CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

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Exam 3, Segment 3L

Life Contingencies and Statistics

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May 8, 2009

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2009 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. Your Candidate ID number is fewer than 5 digits so you will need to include leading zeros. For example, if your Candidate ID number is 1987, consider that your Candidate ID number is 01987, enter a zero on the first row, 1 on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by June 1, 2009.

END OF INSTRUCTIONS

1. An actuary is modeling the failure of a product over the next 20 years. You are given that the product's lifetime follows a constant force model with $\mu(x) = 0.05$.

Calculate the expected lifetime of a new product over the next 20 years.

A. Less than 10.0
B. At least 10.0, but less than 11.0
C. At least 11.0, but less than 12.0
D. At least 12.0, but less than 13.0
E. At least 13.0

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2. You are given:

- ${}_{2|2}q_{49} = 0.182$
- $p_{50} = 0.900$
- ${}_{1|}q_{49} = 0.100$
- $p_{52} = 0.875$
- $l_{50} = 1000$

Calculate l_{53}, the number of lives at age 53.

A. 636
B. 637
C. 638
D. 639
E. 640

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3. A life table is constructed at annual points in time using a constant force of mortality model, $\mu(x) = 0.20$.

Using the uniform distribution of deaths for partial years, calculate the probability that a new life dies during the first 9 months.

A. Less than 0.135
B. At least 0.135, but less than 0.137
C. At least 0.137, but less than 0.139
D. At least 0.139, but less than 0.141
E. At least 0.141

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4. You are given a choice of 2 types of power systems:

- Each power system uses two batteries.
- Type A produces power if at least one of the two batteries is functioning.
- Type B only produces power if both batteries are functioning.
- Battery lifetimes in both power systems are independent with the same density function, given below:

$$f(t)=\frac{2}{(1+t)^3}, t \geq 0$$

Calculate the difference in expected time to failure between the two power systems.

A. Less than 1.4
B. At least 1.4, but less than 1.5
C. At least 1.5, but less than 1.6
D. At least 1.6, but less than 1.7
E. At least 1.7

CONTINUED ON NEXT PAGE

5. You are given two independent lives (x) and (y) and the following information:

- $x = 20$
- $y = 40$
- The lifetimes of (x) and (y) follow De Moivre's law, with $\omega = 100$.

Calculate the probability that at least one life will survive for the next 20 years.

A. Less than 0.88
B. At least 0.88, but less than 0.89
C. At least 0.89, but less than 0.90
D. At least 0.90, but less than 0.91
E. At least 0.91

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6. For a non-homogeneous Markov Chain with two states, Intact and Failed, the following matrix shows the probability of movement between states, where t = 1, 2, 3,...

$$Q_t = \begin{bmatrix} 0.8^{0.5t} & 1-0.8^{0.5t} \\ 0 & 1 \end{bmatrix}$$

Calculate the minimum number of time periods so that the expected percentage of entities in state Intact is less than 10%.

A. 5
B. 6
C. 10
D. 11
E. 21

CONTINUED ON NEXT PAGE

7. You are given the following information about workers' compensation claims:

- Open claims can be classified as Temporary Disability or Permanent Disability until they are closed.
- Claims are reclassified or closed at the end of the year.
- A claim costs $5,000 for each year in which it is open.
- i = 0%.

The following transition matrix describes movement between the states for each year:

	Temporary Disability	Permanent Disability	Closed
Temporary Disability	0.4	0.3	0.3
Permanent Disability	0.0	0.9	0.1
Closed	0.0	0.0	1.0

Calculate the probability that a claim starting as Temporary Disability costs $15,000 or less.

A. Less than 0.40
B. At least 0.40 but less than 0.45
C. At least 0.45 but less than 0.50
D. At least 0.50 but less than 0.55
E. At least 0.55

CONTINUED ON NEXT PAGE

8. Bill receives mail at a Poisson rate of 10 items per day. The contents of the items are randomly distributed:

- 50% of the items are credit card applications.
- 30% of the items are catalogs.
- 20% of the items are letters from friends.

Bill has received 20 credit card applications in two days.

Calculate the probability that for those same two days, he receives at least 3 letters from friends and exactly 5 catalogs.

A. Less than 6%
B. At least 6%, but less than 10%
C. At least 10%, but less than 14%
D. At least 14%, but less than 18%
E. At least 18%

CONTINUED ON NEXT PAGE

9. You are given the following information:

- Policyholder calls to a call center follow a homogenous Poisson process with $\lambda = 250$ per day.
- Policyholders may call for 3 reasons: Endorsement, Cancellation, or Payment.
- The distribution of calls is as follows:

Call Type	Percent of Calls
Endorsement	50%
Cancellation	10%
Payment	40%

Using the normal approximation with continuity correction, calculate the probability of receiving more than 156 calls in a day that are either endorsements or cancellations.

