Question # 47
Answer: C

The maximum likelihood estimate for the Poisson distribution is the sample mean:

$$\hat{\lambda}=\bar{x}=\frac{50(0)+122(1)+101(2)+92(3)}{365}=1.6438.$$

The table for the chi-square test is:

Number of days	Probability	Expected*	Chi-square
0	$e^{-1.6438}=.19324$	70.53	5.98
1	$1.6438e^{-1.6438}=.31765$	115.94	0.32
2	$\frac{1.6438^2 e^{-1.6438}}{2}=.26108$	95.30	0.34
3+	.22803**	83.23	0.92

*365x(Probability) **obtained by subtracting the other probabilities from 1

The sum of the last column is the test statistic of 7.56. Using 2 degrees of freedom (4 rows less 1 estimated parameter less 1) the model is rejected at the 2.5% significance level but not at the 1% significance level.

Question # 48
Answer: D

$$\mu(0) = \frac{.4(0)+.1(1)+.1(2)}{.6} = .5; \quad \mu(1) = \frac{.1(0)+.2(1)+.1(2)}{.4} = 1$$

$$\mu = .6(.5)+.4(1) = .7$$

$$a = .6(.5^2)+.4(1^2)-.7^2 = .06$$

$$v(0) = \frac{.4(0)+.1(1)+.1(4)}{.6} - .5^2 = \frac{7}{12}; \quad v(1) = \frac{.1(0)+.2(1)+.1(4)}{.4} - 1^2 = .5$$

$$v = .6(7/12)+.4(.5) = 11/20$$

$$k = v/a = 55/6; \quad Z = \frac{10}{10+55/6} = \frac{60}{115}$$

Bühlmann credibility premium = $\frac{60}{115}\frac{10}{10}+\frac{55}{115}(.7) = .8565$.

Question # 49
Answer: C

$$\mu = .5(0)+.3(1)+.1(2)+.1(3) = .8$$

$$\sigma^2 = .5(0)+.3(1)+.1(4)+.1(9)-.64 = .96$$

$$E(S_n^2) = \frac{n-1}{n}\sigma^2 = \frac{3}{4}(.96) = .72$$

$$bias = .72-.96 = -.24.$$

Question # 50
Answer: C

The four classes have means .1, .2, .5, and .9 respectively and variances .09, .16, .25, and .09 respectively.

Then,

$$\mu = .25(.1+.2+.5+.9) = .425$$

$$v = .25(.09+.16+.25+.09) = .1475$$

$$a = .25(.01+.04+.25+.81) - .425^2 = .096875$$

$$k = .1475/.096875 = 1.52258$$

$$Z = \frac{4}{4+1.52258} = .7243$$

The estimate is $\left[.7243(2/4)+.2757(.425)\right]\cdot 5 = 2.40$.

Question # 51
Answer: D

The lower limit is determined as the smallest value such that
$\hat{S}(t) \le .25+1.96\sqrt{\hat{V}[\hat{S}(t)]}$.
At t = 50 the two sides are .360 and $.25+1.96(.0470)=.342$ and the inequality does not hold.
At t = 54 the two sides are .293 and $.25+1.96(.0456)=.339$ and the inequality does hold.
The lower limit is 54.

Question # 52
Answer: A

The distribution used for simulation is given by the observed values.

Question # 53
Answer: B

First obtain the distribution of aggregate losses:

Value	Probability
0	1/5
25	(3/5)(1/3) = 1/5
100	(1/5)(2/3)(2/3) = 4/45
150	(3/5)(2/3) = 2/5
250	(1/5)(2)(2/3)(1/3) = 4/45
400	(1/5)(1/3)(1/3) = 1/45

$$\mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105$$

$$\sigma^2 = (1/5)(0^2) + (1/5)(25^2) + (4/45)(100^2) + (2/5)(150^2)$$
$$+ (4/45)(250^2) + (1/45)(400^2) - 105^2 = 8{,}100.$$

Question # 54
Answer: A

Loss Range	Cum. Prob.
0 – 100	0.320
100 – 200	0.530
200 – 400	0.800
400 – 750	0.960
750 – 1000	0.980
1000 – 1500	1.000

At 400, $F(x)$ = 0.8 = $1 - e^{-\frac{400}{\theta}}$; solving gives $\theta = 248.53$.

