# Course 3

**\*\*BEGINNING OF EXAMINATION\*\***

**1.** Given: The survival function $s(x)$, where

$$s(x) = 1, \quad 0 \le x < 1$$

$$s(x) = 1 - \left\{ \left( e^x \right) / 100 \right\}, \quad 1 \le x < 45$$

$$s(x) = 0, \quad 4.5 \le x$$

Calculate $m(4)$.

(A)     0.45

(B)     0.55

(C)     0.80

(D)     1.00

(E)     1.20

**2.** For a triple decrement table, you are given:

(i) $m_x^{(1)}(t) = 0.3$, $t > 0$

(ii) $m_x^{(2)}(t) = 0.5$, $t > 0$

(iii) $m_x^{(3)}(t) = 0.7$, $t > 0$

Calculate $q_x^{(2)}$.

(A)     0.26

(B)     0.30

(C)     0.33

(D)     0.36

(E)     0.39

**3.** You are given:

(i)     the following select-and-ultimate mortality table with 3-year select period:

| $x$ | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | $q_{x+3}$ | $x+3$ |
|---|---|---|---|---|---|
| 60 | 0.09 | 0.11 | 0.13 | 0.15 | 63 |
| 61 | 0.10 | 0.12 | 0.14 | 0.16 | 64 |
| 62 | 0.11 | 0.13 | 0.15 | 0.17 | 65 |
| 63 | 0.12 | 0.14 | 0.16 | 0.18 | 66 |
| 64 | 0.13 | 0.15 | 0.17 | 0.19 | 67 |

(ii)     $i = 0.03$

Calculate $_{2|2}A_{60}^{\cdot}$ , the actuarial present value of a 2-year deferred 2-year term insurance on $[60]$.

(A)     0.156

(B)     0.160

(C)     0.186

(D)     0.190

(E)     0.195

**4.** You are given:

(i)   $m_x(t) = 0.01,$   $0 \le t < 5$

(ii)   $m_x(t) = 0.02,$   $5 \le t$

(iii)   $d = 0.06$

Calculate $\bar{a}_x$.

(A)   12.5

(B)   13.0

(C)   13.4

(D)   13.9

(E)   14.3

**5.**   Actuaries have modeled auto windshield claim frequencies.  They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter $\lambda$ , where $\lambda$  follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

   (A)   0.15

   (B)   0.19

   (C)   0.20

   (D)   0.24

   (E)   0.31

**6.**  The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750.  Individual losses have mean 1101 and standard deviation 70.  The number of claims and the amounts of individual losses are independent.

Using the normal approximation, calculate the probability that SDIC's aggregate auto vandalism losses reported for a month will be less than 100,000.

(A)     0.24

(B)     0.31

(C)     0.36

(D)     0.39

(E)     0.49

**7.** For an allosaur with 10,000 calories stored at the start of a day:

(i) The allosaur uses calories uniformly at a rate of 5,000 per day. If his stored calories reach 0, he dies.

(ii) Each day, the allosaur eats 1 scientist (10,000 calories) with probability 0.45 and no scientist with probability 0.55.

(iii) The allosaur eats only scientists.

(iv) The allosaur can store calories without limit until needed.

Calculate the probability that the allosaur ever has 15,000 or more calories stored.

(A) 0.54

(B) 0.57

(C) 0.60

(D) 0.63

(E) 0.66

**8.** The value of currency in country M is currently the same as in country N (i.e., 1 unit in country M can be exchanged for 1 unit in country N). Let $C(t)$ denote the difference between the currency values in country M and N at any point in time (i.e., 1 unit in country M will exchange for $1+C(t)$ at time $t$). $C(t)$ is modeled as a Brownian motion process with drift 0 and variance parameter 0.01.

An investor in country M currently invests 1 in a risk free investment in country N that matures at 1.5 units in the currency of country N in 5 years. After the first year, 1 unit in country M is worth 1.05 in country N.

Calculate the conditional probability after the first year that when the investment matures and the funds are exchanged back to country M, the investor will receive at least 1.5 in the currency of country M.

(A)     0.3

(B)     0.4

(C)     0.5

(D)     0.6

(E)     0.7

**9.** Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:

(i)    60% of the coins are worth 1 each

(ii)   20% of the coins are worth 5 each

(iii)  20% of the coins are worth 10 each.

Calculate the probability that in the first ten minutes of his walk he finds at least 2 coins worth 10 each, and in the first twenty minutes finds at least 3 coins worth 10 each.

(A)    0.08

(B)    0.12

(C)    0.16

(D)    0.20

(E)    0.24

**10.** You wish to simulate a value, $Y$, from a two point mixture.

With probability 0.3, $Y$ is exponentially distributed with mean 0.5. With probability 0.7, $Y$ is uniformly distributed on $[-3, 3]$. You simulate the mixing variable where low values correspond to the exponential distribution. Then you simulate the value of $Y$, where low random numbers correspond to low values of $Y$. Your uniform random numbers from $[0, 1]$ are 0.25 and 0.69 in that order.

Calculate the simulated value of $Y$.

(A)    0.19

(B)    0.38

(C)    0.59

(D)    0.77

(E)    0.95

**11.** For a fully discrete whole life insurance of 1000 on (60), the annual benefit premium was calculated using the following:

(i)    $i = 0.06$

(ii)   $q_{60} = 0.01376$

(iii)  $1000 A_{60} = 369.33$

(iv)   $1000 A_{61} = 383.00$

A particular insured is expected to experience a first-year mortality rate ten times the rate used to calculate the annual benefit premium. The expected mortality rates for all other years are the ones originally used.

