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## Exam 3, Segment 3L

Life Contingencies and Statistics

October 28, 2008

2.5 HOURS

### INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.
2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.
  - Fill in that it is Fall 2008 and that the exam number is 3L.
  - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter zeroes on the first and second rows, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
  - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
  - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.
4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
  - Verify that you have a copy of "Tables for CAS Exam 3" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.
7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.  
  
If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.  
  
Candidates may obtain a copy of the examination from the CAS Web Site.  
  
All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.
9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 17, 2008.

**END OF INSTRUCTIONS**

Exam 3L, Fall 2008

1.

The number of accidents on a highway from 3:00 PM to 7:00 PM follows a non-homogeneous Poisson process with rate function

$$\lambda = 4 - (t - 2)^2, \text{ where } t \text{ is the number of hours since 3:00 PM.}$$

How many more accidents are expected from 4:00 PM to 5:00 PM than from 3:00 PM to 4:00 PM?

- A. Less than 0.75
- B. At least 0.75, but less than 1.25
- C. At least 1.25, but less than 1.75
- D. At least 1.75, but less than 2.25
- E. At least 2.25

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Exam 3L, Fall 2008

2.

You are given the following:

- Hurricanes occur at a Poisson rate of  $1/4$  per week during the hurricane season.
- The hurricane season lasts for exactly 15 weeks.

Prior to the next hurricane season, a weather forecaster makes the statement, "There will be at least three and no more than five hurricanes in the upcoming hurricane season."

Calculate the probability that this statement will be correct.

- A. Less than 54%
- B. At least 54%, but less than 56%
- C. At least 56%, but less than 58%
- D. At least 58%, but less than 60%
- E. At least 60%

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3.

You are given the following information about a compound Poisson process that describes the number of items purchased by shoppers in a store:

- The rate at which shoppers arrive at the store is 10 per hour.
- The expected number of items purchased by all shoppers entering the store in one hour is 10.
- The variance of the number of items purchased by all shoppers entering the store in one hour is 100.

Calculate the variance of the number of items purchased by an individual shopper.

- A. Less than 6.5
- B. At least 6.5, but less than 7.5
- C. At least 7.5, but less than 8.5
- D. At least 8.5, but less than 9.5
- E. At least 9.5

Exam 3L, Fall 2008

4.

You are given the following information:

- A random variable  $X$  has probability density function:

$$f(x;\theta) = \theta x^{\theta-1}, \text{ where } 0 < x < 1 \text{ and } \theta > 0$$

- A random sample of five observations from this distribution is shown below:

0.25   0.50   0.40   0.80   0.65

Calculate the maximum likelihood estimator for  $\theta$ .

- A. Less than 1.00
- B. At least 1.00, but less than 1.10
- C. At least 1.10, but less than 1.20
- D. At least 1.20, but less than 1.30
- E. At least 1.30

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5.

You are given the following information about a random variable  $X$ .

- $X$  follows a Gamma Distribution
- $\alpha = 3$
- $\theta = 100$

Using the mean square error criterion, calculate the constant value,  $c$ , which is the best estimate of  $X$ .

- A. Less than 50
- B. At least 50, but less than 150
- C. At least 150, but less than 250
- D. At least 250, but less than 350
- E. At least 350

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6.

You are given the following:

- An insurance company provides a coverage which can result in only three loss amounts in the event that a claim is filed: \$0, \$500 or \$1,000.
- The probability,  $p$ , of a loss being \$0 is the same as the probability of it being \$1,000.
- The following 3 claims are observed:  
\$0   \$0   \$1,000

What is the maximum likelihood estimate of  $p$ ?

- A. Less than 0.20
- B. At least 0.20, but less than 0.40
- C. At least 0.40, but less than 0.60
- D. At least 0.60, but less than 0.80
- E. At least 0.80

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7.

You are given that the distribution of the inter-event times between patients entering a waiting room is exponential with mean  $\theta$ . You are testing the following hypotheses:

$$H_0: \theta = 10$$

$$H_1: \theta \neq 10$$

You are given the following five observations:

8      9      9      10      11

Calculate the Likelihood Ratio  $L(H_0)/L(H_1)$ .

- A. Less than 0.990
- B. At least 0.990, but less than 0.992
- C. At least 0.992, but less than 0.994
- D. At least 0.994, but less than 0.996
- E. At least 0.996

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8.

You are given the following:

- A random variable,  $X$ , follows a normal distribution with unknown mean  $\mu$  and known variance  $\sigma^2 = 25$ .
- $H_0: \mu = 0$
- $H_1: \mu = 1$
- The critical region is the best under the Neyman-Pearson lemma of size  $\alpha = 0.05$

What is the minimum sample size such that the power of the test (1 – the probability of a Type II error) is 0.95?

- A. 16
- B. 20
- C. 100
- D. 271
- E. 384

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9.