Question # 55
Answer: B

$$\Pr(class1\,|\,1) = \frac{(1/2)(1/3)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = \frac{3}{4}$$

$$\Pr(class2\,|\,1) = \frac{(1/3)(1/6)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = \frac{1}{4}$$

$$\Pr(class3\,|\,1) = \frac{(1/6)(0)}{(1/2)(1/3) + (1/3)(1/6) + (1/6)(0)} = 0$$

because the prior probabilities for the three classes are 1/2, 1/3, and 1/6 respectively.

The class means are

$$\mu(1) = (1/3)(0) + (1/3)(1) + (1/3)(2) = 1$$
$$\mu(2) = (1/6)(1) + (2/3)(2) + (1/6)(3) = 2.$$

The expectation is

$$E(X_2 \mid 1) = (3/4)(1) + (1/4)(2) = 1.25.$$

Question # 56
Answer: **E**

The first, second, third, and sixth payments were observed at their actual value and each contributes *f*(*x*) to the likelihood function. The fourth and fifth payments were paid at the policy limit and each contributes 1 – *F*(*x*) to the likelihood function. This is answer (E).

Question #57
Answer is E

For an interval running from *c* to *d*, the uniform density function is $f(x) = g/[n(d-c)]$ where *g* is the number of observations in the interval and *n* is the sample size. The contribution to the second raw moment for this interval is:

$$\int_c^d x^2 \frac{g}{n(d-c)}dx = \left.\frac{gx^3}{3n(d-c)}\right|_c^d = \frac{g(d^3-c^3)}{3n(d-c)}.$$

For this problem, the second raw moment is:

$$\frac{1}{90}\left[\frac{30(25^3-0^3)}{3(25-0)} + \frac{32(50^3-25^3)}{3(50-25)} + \frac{20(100^3-50^3)}{3(100-50)} + \frac{8(200^3-100^3)}{3(200-100)}\right] = 3958.33.$$

Question #58
Answer is B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $\mu(\lambda) = v(\lambda) = \lambda$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$$\mu = E(\lambda) = 50/500 = 0.1$$

$$v = E(\lambda) = 0.1$$

$$a = Var(\lambda) = 50/500^2 = 0.0002$$

$$k = v/a = 500$$

$$Z = 1500/(1500+500) = 0.75$$

$$\overline{X} = \frac{75+210}{600+900} = 0.19.$$

The credibility estimate is $0.75(0.19)+0.25(0.1)=0.1675$. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter λ times the number of policies)
$\frac{e^{-600\lambda}(600\lambda)^{75}}{75!}\frac{e^{-900\lambda}(900\lambda)^{210}}{210!}\frac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)}\propto\lambda^{335}e^{-2000\lambda}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.

Question #59
Answer is E

The *q-q* plot takes the ordered values and plots the *j*th point at *j*/(*n*+1) on the horizontal axis and at $F(x_j;\theta)$ on the vertical axis. For small values, the model assigns more probability to being below that value than occurred in the sample. This indicates that the model has a heavier left tail than the data. For large values, the model again assigns more probability to being below that value (and so less probability to being above that value). This indicates that the model has a lighter right tail than the data. Of the five answer choices, only E is consistent with these observations. In addition, note that as you go from 0.4 to 0.6 on the horizontal axis (thus looking at the middle 20% of the data), the *q-q* plot increases from about 0.3 to 0.4 indicating that the model puts only about 10% of the probability in this range, thus confirming answer E.

Question #60
Answer is C

The posterior probability of having one of the coins with a 50% probability of heads is proportional to (.5)(.5)(.5)(.5)(4/6) = 0.04167. This is obtained by multiplying the probabilities of making the successive observations 1, 1, 0, and 1 with the 50% coin times the prior probability of 4/6 of selecting this coin. The posterior probability for the 25% coin is proportional to (.25)(.25)(.75)(.25)(1/6) = 0.00195 and the posterior probability for the 75% coin is proportional to (.75)(.75)(.25)(.75)(1/6) = 0.01758. These three numbers total 0.06120. Dividing by this sum gives the actual posterior probabilities of 0.68088, 0.03186, and 0.28726. The expected value for the fifth toss is then (.68088)(.5) + (.03186)(.25) + (.28726)(.75) = 0.56385.