Calculate the expected loss at issue for this insured, based on the original benefit premium.

(A)    72

(B)    86

(C)    100

(D)    114

(E)    128

**12.** For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table.

(iii) $\ddot{a}_{40:\overline{10}|} = 7.70$

(iv) $\ddot{a}_{50:\overline{10}|} = 7.57$

(v) $1000 A^1_{40:\overline{20}|} = 60.00$

At the end of the tenth year, the insured elects an option to retain the coverage of 1000 for life, but pay premiums for the next ten years only.

Calculate the revised annual benefit premium for the next 10 years.

(A) 11

(B) 15

(C) 17

(D) 19

(E) 21

**13.** For a double-decrement table where cause 1 is death and cause 2 is withdrawal, you are given:

(i) Deaths are uniformly distributed over each year of age in the single-decrement table.

(ii) Withdrawals occur only at the end of each year of age.

(iii) $l_x^{(t)} = 1000$

(iv) $q_x^{(2)} = 0.40$

(v) $d_x^{(1)} = 0.45\ d_x^{(2)}$

Calculate $p_x'^{(2)}$.

(A)    0.51

(B)    0.53

(C)    0.55

(D)    0.57

(E)    0.59

**14.** You intend to hire 200 employees for a new management-training program. To predict the number who will complete the program, you build a multiple decrement table. You decide that the following associated single decrement assumptions are appropriate:

(i)   Of 40 hires, the number who fail to make adequate progress in each of the first three years is 10, 6, and 8, respectively.

(ii)   Of 30 hires, the number who resign from the company in each of the first three years is 6, 8, and 2, respectively.

(iii)   Of 20 hires, the number who leave the program for other reasons in each of the first three years is 2, 2, and 4, respectively.

(iv)   You use the uniform distribution of decrements assumption in each year in the multiple decrement table.

Calculate the expected number who fail to make adequate progress in the third year.

(A)   4

(B)   8

(C)   12

(D)   14

(E)   17

**15.** Bob is an overworked underwriter. Applications arrive at his desk at a Poisson rate of 60 per day. Each application has a 1/3 chance of being a "bad" risk and a 2/3 chance of being a "good" risk.

Since Bob is overworked, each time he gets an application he flips a fair coin. If it comes up heads, he accepts the application without looking at it. If the coin comes up tails, he accepts the application if and only if it is a "good" risk. The expected profit on a "good" risk is 300 with variance 10,000. The expected profit on a "bad" risk is −100 with variance 90,000.

Calculate the variance of the profit on the applications he accepts today.

(A)   4,000,000

(B)   4,500,000

(C)   5,000,000

(D)   5,500,000

(E)   6,000,000

**16.** Prescription drug losses, *S*, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

Calculate $E\left[(S-100)_+\right]$.

(A)    60

(B)    82

(C)    92

(D)    114

(E)    146

**17.** For a temporary life annuity-immediate on independent lives (30) and (40):

(i)    Mortality follows the Illustrative Life Table.

(ii)    $i = 0.06$

Calculate $a_{30:40:\overline{10}|}$.

(A)    6.64

(B)    7.17

(C)    7.88

(D)    8.74

(E)    9.86

**18.** For a special whole life insurance on (35), you are given:

(i)  The annual benefit premium is payable at the beginning of each year.

(ii)  The death benefit is equal to 1000 plus the return of all benefit premiums paid in the past without interest.

(iii)  The death benefit is paid at the end of the year of death.

(iv)  $A_{35} = 0.42898$

(v)  $(IA)_{35} = 6.16761$

(vi)  $i = 0.05$

Calculate the annual benefit premium for this insurance.

(A)  73.66

(B)  75.28

(C)  77.42

(D)  78.95

(E)  81.66

**19.** For a fully discrete whole life insurance of 1000 on Glenn:

(i)     Glenn is now age 80.  The insurance was issued 30 years ago, at a contract premium of 20 per year.

(ii)    Mortality follows the Illustrative Life Table.

(iii)   $i = 0.06$

(iv)    You are simulating $_{30}L$, the prospective loss random variable at time 30 for this insurance based on the contract premium.  You are using the inverse transform method, where low random numbers correspond to early deaths (soon after age 80).

(v)     Your first random number from the uniform distribution on [0, 1] is 0.42.

Calculate your first simulated value of $_{30}L$.

(A)     532

(B)     555

(C)     578

(D)     601

(E)     624

**20.** Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The types of each train are independent. An express gets you to work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.

Which of the following is true?

(A)    Your expected arrival time is 6 minutes earlier than your co-worker's.

(B)    Your expected arrival time is 4.5 minutes earlier than your co-worker's.

(C)    Your expected arrival times are the same.

(D)    Your expected arrival time is 4.5 minutes later than your co-worker's.

(E)    Your expected arrival time is 6 minutes later than your co-worker's.

**21.** An insurance company is established on January 1.

(i) The initial surplus is 2.

(ii) At the 5$^{th}$ of every month a premium of 1 is collected.

(iii) At the middle of every month the company pays a random claim amount with distribution as follows

| $x$ | $p(x)$ |
|-----|--------|
| 1 | 0.90 |
| 2 | 0.09 |
| 3 | 0.01 |

(iv) The company goes out of business if its surplus is 0 or less at any time.

(v) $i = 0$

Calculate the largest number $m$ such that the probability that the company will be out of business at the end of $m$ complete months is less than 5%.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

**22.** A stream fills a lake at a constant rate of 500 liters per day.  Deer arrive at the lake at a Poisson rate of 250 per day.  The amount of water each deer drinks per arrival is uniformly distributed between 1 and 2 liters.