A. Less than 27%
B. At least 27%, but less than 29%
C. At least 29%, but less than 31%
D. At least 31%, but less than 33%
E. At least 33%

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10. The premium production of an insurance agent can be modeled by the following process:

- The number of policies the agent sells follows a Poisson distribution with $\lambda = 13$ policies per day.
- The amount of premium (in \$) on any given policy follows a gamma distribution with $\alpha = 0.5$ and $\theta = 1000$.

Calculate the standard deviation of the total amount of premium written by the agent over a five-day period.

A. Less than \$6,000
B. At least \$6,000, but less than \$7,000
C. At least \$7,000, but less than \$8,000
D. At least \$8,000, but less than \$9,000
E. At least \$9,000

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11. You are given the following information about a continuous 20-year term life insurance policy, with benefit b_t, on a life aged x:

- $s(x+t) = e^{-0.25t}$
- $\delta = 0.05$
- $b_t = e^{0.15t}$

Calculate the actuarial present value.

A. Less than 1.59
B. At least 1.59, but less than 1.60
C. At least 1.60, but less than 1.61
D. At least 1.61, but less than 1.62
E. At least 1.62

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12. You are given:

- The force of mortality is constant.
- $\bar{A}_x = 0.667$
- $\bar{a}_x = 8.333$

Calculate $Var(\bar{a}_{\overline{T}|})$.

A. Less than 32
B. At least 32, but less than 34
C. At least 34, but less than 36
D. At least 36, but less than 38
E. At least 38

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13. John is permanently disabled at age 65. His workers' compensation plan offers him a choice of two benefits:

- Option I: Payments of $45,000 at the beginning of each year (starting now) until he reaches age 70 or dies, whichever occurs first.

- Option II: A single lump sum payment, paid immediately.

You are given:

- $i = 5\%$
- De Moivre mortality with $\omega = 100$.
- Options I and II are actuarially equivalent.

Calculate the amount of the lump sum payment.

A. Less than $195,000
B. At least $195,000, but less than $205,000
C. At least $205,000, but less than $215,000
D. At least $215,000, but less than $225,000
E. At least $225,000

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14. For a fully discrete five-year endowment insurance of benefit 1 on a life aged 35, you are given:

- $\ddot{a}_{35:\overline{5}|} = 4.1$
- $q_{38} = 0.10$
- $d = 0.05$
- Premiums are paid at the beginning of the year.
- Benefits are paid at the end of the year.

Calculate ${}_3V_{35:\overline{5}|}$, the benefit reserve at t = 3.

A. Less than 0.550
B. At least 0.550, but less than 0.575
C. At least 0.575, but less than 0.600
D. At least 0.600, but less than 0.625
E. At least 0.625

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15. A credit card company provides credit life insurance to its cardholders. In the event that the cardholder dies, the company pays the credit card balance at the moment of death.

For the average cardholder, you are given:

- Annual insurance premium is \$12, payable continuously.
- Credit card balance is \$1,500.
- Constant force of mortality, $\mu = 0.01$
- Constant force of interest, $\delta = 0.06$

Calculate the credit life insurance reserve for the average cardholder.

A. Less than \$40
B. At least \$40, but less than \$41
C. At least \$41, but less than \$42
D. At least \$42, but less than \$43
E. At least \$43

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16. An insurance company offers two different three-year auto insurance policies to drivers who have not had an accident in the previous year:

- Policy #1 charges premium X at the start of each year.
- Policy #2 charges premium Y at the start of each year, but also charges a penalty of $100 at the end of each year in which an accident occurs.

You are also given:

- The probability that an accident occurs in a year is 20% if no accidents occurred in the previous year.
- The probability that an accident occurs in a year is 50% if at least one accident occurred in the previous year.
- $i = 4\%$
- Premiums are calculated using the equivalence principle.

Calculate the difference $X - Y$.

A. Less than $55
B. At least $55, but less than $60
C. At least $60, but less than $65
D. At least $65, but less than $70
E. At least $70

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17. A random variable, X, follows a lognormal distribution. You are given a sample of size n and the following information:

$$\frac{\sum x_i}{n} = 1.8682$$

$$\frac{\sum x_i^2}{n} = 4.4817$$

Use the method of moments to estimate the lognormal parameter σ.

A. Less than 0.4
B. At least 0.4, but less than 0.8
C. At least 0.8, but less than 1.2
D. At least 1.2, but less than 1.6
E. At least 1.6

CONTINUED ON NEXT PAGE

18. You are given the following information about a random sample of claim amounts that are independent and identically distributed:

- Claim severity follows a two-parameter Pareto distribution with $\alpha = 3$ and $\theta = 50$.
- The sample size is 100.