Question #61
Answer is A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,

$$L(\theta) = \frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1-F(100)]^5} = \frac{\theta^{-1}e^{-125/\theta}\theta^{-1}e^{-150/\theta}\theta^{-1}e^{-165/\theta}\theta^{-1}e^{-175/\theta}\theta^{-1}e^{-250/\theta}}{(e^{-100/\theta})^5}$$
$$= \theta^{-5}e^{-365/\theta}.$$

Taking logarithms and then a derivative gives

$$l(\theta) = -5\ln(\theta) - 365/\theta, \; l'(\theta) = -5/\theta + 365/\theta^2 = 0.$$

The solution is $\hat{\theta} = 365/5 = 73.$

Question #62
Answer is D

The number of claims for each insured has a binomial distribution with *n* = 1 and *q* unknown. We have

$$\mu(q) = q, \; v(q) = q(1-q)$$
$$\mu = E(q) = 0.1, \text{ given in item (iv)}$$
$$a = Var(q) = E(q^2) - E(q)^2 = E(q^2) - 0.01 = 0.01, \text{ given in item (v)}$$
$$\text{Therefore, } E(q^2) = 0.02$$
$$v = E(q - q^2) = 0.1 - 0.02 = 0.08$$
$$k = v/a = 8, \; Z = \frac{10}{10+8} = 5/9.$$

Then the expected number of claims in the next one year is (5/9)(0) + (4/9)(0.1) = 2/45 and the expected number of claims in the next five years is 5(2/45) = 2/9 = 0.22.

Question #63
Answer is A

Using the set-up as in the text, the solution proceeds as follows: Taking one year as the unit of time, we have $\tau = 3$, X is the time between the issue and the first claim on a policy, and we want to estimate $P(X < 2 | X \le 3)$.

No. Of Policies	T_i	X_i	R_i	d_i	Y_i	$P(X < x_i \mid X \le 3)$
5	0	1	2			
6	1	1	2			
7	2	1	2	18	18	0
9	0	2	1			
10	1	2	1	19	30	$\left(14/27\right) \times \left(11/30\right) = 0.1901$
13	0	3	0	13	27	$\left(14/27\right) = 0.5185$

The answer is the middle number in the last column, namely 0.1901.

Alternatively, perhaps all that one remembers is that for right-truncated data the Kaplan-Meier estimate can be used provided we work with, in this case, the variable R = 3 - X. Then the observations become left truncated. The probability we seek is $\Pr(X < 2 | X \le 3) = \Pr(3 - R < 2 | 3 - R \le 3) = \Pr(R > 1 | R \ge 0)$ and because R cannot be negative, this reduces to $\Pr(R > 1)$.

The six entries in the original table can be identified as follows:

Number of entries	Left truncation point	Value of *R*
5	0	2
9	0	1
13	0	0
6	1	2
10	1	1
7	2	2

The left truncation point is three minus the right truncation point. For example the entries in the second row of the original table could have X values of 1 or 2, but no higher. So they have a right truncation point 2 which for R is a left truncation point of 3 - 2 = 1. We then observe that the risk set at time 0 is 27 (the observations with a left truncation point at 0) and of them, there were 13 deaths. The Kaplan-Meier estimate of surviving past time 0 is then (14/27). At time 1 the risk set has 30 members (the 43 who were left truncated at 0 or 1 less the 13 who died at time 0) of which 19 died (had an R value of 1). The Kaplan-Meier estimate of surviving past time 1 is (14/27)(11/30) = 0.1901.

Question #64
Answer is E

The model distribution is $f(x|\theta) = 1/\theta, 0 < x < \theta$. Then the posterior distribution is proportional to

$$\pi(\theta | 400, 600) \propto \frac{1}{\theta}\frac{1}{\theta}\frac{500}{\theta^2} \propto \theta^{-4}, \theta > 600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 in order for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} \theta^{-4} d\theta = \frac{1}{3(600)^3}$ and thus the exact posterior density is

$\pi(\theta | 400, 600) = 3(600)^3 \theta^{-4}, \theta > 600$. The posterior probability of an observation exceeding 550 is

$$\Pr(X_3 > 550 | 400, 600) = \int_{600}^{\infty} \Pr(X_3 > 550 | \theta)\pi(\theta | 400, 600)d\theta$$

$$= \int_{600}^{\infty} \frac{\theta - 550}{\theta} 3(600)^3 \theta^{-4} d\theta = 0.3125$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.