Calculate the probability that the lake level is ever lower than it is right now.

(A)     0.00

(B)     0.25

(C)     0.50

(D)     0.75

(E)     1.00

**23.** Your company insures a risk that is modeled as a surplus process as follows:

(i) Interarrival times for claims are independent and exponentially distributed with mean 1/3.

(ii) Claim size equals $10^t$, where $t$ equals the time the claim occurs.

(iii) Initial surplus equals 5.

(iv) Premium is collected continuously at rate $ct^4$.

(v) $i = 0$

You simulate the interarrival times for the first three claims by using 0.5, 0.2, and 0.1, respectively, from the uniform distribution on $[0, 1]$, where small random numbers correspond to long interarrival times.

Of the following, which is the smallest $c$ such that your company does not become insolvent from any of these three claims?

(A) 22

(B) 35

(C) 49

(D) 113

(E) 141

**24.** For a special fully continuous whole life insurance of 1 on the last-survivor of $(x)$ and $(y)$, you are given:

(i) $T(x)$ and $T(y)$ are independent.

(ii) $m_x(t) = m_y(t) = 0.07, \quad t > 0$

(iii) $d = 0.05$

(iv) Premiums are payable until the first death.

Calculate the level annual benefit premium for this insurance.

(A) 0.04

(B) 0.07

(C) 0.08

(D) 0.10

(E) 0.14

**25.** For a fully discrete whole life insurance of 1000 on (20), you are given:

(i) $1000\,P_{20} = 10$

(ii) $1000\,{}_{20}V_{20} = 490$

(iii) $1000\,{}_{21}V_{20} = 545$

(iv) $1000\,{}_{22}V_{20} = 605$

(v) $q_{40} = 0.022$

Calculate $q_{41}$.

(A) 0.024

(B) 0.025

(C) 0.026

(D) 0.027

(E) 0.028

**26.** For a fully discrete whole life insurance of 1000 on (60), you are given:

(i)     $i = 0.06$

(ii)    Mortality follows the Illustrative Life Table, except that there are extra mortality risks at age 60 such that $q_{60} = 0.015$.

Calculate the annual benefit premium for this insurance.

(A)     31.5

(B)     32.0

(C)     32.1

(D)     33.1

(E)     33.2

**27.** At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome $N$. The player then rolls $N$ dice and wins an amount equal to the total of the numbers showing on the $N$ dice. All dice have 6 sides and are fair.

Using the normal approximation, calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds.

(A)    0.01

(B)    0.04

(C)    0.06

(D)    0.09

(E)    0.12

**28.** $X$ is a discrete random variable with a probability function which is a member of the $(a,b,0)$ class of distributions.

You are given:

(i)   $P(X = 0) = P(X = 1) = 0.25$

(ii)   $P(X = 2) = 0.1875$


Calculate $P(X = 3)$.


(A)   0.120

(B)   0.125

(C)   0.130

(D)   0.135

(E)   0.140

**29.** Homerecker Insurance Company classifies its insureds based on each insured's credit rating, as one of Preferred, Standard or Poor. Individual transition between classes is modeled as a discrete Markov process with a transition matrix as follows:

|          | Preferred | Standard | Poor |
|----------|-----------|----------|------|
| Preferred | 0.95 | 0.04 | 0.01 |
| Standard | 0.15 | 0.80 | 0.05 |
| Poor | 0.00 | 0.25 | 0.75 |

Calculate the percentage of insureds in the Preferred class in the long run.

(A)  33%

(B)  50%

(C)  69%

(D)  75%

(E)  92%

**30.** Nancy reviews the interest rates each year for a 30-year fixed mortgage issued on July 1. She models interest rate behavior by a Markov model assuming:

(i)     Interest rates always change between years.

(ii)    The change in any given year is dependent on the change in prior years as follows:

| from year $t-3$ to year $t-2$ | from year $t-2$ to year $t-1$ | Probability that year $t$ will increase from year $t-1$ |
|:---:|:---:|:---:|
| Increase | Increase | 0.10 |
| Decrease | Decrease | 0.20 |
| Increase | Decrease | 0.40 |
| Decrease | Increase | 0.25 |

She notes that interest rates decreased from year 2000 to 2001 and from year 2001 to 2002.

Calculate the probability that interest rates will decrease from year 2003 to 2004.

(A)     0.76

(B)     0.79

(C)     0.82

(D)     0.84

(E)     0.87

**31.** For a 20-year deferred whole life annuity-due of 1 per year on (45), you are given:

(i)    Mortality follows De Moivre's law with $w = 105$.

(ii)    $i = 0$

Calculate the probability that the sum of the annuity payments actually made will exceed the actuarial present value at issue of the annuity.

(A)    0.425

(B)    0.450

(C)    0.475

(D)    0.500

(E)    0.525

**32.** For a continuously increasing whole life insurance on $(x)$, you are given:

(i) The force of mortality is constant.

(ii) $d = 0.06$

(iii) $^2\overline{A}_x = 0.25$

Calculate $\left(\overline{IA}\right)_x$.

(A) 2.889

(B) 3.125

(C) 4.000

(D) 4.667

(E) 5.500

**33.** XYZ Co. has just purchased two new tools with independent future lifetimes. Each tool has its own distinct De Moivre survival pattern. One tool has a 10-year maximum lifetime and the other a 7-year maximum lifetime.

Calculate the expected time until both tools have failed.

(A)   5.0

(B)   5.2

(C)   5.4

(D)   5.6

(E)   5.8

**34.** XYZ Paper Mill purchases a 5-year special insurance paying a benefit in the event its machine breaks down. If the cause is "minor" (1), only a repair is needed. If the cause is "major" (2), the machine must be replaced.