Calculate the probability that the sample mean will lie between 30 and 35 by using the limiting distribution for the sample mean.

A. Less than 5%
B. At least 5%, but less than 6%
C. At least 6%, but less than 7%
D. At least 7%, but less than 8%
E. At least 8%

CONTINUED ON NEXT PAGE

19. You are given the following:

- A random variable, X, has the following probability density function:
  $f(x,\theta) = \theta x^{\theta-1}$, $0< x < 1$, $0< \theta < \infty$

- A random sample from this distribution is shown below:
  0.10 0.25 0.50 0.60 0.70

Calculate the maximum likelihood estimate of θ.

A. Less than 0.4
B. At least 0.4, but less than 0.6
C. At least 0.6, but less than 0.8
D. At least 0.8, but less than 1.0
E. At least 1.0

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20. You are given the following information about a random sample of 100 claims:

- Claim severity follows an exponential distribution with mean θ.
- H_0: $\theta = 9{,}500$
- H_1: $\theta = 12{,}500$
- Reject H_0 if the sample mean is greater than 11,000.

Calculate the probability of a Type II error.

A. Less than 2.5%
B. At least 2.5%, but less than 5.0%
C. At least 5.0%, but less than 7.5%
D. At least 7.5%, but less than 10.0%
E. At least 10.0%

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21. You are given the following information:

- $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean μ_x and standard deviation σ_x.
- $Y_1, \ldots, Y_n$ is a random sample from a normal distribution with mean μ_y and standard deviation σ_y.
- The unbiased sample variances S_x^2 and S_y^2 are 4.0 and 1.3, respectively.
- $H_0 : \sigma_x^2 = \sigma_y^2$
- $H_1 : \sigma_x^2 > \sigma_y^2$
- Use of the F test results in rejection of the null hypothesis at a significance level of 5%.

Calculate the minimum possible value of sample size, n.

A. 7
B. 8
C. 9
D. 10
E. 11

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22. You are given the following information about a one-sided test that is being performed on a random sample taken from a normal distribution:

- The unbiased sample variance is 9.
- The sample size is 16.
- H_0: $\mu = 80$
- H_1: $\mu > 80$

Calculate the highest possible sample mean that would still result in the null hypothesis not being rejected at the 1% significance level.

A. Less than 81.00
B. At least 81.00, but less than 81.50
C. At least 81.50, but less than 82.00
D. At least 82.00, but less than 82.50
E. At least 82.50

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23. Today, you bought three new plasma screen televisions. You have been told that the failure time (time until the televisions need to be replaced) for these televisions follows an exponential distribution with $\theta = 12$.

Calculate the probability that at least two of the three televisions will last 10 years or more.

A. Less than 35%
B. At least 35%, but less than 37%
C. At least 37%, but less than 39%
D. At least 39%, but less than 41%
E. At least 41%

CONTINUED ON NEXT PAGE

24. You are given the following information:

- Claim severity follows a uniform distribution over the interval $(0,\alpha)$.
- A random sample of n claims is observed.
- The probability density function of the order statistic Y_k with a sample size n is:
  $$\frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y).$$
- Y_n, the largest claim amount in the sample, is used as an estimator of α.

Calculate the minimal sample size, n, necessary to ensure that the absolute value of the bias of the estimator Y_n is less than 5% of α.

A. 19
B. 20
C. 21
D. 22
E. 23

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25. You are modeling your company's yearly premium as a function of time using the linear model:

- $Y = \alpha + \beta X$
- Y represents premium (in millions of dollars).
- X represents the number of years since 2000 (so that 1 represents 2001, 2 represents 2002, etc.).

You use the following summary statistics to perform a least squares regression:

$$n = 6$$

$$\sum_{i=1}^{6} X_i = 15 \qquad \sum_{i=1}^{6} X_i^2 = 55$$

$$\sum_{i=1}^{6} Y_i = 468 \qquad \sum_{i=1}^{6} X_i Y_i = 1{,}363$$

Determine the first year that the estimated premium exceeds $150 million.

A. 2007
B. 2008
C. 2009
D. 2010
E. 2011

END OF EXAMINATION

Answer Key, Spring 2009 Exam 3L

1. D
2. Invalid
3. B
4. A
5. E
6. B
7. D
8. C
9. C
10. B
11. A
12. C
13. A,D
14. A
15. D
16. A,D
17. B
18. E
19. D
20. E
21. E
22. C
23. D
24. B
25. C,D