Question #65
Answer is C

$$E(N) = r\beta = 0.40$$

$$Var(N) = r\beta(1+\beta) = 0.48$$

$$E(Y) = \theta/(\alpha - 1) = 500$$

$$Var(Y) = \theta^2\alpha / \left[(\alpha-1)^2(\alpha-2)\right] = 750,000$$

Therefore,

$$E(X) = 0.40(500) = 200$$

$$Var(X) = 0.40(750,000) + 0.48(500)^2 = 420,000$$

The full credibility standard is $n = \left(\frac{1.645}{0.05}\right)^2 \frac{420,000}{200^2} = 11,365$ and then

$$Z = \sqrt{2500/11,365} = 0.47.$$

Question #66
Answer is E

The sample variance is $s^2 = \dfrac{(1-3)^2+(2-3)^2+(3-3)^2+(4-3)^2+(5-3)^2}{4} = 2.5.$ The estimator of $E[X]$ is the sample mean and the variance of the sample mean is the variance divided by the sample size, estimated here as $2.5/n$. Setting the standard deviation of the estimator equal to 0.05 gives the equation $\sqrt{2.5/n} = 0.05$ which yields $n = 1000$.

Question #67
Answer is E

$$\mu(r) = E(X \mid r) = E(N)E(Y) = r\beta\theta/(\alpha-1) = 100r$$
$$\begin{aligned} v(r) &= Var(X \mid r) = Var(N)E(Y)^2 + E(N)Var(Y) \\ &= r\beta(1+\beta)\theta^2/(\alpha-1)^2 + r\beta\alpha\theta^2/[(\alpha-1)^2(\alpha-2)] = 210{,}000r. \end{aligned}$$

$$v = E(210{,}000r) = 210{,}000(2) = 420{,}000$$
$$a = Var(100r) = (100)^2(4) = 40{,}000$$
$$k = v/a = 10.5$$
$$Z = 100/(100+10.5) = 0.905.$$

Question #68
Answer is B

Using all participants, $S^{\mathrm{T}}(4) = \left(1-\dfrac{35}{300}\right)\left(1-\dfrac{74}{265}\right)\left(1-\dfrac{34}{191}\right)\left(1-\dfrac{32}{157}\right) = 0.41667.$

Using only Country B, $S^{\mathrm{B}}(4) = \left(1-\dfrac{15}{100}\right)\left(1-\dfrac{20}{85}\right)\left(1-\dfrac{20}{65}\right)\left(1-\dfrac{10}{45}\right) = 0.35.$

The difference is, $S^{\mathrm{T}}(4) - S^{\mathrm{B}}(4) = 0.41667 - 0.35 = 0.0667 = 0.07.$

Question #69
Answer is B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have

$$E(\bar{X}) = E(X) = \theta, Var(\bar{X}) = Var(X)/n = \theta^2/n.$$
$$cv = SD(\bar{X})/E(\bar{X}) = [\theta/\sqrt{n}]/\theta = 1/\sqrt{n} = 1/\sqrt{5} = 0.447.$$

If the above facts are not known, the loglikelihood function can be used:

$L(\theta)=\theta^{-n}e^{-\Sigma x_j/\theta},\quad l(\theta)=-n\ln\theta-n\bar{X}/\theta,\quad l'(\theta)=-n\theta^{-1}+n\bar{X}\theta^{-2}=0\Rightarrow\hat{\theta}=\bar{X}.$

$l''(\theta)=n\theta^{-2}-2n\bar{X}\theta^{-3},\quad I(\theta)=E[-n\theta^{-2}+2n\bar{X}\theta^{-3}]=n\theta^{-2}.$

Then, $Var(\hat{\theta})=\theta^2/n.$

Question #70
Answer is D

Because the total expected claims for business use is 1.8, it must be that 20% of business users are rural and 80% are urban. Thus the unconditional probabilities of being business-rural and business-urban are 0.1 and 0.4 respectively. Similarly the probabilities of being pleasure-rural and pleasure-urban are also 0.1 and 0.4 respectively. Then,