Given:

(i)     The benefit for cause (1) is 2000 payable at the moment of breakdown.

(ii)    The benefit for cause (2) is 500,000 payable at the moment of breakdown.

(iii)   Once a benefit is paid, the insurance contract is terminated.

(iv)    $\mu^{(1)}(t) = 0.100$ and $\mu^{(2)}(t) = 0.004$, for $t > 0$

(v)     $\delta = 0.04$

Calculate the actuarial present value of this insurance.

(A)     7840

(B)     7880

(C)     7920

(D)     7960

(E)     8000

**35.** You are given:

(i) $R = 1 - e^{-\int_0^1 m_x(t)\,dt}$

(ii) $S = 1 - e^{-\int_0^1 (m_x(t)+k)\,dt}$

(iii) $k$ is a constant such that $S = 0.75R$

Determine an expression for $k$.

(A) $\ln\big((1-q_x)/(1-0.75q_x)\big)$

(B) $\ln\big((1-0.75q_x)/(1-p_x)\big)$

(C) $\ln\big((1-0.75p_x)/(1-p_x)\big)$

(D) $\ln\big((1-p_x)/(1-0.75q_x)\big)$

(E) $\ln\big((1-0.75q_x)/(1-q_x)\big)$

**36.** The number of claims in a period has a geometric distribution with mean 4. The amount of each claim $X$ follows $P(X = x) = 0.25$, $x = 1,2,3,4$. The number of claims and the claim amounts are independent. $S$ is the aggregate claim amount in the period.

Calculate $F_s(3)$.

(A)   0.27

(B)   0.29

(C)   0.31

(D)   0.33

(E)   0.35

**37.** Insurance agent Hunt N. Quotum will receive no annual bonus if the ratio of incurred losses to earned premiums for his book of business is 60% or more for the year.  If the ratio is less than 60%, Hunt's bonus will be a percentage of his earned premium equal to 15% of the difference between his ratio and 60%.  Hunt's annual earned premium is 800,000.

Incurred losses are distributed according to the Pareto distribution, with $q = 500{,}000$ and $a = 2$.

Calculate the expected value of Hunt's bonus.

(A)    13,000

(B)    17,000

(C)    24,000

(D)    29,000

(E)    35,000

**38.** A large machine in the ABC Paper Mill is 25 years old when ABC purchases a 5-year term insurance paying a benefit in the event the machine breaks down.

Given:

(i) Annual benefit premiums of 6643 are payable at the beginning of the year.

(ii) A benefit of 500,000 is payable at the moment of breakdown.

(iii) Once a benefit is paid, the insurance contract is terminated.

(iv) Machine breakdowns follow De Moivre's law with $l_x = 100 - x$.

(v) $i = 0.06$

Calculate the benefit reserve for this insurance at the end of the third year.

(A)  –91

(B)   0

(C)  163

(D)  287

(E)  422

**39.** For a whole life insurance of 1 on $(x)$, you are given:

(i)      The force of mortality is $\boldsymbol{m}_x(t)$.

(ii)     The benefits are payable at the moment of death.

(iii)    $\boldsymbol{d} = 0.06$

(iv)    $\overline{A}_x = 0.60$

Calculate the revised actuarial present value of this insurance assuming $\boldsymbol{m}_x(t)$ is increased by 0.03 for all $t$ and $\boldsymbol{d}$ is decreased by 0.03.

(A)    0.5

(B)    0.6

(C)    0.7

(D)    0.8

(E)    0.9

**40.** A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs. The light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.

You are given:

(i) For new light bulbs, $q_0 = 0.10$
$q_1 = 0.30$
$q_2 = 0.50$

(ii) Each light bulb costs 1.

(iii) $i = 0.05$

Calculate the actuarial present value of this contract.

(A) 6700

(B) 7000

(C) 7300

(D) 7600

(E) 8000

**\*\*END OF EXAMINATION\*\***

# Course 3
# FALL 2002

# PRELIMINARY ANSWER KEY

| Question # | Answer |  | Question # | Answer |
|---|---|---|---|---|
| 1 | E |  | 21 | B |
| 2 | A |  | 22 | D |
| 3 | D |  | 23 | C |
| 4 | B |  | 24 | C |
| 5 | E |  | 25 | E |
| 6 | A |  | 26 | E |
| 7 | C |  | 27 | E |
| 8 | B |  | 28 | B |
| 9 | D |  | 29 | C |
| 10 | C |  | 30 | B |
| 11 | D |  | 31 | B |
| 12 | D |  | 32 | C |
| 13 | A |  | 33 | E |
| 14 | D |  | 34 | A |
| 15 | C |  | 35 | A |
| 16 | C |  | 36 | E |
| 17 | B |  | 37 | E |
| 18 | A |  | 38 | D |
| 19 | D |  | 39 | D |
| 20 | C |  | 40 | A |