You are given the following information on two groups of policyholders, Group 1 and Group 2:

- Group 1 and Group 2 claim counts follow the binomial distribution.
- A policyholder has either no claim or one claim during the policy period.
- A table summarizing the experience for one policy period is given below:

Policyholder Group	Policyholders With No Claims	Policyholders With One Claim
Group 1	40	10
Group 2	120	30

- $H_0$ : The probability of a policyholder having a claim is the same for the two groups.
- $H_1$ : The probability of a policyholder having a claim is not the same for the two groups.

Calculate the absolute value of the difference between the Chi-Square test statistic to test the null hypothesis and the critical region value at a 5% significance level.

- A. Less than 2.5
- B. At least 2.5, but less than 3.0
- C. At least 3.0, but less than 3.5
- D. At least 3.5, but less than 4.0
- E. At least 4.0

10.

Let  $Y_1$ ,  $Y_2$  and  $Y_3$  be the order statistics of a random sample of size 3 from a uniform distribution on  $[0,1]$ .

Calculate the probability that  $Y_2 < 1/3$ .

- A. Less than 15%
- B. At least 15%, but less than 20%
- C. At least 20%, but less than 25%
- D. At least 25%, but less than 30%
- E. At least 30%

11.

You are given the following:

- $X_1 \dots X_{11}$  is a random sample of size 11 from a normal distribution with mean =  $\mu_1$  and variance =  $\sigma_1^2$ , with the sample mean = 2 and the unbiased sample variance = 4.
- $Y_1 \dots Y_{13}$  is a random sample of size 13 from a normal distribution with mean =  $\mu_2$  and variance =  $\sigma_2^2$ , with the sample mean = 1.5 and the unbiased sample variance = 5.
- $H_0: \mu_1 = \mu_2$
- $H_1: \mu_1 > \mu_2$
- The variance is unknown but is assumed to be the same between samples. (i.e.  $\sigma_1^2 = \sigma_2^2$ )

Calculate the test statistic to evaluate the null hypothesis and select the answer below that contains the observed significance level or p-value.

- A. Less than 0.5%
- B. At least 0.5%, but less than 1.0%
- C. At least 1.0%, but less than 2.5%
- D. At least 2.5%, but less than 5.0%
- E. At least 5.0%

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12.

You are given the force of mortality,  $\mu(x) = \frac{1}{3(100-x)}$ .

Calculate the probability that a life aged 70 will die between ages 75 and 80.

- A. Less than 0.04
- B. At least 0.04, but less than 0.06
- C. At least 0.06, but less than 0.08
- D. At least 0.08, but less than 0.10
- E. At least 0.10

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13.

You are given the curtate expectation of life at age  $x$  and  $x+1$ :

- $e_x = 5.000$
- $e_{x+1} = 4.530$

Calculate  $q_x$ .

- A. Less than 0.091
- B. At least 0.091, but less than 0.093
- C. At least 0.093, but less than 0.095
- D. At least 0.095, but less than 0.097
- E. At least 0.097

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14.

You are given the following life table:

Age ( $x$ )	$l_x$	$d_x$
0		100
1		
2		
3		
4	125	75
5		50

You are also given:

- ${}_{3|2}q_1 = 0.329$
- ${}_3p_0 = 0.427$

Calculate the number of lives in the cohort at age 3.

- A. Less than 200
- B. At least 200, but less than 210
- C. At least 210, but less than 220
- D. At least 220, but less than 230
- E. At least 230

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15.

Lives  $(x)$  and  $(y)$  follow separate life tables. You are given excerpts from those two life tables below:

Age for $(x)$	$l_x$
20	100
21	96

Age for $(y)$	$l_y$
30	100
31	94

Assume a uniform distribution of deaths.

Calculate  $0.5q_{20.5:30.5}$  where age 20.5 is for  $(x)$  and age 30.5 is for  $(y)$ .

- A. Less than 0.049
- B. At least 0.049, but less than 0.050
- C. At least 0.050, but less than 0.051
- D. At least 0.051, but less than 0.052
- E. At least 0.052

16.

For lives  $(x)$  and  $(y)$ , you are given:

- ${}_nq_{xy} = 0.086$
- ${}_nq_x = 0.15$
- ${}_{n+1}P_{xy} = 0.56$

Calculate  ${}_nP_y$

- A. Less than 0.73
- B. At least 0.73, but less than 0.75
- C. At least 0.75, but less than 0.77
- D. At least 0.77, but less than 0.79
- E. At least 0.79

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17.

A piece of electronic equipment is built with two cooling fans. If at least one of the two fans is working, then the equipment will function.

The hazard rates of the fans are independent, constant and identical. The probability that both fans fail over a two year period is 0.050.

Calculate the hazard rate for one of the fans.