$\mu=0.1(1.0)+0.4(2.0)+0.1(1.5)+0.4(2.5)=2.05$

$v=0.1(0.5)+0.4(1.0)+0.1(0.8)+0.4(1.0)=0.93$

$a=0.1(1.0^2)+0.4(2.0^2)+0.1(1.5^2)+0.4(2.5^2)-2.05^2\ =0.2225$

$k=v/a=4.18$

$Z\ =1/(1+4.18)=0.193.$

Question #71
Answer is A

No. claims	Hypothesized	Observed	Chi-square
1	250	235	$15^2/250$ = 0.90
2	350	335	$15^2/350$ = 0.64
3	240	250	$10^2/240$ = 0.42
4	110	111	$1^2/110$ = 0.01
5	40	47	$7^2/40$ = 1.23
6+	10	22	$12^2/10$ = 14.40

The last column sums to the test statistic of 17.60 with 5 degrees of freedom (there were no estimated parameters), so from the table reject at the 0.005 significance level.

Question #72
Answer is C

In part (ii) you are given that $\mu = 20$. In part (iii) you are given that $a = 40$. In part (iv) you are given that $v = 8,000$. Therefore, $k = v/a = 200$. Then,

$$\bar{X} = \frac{800(15) + 600(10) + 400(5)}{1800} = \frac{100}{9}$$

$$Z = \frac{1800}{1800 + 200} = 0.9$$

$$P_c = 0.9(100/9) + 0.1(20) = 12.$$

Question #73
Answer is C

$$\Pr(X > 30,000) = S(30,000) = \left(1 - \frac{1}{10 - 2/2}\right)\left(1 - \frac{1}{7 - 2/2}\right) = 20/27 = 0.741.$$

Question #74
Answer is C

The formulas are from Section 5.5 of *Loss Models*.

$$\hat{v} = \frac{3(0.536 + 0.125 + 0.172)}{3 + 3 + 3} = 0.27767.$$

$$\hat{a} = \frac{0.887 + 0.191 + 1.348 - 2(0.27767)}{500 - \frac{1}{500}(50^2 + 300^2 + 150^2)} = 0.00693.$$

Then,

$$k = 0.27767/0.00693 = 40.07, \; Z_1 = \frac{50}{50 + 40.07} = 0.55512, \; Z_2 = \frac{300}{300 + 40.07} = 0.88217,$$

$$Z_3 = \frac{150}{150 + 40.07} = 0.78918.$$

The credibility weighted mean is,

$$\hat{\mu} = \frac{0.55512(1.406) + 0.88217(1.298) + 0.78918(1.178)}{0.55512 + 0.88217 + 0.78918} = 1.28239.$$

The credibility premium for state 1 is

$$P_c = 0.55512(1.406) + 0.44488(1.28239) = 1.351.$$

Question #75
Answer is D

$$E(X)=\int_{\delta}^{\infty}\frac{x}{\theta}e^{-(x-\delta)/\theta}dx=\int_{0}^{\infty}\frac{y+\delta}{\theta}e^{-y/\theta}dx=\theta+\delta$$

$$E(X^2)=\int_{\delta}^{\infty}\frac{x^2}{\theta}e^{-(x-\delta)/\theta}dx=\int_{0}^{\infty}\frac{y^2+2y\delta+\delta^2}{\theta}e^{-y/\theta}dx=2\theta^2+2\theta\delta+\delta^2.$$

Both derivations use the substitution $y=x-\delta$ and then recognize that the various integrals are requesting moments from an ordinary exponential distribution. The method of moments solves the two equations

$\theta+\delta=10$

$2\theta^2+2\theta\delta+\delta^2=130.6$

producing $\hat{\delta}=4.468$.

It is faster to do the problem if it is noted that $X=Y+\delta$ where *Y* has an ordinary exponential distribution. Then $E(X)=E(Y)+\delta=\theta+\delta$ and $Var(X)=Var(Y)=\theta^2$.

Question #76
Answer is D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $\pi(\theta\,|\,N>0)\propto\Pr(N>0\,|\,\theta)\pi(\theta)=(1-e^{-\theta})\theta e^{-\theta}$.

The constant of proportionality is obtained from $\int_0^\infty \theta e^{-\theta}-\theta e^{-2\theta}d\theta=\frac{1}{1^2}-\frac{1}{2^2}=0.75.$

The posterior density is $\pi(\theta\,|\,N>0)=(4/3)(\theta e^{-\theta}-\theta e^{-2\theta})$.