**Question #1**
**Answer: E**

$$\mu(4) = -s'(4)/s(4)$$

$$= \frac{-\left(-e^4/100\right)}{1-e^4/100}$$

$$= \frac{e^4/100}{1-e^4/100}$$

$$= \frac{e^4}{100-e^4}$$

$$= 1.202553$$

**Question # 2**
**Answer: A**

$$q_x^{(i)} = q_x^{(t)} \left[ \frac{\ln p_x'^{(i)}}{\ln p_x^{(t)}} \right] = q_x^{(t)} \left[ \frac{\ln e^{-m^{(i)}}}{\ln e^{-m^{(t)}}} \right]$$

$$= q_x^{(t)} \times \frac{m^{(i)}}{m^{(t)}}$$

$$m_x^{(t)} = m_x^{(1)} + m_x^{(2)} + m_x^{(3)} = 15$$

$$q_x^{(t)} = 1 - e^{-m(t)} = 1 - e^{-1.5}$$

$$= 0.7769$$

$$q_x^{(2)} = \frac{(0.7769) m^{(2)}}{m^{(t)}} = \frac{(0.5)(0.7769)}{15}$$

$$= 0.2590$$

**Question # 3**
**Answer: D**

$$_{2|2}A_{[60]} = v^3 \quad \times \quad _2p_{[60]} \quad \times \quad q_{[60]+2} \quad +$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

pay at end      live      then die

of year 3     2 years   in year 3

$$+v^4 \quad \times \quad _3p_{[60]} \quad \times \quad q_{60+3}$$

pay at end      live      then die

of year 4     3 years   in year 4

$$= \frac{1}{(1.03)^3}(1-0.09)(1-0.11)(0.13) + \frac{1}{(1.03)^4}(1-0.09)(1-0.11)(1-0.13)(0.15)$$

$$= 0.19$$

**Question # 4**
**Answer: B**

$$\bar{a}_x = \bar{a}_{x:\overline{5}|} + {}_5E_x\,\bar{a}_{x+5}$$

$$\bar{a}_{x:\overline{5}|} = \frac{1-e^{-0.07(5)}}{0.07} = 4.219 \text{, where } 0.07 = m+d \text{ for } t<5$$

$${}_5E_x = e^{-0.07(5)} = 0.705$$

$$\bar{a}_{x+5} \quad = \quad \frac{1}{0.08} \quad = \quad 12.5 \text{, where } 0.08 = m+d \text{ for } t \geq 5$$

$$\therefore \bar{a}_x = 4.219 + (0.705)(12.5) = 13.03$$

**Question # 5**
**Answer: E**

The distribution of claims (a gamma mixture of Poissons) is negative binomial.

$$E(N) = E_\wedge\big(E(N|\Lambda)\big) = E_\wedge(\Lambda) = 3$$
$$Var(N) = E_\wedge\big(Var(N|\Lambda)\big) + Var_\wedge\big(E(N|\Lambda)\big)$$
$$= E_\wedge(\Lambda) + Var_\wedge(\Lambda) = 6$$
$$r\boldsymbol{b} = 3$$
$$r\boldsymbol{b}(1+\boldsymbol{b}) = 6$$
$$(1+\boldsymbol{b}) = 6/3 = 2\;;\;\;\boldsymbol{b} = 1$$
$$r\boldsymbol{b} = 3$$
$$r = 3$$

$$p_0 = (1+\boldsymbol{b})^{-r} = 0.125$$
$$p_1 = \frac{r\boldsymbol{b}}{(1+\boldsymbol{b})^{r+1}} = 0.1875$$
$$\text{Prob(at most 1)} = p_0 + p_1$$
$$= 0.3125$$

**Question # 6**
**Answer: A**

$$E(S) = E(N) \times E(X) = 110 \times 1{,}101 = 121{,}110$$

$$Var(S) = E(N) \times Var(X) + E(X)^2 \times Var(N)$$
$$= 110 \times 70^2 + 1101^2 \times 750$$
$$= 909{,}689{,}750$$

$$\text{Std Dev }(S) = 30{,}161$$

$$\Pr(S < 100{,}000) = \Pr\big(Z < (100{,}000 - 121{,}110)/30{,}161\big) \text{ where } Z \text{ has standard normal distribution}$$
$$= \Pr(Z < -0.70) = 0.242$$

**Question # 7**
**Answer: C**


This is just the Gambler's Ruin problem, in units of 5,000 calories.
Each day, up one with $p = 0.45$; down 1 with $q = 0.55$
Will Allosaur ever be up 1 before being down 2?

$$P_2 = \frac{\left(1 - (0.55/0.45)^{\wedge}2\right)}{\left(1 - (0.55/0.45)^{\wedge}3\right)} = 0.598$$

Or, by general principles instead of applying a memorized formula:
Let $P_1$ = probability of ever reaching 3 (15,000 calories) if at 1 (5,000 calories).
Let $P_2$ = probability of ever reaching 3 (15,000 calories) if at 2 (10,000 calories).

From either, we go up with $p = 0.45$, down with $q = 0.55$

P(reaching 3) = P(up) $\times$ P(reaching 3 after up) + P(down) $\times$ P(reaching 3 after down)

$P_2 = 0.45 \times 1 + 0.55 \times P_1$
$P_1 = 0.45 \times P_2 + 0.55 \times 0 = 0.45 \times P_2$
$P_2 = 0.45 + 0.55 \times P_1 = 0.45 + 0.55 \times 0.45 \times P_2 = 0.45 + 0.2475 P_2$
$P_2 = 0.45 / (1 - 0.2475) = 0.598$

Here is another approach, feasible since the number of states is small.

Let states 0,1,2,3 correspond to 0; 5,000; 10,000; ever reached 15,000 calories. For purposes of this problem, state 3 is absorbing, since once the allosaur reaches 15,000 we don't care what happens thereafter.