- A. Less than 0.124
- B. At least 0.124, but less than 0.125
- C. At least 0.125, but less than 0.126
- D. At least 0.126, but less than 0.127
- E. At least 0.127

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18.

An entity aged ( $x$ ) is subject to two possible decrements.

You are given:

- $m_x$  is defined as the central death rate over the interval  $x$  to  $x+1$
- The total decrement is uniformly distributed over each year
- $l_x = 1,000$
- $l_{x+1} = 800$
- $m_x^{(1)} = 0.1$

Calculate  $q_x^{(2)}$

- A. Less than 0.105
- B. At least 0.105, but less than 0.115
- C. At least 0.115, but less than 0.125
- D. At least 0.125, but less than 0.135
- E. At least 0.135

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19.

Workers' compensation claims occupy one of four states in the table shown below. Claims transition between states from one year to the next according to the following matrix:

	Medical Only	Temporary Disability	Permanent Disability	Fatal
Medical Only	0.60	0.35	0.04	0.01
Temporary Disability	0.00	0.60	0.36	0.04
Permanent Disability	0.00	0.00	0.90	0.10
Fatal	0.00	0.00	0.00	1.00

If a new claim begins as a Medical Only claim, calculate the probability that it is a Permanent Disability claim after three years.

- A. Less than 0.28
- B. At least 0.28 but less than 0.30
- C. At least 0.30 but less than 0.32
- D. At least 0.32 but less than 0.34
- E. At least 0.34

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20.

You are given that mortality follows the Illustrative Life Table and that you may use an interest rate,  $i = 6\%$ .

Calculate the present value for a discrete 3-year annuity-immediate on a life aged 40.

- A. Less than 2.70
- B. At least 2.70, but less than 2.75
- C. At least 2.75, but less than 2.80
- D. At least 2.80, but less than 2.85
- E. At least 2.85

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21.

For a 40-year continuous annuity with a benefit 1 on  $(x)$ , you are given:

- $Z$  is the present value random variable for this annuity
- $\delta = 0.06$
- $\mu = 0.05$

Calculate the variance of  $Z$ .

- A. Less than 22
- B. At least 22, but less than 23
- C. At least 23, but less than 24
- D. At least 24, but less than 25
- E. At least 25

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22.

For a fully continuous whole life policy with benefit 1 on a life aged 40, you are given:

- Survival follows DeMoivre's law with  $\omega = 110$
- $i = 0.071$

Calculate the benefit premium.

- A. Less than 0.018
- B. At least 0.018, but less than 0.019
- C. At least 0.019, but less than 0.020
- D. At least 0.020, but less than 0.021
- E. At least 0.021

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23.

For a fully continuous whole life policy on a life aged  $x$ , you are given:

- Mortality follows an exponential distribution with parameter  $\mu$
- $\delta$  is the force of interest
- The death benefit is  $\delta$

Calculate  ${}_1\overline{V}_x$ , the benefit reserve at  $t = 1$ .

- A. 0
- B.  $\delta$
- C.  $\mu$
- D.  $\frac{\delta}{\mu + \delta}$
- E.  $\frac{\mu}{\mu + \delta}$

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24.

For a whole life insurance policy with benefit 1 on a life aged (55), you are given:

- $i = 0.05$
- $p_{60} = p_{61} = p_{62}$
- ${}_5V_{55} = {}_8V_{55}$
- $\ddot{a}_{60} = 14$

Calculate  $p_{60}$

- A. 0.945
- B. 0.954
- C. 0.975
- D. 0.980
- E. 0.984

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25.

A professional athlete is considering two different two-year contracts:

- Contract A guarantees \$1 million to be paid to the athlete both at the end of year 1 and at the end of year 2, regardless of whether the athlete is healthy or injured.
- Contract B pays \$600,000 at time = 0, \$700,000 at the end of year 1, and \$1 million at the end of year 2, but it only pays each of these amounts if the athlete is healthy at the time of the pay-out.

You are given the following assumptions:

- At time = 0, the athlete is healthy.
- During each year, there is a 15% chance that the athlete becomes injured and once the athlete becomes injured, he remains injured forever.
- $i = 5\%$

Calculate the absolute value of the difference between the actuarial present values of the contracts.

- A. Less than \$30,000
- B. At least \$30,000, but less than \$35,000
- C. At least \$35,000, but less than \$40,000
- D. At least \$40,000, but less than \$45,000
- E. At least \$45,000

Exam Question      Answer

- 1 D
- 2 B
- 3 D
- 4 E
- 5 D
- 6 C
- 7 B
- 8 D
- 9 D
- 10 D
- 11 E
- 12 C
- 13 D
- 14 B
- 15 C
- 16 C
- 17 D
- 18 B
- 19 D
- 20 A
- 21 B
- 22 A
- 23 A
- 24 C
- 25 C