Then,

$$\Pr(N_2>0\,|\,N_1>0)=\int_0^\infty \Pr(N_2>0\,|\,\theta)\pi(\theta\,|N_1>0)d\theta=\int_0^\infty(1-e^{-\theta})(4/3)(\theta e^{-\theta}-\theta e^{-2\theta})d\theta$$

$$=\frac{4}{3}\int_0^\infty \theta e^{-\theta}-2\theta e^{-2\theta}+\theta e^{-3\theta}d\theta=\frac{4}{3}\left(\frac{1}{1^2}-\frac{2}{2^2}+\frac{1}{3^2}\right)=0.8148.$$

Question #77
Answer is E

The interval is centered at 2.09 and the plus/minus term is 0.46 which must equal $1.96\hat{\sigma}$ and so $\hat{\sigma}=0.2347$. For the log-transformed interval we need $\phi=e^{1.96(0.2347)/2.09}=1.2462$. The lower limit is 2.09/1.2462 = 1.68 and the upper limit is 2.09(1.2462) = 2.60.

Question #78
Answer is B

From item (ii), $\mu = 1000$ and *a* = 50. From item (i), *v* = 500. Therefore, *k* = *v*/*a* = 10 and
Z = 3/(3+10) = 3/13. Also, $\bar{X} = (750 + 1075 + 2000)/3 = 1275$. Then
$P_c = (3/13)(1275) + (10/13)(1000) = 1063.46.$

Question #79
Answer is C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10,000}e^{-x/10,000}$$

$$\begin{aligned} L(100,200) &= f(100)f(2000) \\ &= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right) \end{aligned}$$

Question #80
Key: C

Model Solution:

For a binomial random variable with $n = 100$ and $p = q_{70} = 0.03318,$ simulate number of deaths:

$i = 0: (1-p)^{100} = 0.03424 = f(0) = F(0)$

Since $0.18 > F(0),$ continue

$$\begin{aligned} i = 1: f(1) &= f(0)(n)(p)/(1-p) \\ &= (0.03424)(100)(0.03318)/(0.96682) \\ &= 0.11751 \end{aligned}$$

$F(1) = F(0) + f(1) = 0.03424 + 0.11751 = 0.15175$
Since $0.18 > F(1)$, continue

$i = 2\text{:}\ f(2) = f(1)\left[(n-1)/2\right](p)/(1-p)$

$$= (0.11751)(99/2)(0.03318/0.96682)$$
$$= 0.19962$$

$F(2) = F(1) + f(2) = 0.15175 + 0.19962 = 0.35137$

Since $0.18 < F(2)$, number of claims $= 2$, so claim amount $= 20.$

Question # 81
Answer: C

Which distribution is it from?

0.25 < 0.30, so it is from the exponential.

Given that Y is from the exponential, we want

$$\Pr(Y \le y) = F(y) = 0.69$$
$$1 - e^{-y/\theta} = 0.69$$
$$1 - e^{-y/0.5} = 0.69 \text{ since mean = 0.5}$$
$$\frac{-y}{0.5} = \ln(1-0.69) = -1.171$$
$$y = 0.5855$$

Question #82
Key: B

If you happen to remember this distribution from the Simulation text (example 4d in third edition), you could use:

$$n = Int\left(\frac{\log(1-u)}{\log q}\right) + 1 = Int\frac{\log 0.95}{\log 0.1} + 1 = 0 + 1 = 1$$

For mere mortals, you get the simulated value of *N* from the definition of the inverse transformation method:

f(1) = *F*(1) = 0.9

0.05 $\le$ 0.9 so *n* = 1

$$x_1 = \frac{1}{\lambda}\log^{(1-v_1)} = -\frac{1}{0.01}\log 0.7 = 35.67$$

The amount of total claims during the year $= 35.67$

Question #83
Key: B

$F(0) = 0.8$
$F(t) = 0.8 + 0.00025(t\text{-}1000)$, $1000 \le t \le 5000$

$0.75 \Rightarrow$ 0 found since $F(0) \ge 0.75$

$0.85 \Rightarrow$ 2000 found since $F(2000) = 0.85$

Average of those two outcomes is 1000.