The transition matrix is
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.55 & 0 & 0.45 & 0 \\ 0 & 0.55 & 0 & 0.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Starting with the allosaur in state 2;

| [0 | 0 | 1 | 0] | at inception |
|---|---|---|---|---|
| [0 | 0.55 | 0 | 0.45] | after 1 |
| [0.3025 | 0 | 0.2475 | 0.45] | after 2 |
| [0.3025 | 0.1361 | 0 | 0.5614] | after 3 |
| [0.3774 | 0 | 0.0612 | 0.5614] | after 4 |
| [0.3774 | 0.0337 | 0 | 0.5889] | after 5 |
| [0.3959 | 0 | 0.0152 | 0.5889] | after 6 |

By this step, if not before, Prob(state 3) must be converging to 0.60. It's already closer to 0.60 than 0.57, and its maximum is 0.5889 + 0.0152

**Question # 8**
**Answer: B**


The investor will receive at least 1.5 if and only if $C(5) \leq 0$.

$C(5) - C(1)$ is normally distributed with mean $(4)(0) = 0$,
variance $= (4)(0.01) = 0.04$, standard deviation $= 0.2$.

$\Pr\left(C(5) \leq 0 \big| C(1) = 0.05\right) = \Pr\left(C(5) - C(1) \leq -0.05\right) =$
$= \Pr\left(\left(C(5) - C(1)\right) / 0.2 \leq -0.05 / 0.2\right)$
$= 1 - \Phi(0.25) = 0.4$

**Question # 9**
**Answer: D**


Per 10 minutes, find coins worth exactly 10 at Poisson rate $(0.5)(0.2)(10) = 1$

Per 10 minutes,

| | |
|---|---|
| $f(0) = 0.3679$ | $F(0) = 0.3679$ |
| $f(1) = 0.3679$ | $F(1) = 0.7358$ |
| $f(2) = 0.1839$ | $F(2) = 0.9197$ |
| $f(3) = 0.0613$ | $F(3) = 0.9810$ |

Let Period 1 = first 10 minutes; period 2 = next 10.

Method 1, succeed with 3 or more in period 1; or exactly 2, then one or more in period 2

$$P = (1 - F(2)) + f(2)(1 - F(0)) = (1 - 0.9197) + (0.1839)(1 - 0.3679)$$
$$= 0.1965$$

Method 2: fail in period 1 if $< 2$; $\qquad\qquad$ $\text{Prob} = F(1) = 0.7358$

fail in period 2 if exactly 2 in period 1, then 0; $\qquad$ $\text{Prob} = f(2)f(0)$
$$= (0.1839)(0.3679) = 0.0677$$

Succeed if fail neither period; $\qquad\qquad$ $\text{Prob} = 1 - 0.7358 - 0.0677$
$$= 0.1965$$


(Method 1 is attacking the problem as a stochastic process model; method 2 attacks it as a ruin model.)

**Question # 10**
**Answer: C**


Which distribution is it from?

$0.25 < 0.30$, so it is from the exponential.

Given that $Y$ is from the exponential, we want
$$\Pr(Y \le y) = F(y) = 0.69$$
$$1 - e^{-y/q} = 0.69$$
$$1 - e^{-y/0.5} = 0.69 \text{ since mean} = 0.5$$
$$\frac{-y}{0.5} = \ln(1 - 0.69) = -1.171$$
$$y = 0.5855$$


**Question # 11**
**Answer: D**


Use Mod to designate values unique to this insured.

$$\ddot{a}_{60} = (1 - A_{60})/d = (1 - 0.36933)/\left[(0.06)/(1.06)\right] = 11.1418$$

$$1000 P_{60} = 1000 A_{60} / \ddot{a}_{60} = 1000(0.36933/11.1418) = 33.15$$

$$A_{60}^{Mod} = v\left(q_{60}^{Mod} + p_{60}^{Mod} A_{61}\right) = \frac{1}{1.06}\left[0.1376 + (0.8624)(0.383)\right] = 0.44141$$

$$\ddot{a}^{Mod} = \left(1 - A_{60}^{Mod}\right)/d = (1 - 0.44141)/\left[0.06/1.06\right] = 9.8684$$

$$E\left[{}_0 L^{Mod}\right] = 1000\left(A_{60}^{Mod} - P_{60}\ddot{a}_{60}^{Mod}\right)$$

$$= 1000\left[0.44141 - 0.03315(9.8684)\right]$$

$$= 114.27$$

**Question # 12**
**Answer: D**

The prospective reserve at age 60 per 1 of insurance is $A_{60}$, since there will be no future premiums. Equating that to the retrospective reserve per 1 of coverage, we have:

$$A_{60} = P_{40} \frac{\ddot{s}_{40:\overline{10}|}}{{}_{10}E_{50}} + P_{50}^{Mod} \, \ddot{s}_{50:\overline{10}|} - {}_{20}k_{40}$$

$$A_{60} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40} \, {}_{10}E_{50}} + P_{50}^{Mod} \frac{\ddot{a}_{50:\overline{10}|}}{{}_{10}E_{50}} - \frac{A^1_{40:\overline{20}|}}{{}_{20}E_{40}}$$

$$0.36913 = \frac{0.16132}{14.8166} \times \frac{7.70}{(0.53667)(0.51081)} + P_{50}^{Mod} \frac{7.57}{051081} - \frac{0.06}{0.27414}$$

$$0.36913 = 0.30582 + 14.8196 \, P_{50}^{Mod} - 0.21887$$

$$1000 \, P_{50}^{Mod} = 19.04$$

Alternatively, you could equate the retrospective and prospective reserves at age 50. Your equation would be:

$$A_{50} - P_{50}^{Mod} \, \ddot{a}_{50:\overline{10}|} = \frac{A_{40}}{\ddot{a}_{40}} \cdot \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}} - \frac{A^1_{40:\overline{10}|}}{{}_{10}E_{40}}$$

where $A^1_{40:\overline{10}|} = A_{40} - {}_{10}E_{40} \, A_{50}$

$$= 0.16132 - (053667)(0.24905)$$
$$= 0.02766$$

$$0.24905 - (P_{50}^{Mod})(7.57) = \frac{0.16132}{14.8166} \times \frac{7.70}{053667} - \frac{0.02766}{053667}$$

$$1000 P_{50}^{Mod} = \frac{(1000)(0.14437)}{7.57} = 19.07$$

Alternatively, you could set the actuarial present value of benefits at age 40 to the actuarial present value of benefit premiums. The change at age 50 did not change the benefits, only the pattern of paying for them.

$$A_{40} = P_{40} \, \ddot{a}_{40:\overline{10}|} + P_{50}^{Mod} \, {}_{10}E_{40} \, \ddot{a}_{50:\overline{10}|}$$

$$0.16132 = \left( \frac{0.16132}{14.8166} \right)(7.70) + (P_{50}^{Mod})(0.53667)(7.57)$$

$$1000 P_{50}^{Mod} = \frac{(1000)(0.07748)}{4.0626} = 19.07$$

**Question # 13**
**Answer: A**


$$d_x^{(2)} = q_x^{(2)} \times l_x^{(t)} = 400$$

$$d_x^{(1)} = 0.45(400) = 180$$

$$q_x'^{(2)} = \frac{d_x^{(2)}}{l_x^{(t)} - d_x^{(1)}} = \frac{400}{1000 - 180} = 0.488$$

$$p_x'^{(2)} = 1 - 0.488 = 0.512$$

Note: The UDD assumption was not critical except to have all deaths during the year so that 1000 - 180 lives are subject to decrement 2.

**Question #14**
**Answer: D**


Use "age" subscripts for years completed in program. E.g., $p_0$ applies to a person newly hired ("age" 0).

Let decrement 1 = fail, 2 = resign, 3 = other.

Then $q_0'^{(1)} = \frac{1}{4}$, $q_1'^{(1)} = \frac{1}{5}$, $q_2'^{(1)} = \frac{1}{3}$

$\quad\quad q_0'^{(2)} = \frac{1}{5}$, $q_1'^{(2)} = \frac{1}{3}$, $q_2'^{(2)} = \frac{1}{8}$

$\quad\quad q_0'^{(3)} = \frac{1}{10}$, $q_1'^{(3)} = \frac{1}{9}$, $q_2'^{(3)} = \frac{1}{4}$

This gives $p_0^{(t)} = (1-1/4)(1-1/5)(1-1/10) = 0.54$

$\quad\quad p_1^{(t)} = (1-1/5)(1-1/3)(1-1/9) = 0.474$

$\quad\quad p_2^{(t)} = (1-1/3)(1-1/8)(1-1/4) = 0.438$

So $1_0^{(t)} = 200$, $1_1^{(t)} = 200(0.54) = 108$, and $1_2^{(t)} = 108(0.474) = 51.2$

$q_2^{(1)} = \left[\log p_2'^{(1)} / \log p_2^{(t)}\right] q_2^{(t)}$

$q_2^{(1)} = \left[\log\left(\frac{2}{3}\right) / \log(0.438)\right][1-0.438]$

$\quad\quad = (0.405 / 0.826)(0.562)$

$\quad\quad = 0.276$

$d_2^{(1)} = 1_2^{(t)} q_2^{(1)}$

$\quad\quad = (51.2)(0.276) = 14$

**Question #15**
**Answer: C**


Let:    $N$ = number
        $X$ = profit
        $S$ = aggregate profit
        subscripts  $G$ = "good",  $B$ = "bad",  $AB$ = "accepted bad"

$$l_G = \left(\tfrac{2}{3}\right)(60) = 40$$

$l_{AB} = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right)(60) = 10$  (If you have trouble accepting this, think instead of a heads-tails rule, that the application is accepted if the applicant's government-issued identification number, e.g. U.S. Social Security Number, is odd. It is <u>not</u> the same as saying he automatically alternates accepting and rejecting.)

$$Var(S_G) = E(N_G) \times Var(X_G) + Var(N_G) \times E(X_G)^2$$
$$= (40)(10{,}000) + (40)(300^2) = 4{,}000{,}000$$

$$Var(S_{AB}) = E(N_{AB}) \times Var(X_{AB}) + Var(N_{AB}) \times E(X_{AB})^2$$
$$= (10)(90{,}000) + (10)(-100)^2 = 1{,}000{,}000$$

$S_G$ and $S_{AB}$ are independent, so
$$Var(S) = Var(S_G) + Var(S_{AB}) = 4{,}000{,}000 + 1{,}000{,}000$$
$$= 5{,}000{,}000$$


If you don't treat it as three streams ("goods", "accepted bads", "rejected bads"), you can compute the mean and variance of the profit <u>per "bad" received</u>.
$$l_B = \left(\tfrac{1}{3}\right)(60) = 20$$

If all "bads" were accepted, we would have  $E(X^2{}_B) = Var(X_B) + E(X_B)^2$

$$= 90{,}000 + (-100)^2 = 100{,}000$$


Since the probability a "bad" will be accepted is only 50%,
$$E(X_B) = \text{Prob(accepted)} \cdot E(X_B|\text{accepted}) + \text{Prob(not accepted)} \cdot E(X_B|\text{not accepted})$$
$$= (0.5)(-100) + (0.5)(0) = -50$$
$$E(X_B^2) = (0.5)(100{,}000) + (0.5)(0) = 50{,}000$$


Likewise,
Now    $Var(S_B) = E(N_B) \times Var(X_B) + Var(N_B) \times E(X_B)^2$
$$= (20)(47{,}500) + (20)(50^2) = 1{,}000{,}000$$


$S_G$ and $S_B$ are independent, so
$$Var(S) = Var(S_G) + Var(S_B) = 4{,}000{,}000 + 1{,}000{,}000$$
$$= 5{,}000{,}000$$

**Question # 16**
**Answer: C**

Let $N$ = number of prescriptions then $S = N \times 40$

| $n$ | $f_N(n)$ | $F_N(n)$ | $1 - F_N(n)$ |
|---|---|---|---|
| 0 | 0.2000 | 0.2000 | 0.8000 |
| 1 | 0.1600 | 0.3600 | 0.6400 |
| 2 | 0.1280 | 0.4880 | 0.5120 |
| 3 | 0.1024 | 0.5904 | 0.4096 |

$$E(N) = 4 = \sum_{j=0}^{\infty} (1 - F(j))$$

$$E\left[(S-80)_+\right] = 40 \times E\left[(N-2)_+\right] = 40 \times \sum_{j=2}^{\infty}(1 - F(j))$$

$$= 40 \times \left[\sum_{j=0}^{\infty}(1 - F(j)) - \sum_{j=0}^{1}(1 - F(j))\right]$$

$$= 40(4 - 1.44) = 40 \times 2.56 = 102.40$$

$$E\left[(S-120)_+\right] = 40 \times E\left[(N-3)_+\right] = 40 \times \sum_{j=3}^{\infty}(1 - F(j))$$

$$= 40 \times \left[\sum_{j=0}^{\infty}(1 - F(j)) - \sum_{j=0}^{2}(1 - F(j))\right]$$

$$= 40(4 - 1.952) = 40 \times 2.048 = 81.92$$

Since no values of S between 80 and 120 are possible,

$$E\left[(S-100)_+\right] = \frac{(120-100) \times E\left[(S-80)_+\right] + (100-80) \times E\left[(S-120)_+\right]}{120} = 92.16$$

Alternatively,

$$E\left[(S-100)_+\right] = \sum_{j=0}^{\infty}(40j - 100)f_N(j) + 100 f_N(0) + 60 f_N(1) + 20 f_N(2)$$

(The correction terms are needed because $(40j - 100)$ would be negative for j = 0, 1, 2; we need to add back the amount those terms would be negative)

$$= 40\sum_{j=0}^{\infty} j \cdot f_N(j) - 100\sum_{j=0}^{\infty} f_N(j) + (100)(0.200) + (0.16)(60) + (0.128)(20)$$

$$= 40 E(N) - 100 + 20 + 9.6 + 2.56$$

$$= 160 - 67.84 = 92.16$$

**Question #17**
**Answer: B**

$$_{10}E_{30:40} = {}_{10}p_{30}\ {}_{10}p_{40}\ v^{10} = \left({}_{10}p_{30}v^{10}\right)\left({}_{10}p_{40}v^{10}\right)(1+i)^{10}$$
$$= \left({}_{10}E_{30}\right)\left({}_{10}E_{40}\right)(1+i)^{10}$$
$$= (0.54733)(0.53667)(1.79085)$$
$$= 0.52604$$

The above is only one of many possible ways to evaluate $_{10}p_{30}\ {}_{10}p_{40}\ v^{10}$, all of which should give 0.52604

$$a_{30:40:\overline{10}|} = a_{30:40} - {}_{10}E_{30:40}\ a_{30+10:40+10}$$
$$= \left(\ddot{a}_{30:40} - 1\right) - (0.52604)\left(\ddot{a}_{40:50} - 1\right)$$
$$= (13.2068) - (0.52604)(11.4784)$$
$$= 7.1687$$

**Question #18**
**Answer: A**

Equivalence Principle, where $p$ is annual benefit premium, gives

$$1000\left(A_{35} + (IA)_{35} \times p\right) = \ddot{a}_x p$$

$$p = \frac{1000A_{35}}{\left(\ddot{a}_{35} - (IA)_{35}\right)} = \frac{1000 \cdot 0.42898}{(11.99143 - 6.16761)}$$
$$= \frac{428.98}{5.82382}$$
$$= 73.66$$

We obtained $\ddot{a}_{35}$ from

$$\ddot{a}_{35} = \frac{1 - A_{35}}{d} = \frac{1 - 0.42898}{0.047619} = 11.99143$$

**Question #19**
**Answer: D**

Low random $\Rightarrow$ early deaths, so we want $_t q_{80} = 0.42$ or $l_{80+t} = 0.58 l_{80}$

Using the Illustrative Life Table, $l_{80+t} = (0.58)3{,}914{,}365$
$$= 2{,}270{,}332$$

$l_{80+5} > l_{80+t} > l_{80+6}$

so K = curtate future lifetime = 5

$_{30}L = 1000v^{k+1} - \text{(Contract premium)}\ \ddot{a}_{\overline{k+1}|}$

$\quad = 705 - (20)(5.2124)$
$\quad = 601$

**Question #20**
**Answer: C**

Time until arrival = waiting time plus travel time.

Waiting time is exponentially distributed with mean $\dfrac{1}{1}$. The time you may already have been waiting is irrelevant: exponential is memoryless.

You:  E (wait) = $\frac{1}{20}$ hour = 3 minutes
       E (travel) = $(0.25)(16) + (0.75)(28) = 25$ minutes
       E (total) = 28 minutes

Co-worker:   E (wait) = $\frac{1}{5}$ hour = 12 minutes
              E (travel) = 16 minutes
              E (total) = 28 minutes

**Question #21**
**Answer: B**

Bankrupt has 3 states 0, 1, 2, corresponding to surplus = 0, 1, 2

Transition matrix is

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.9 & 0 \\ 0.01 & 0.09 & 0.9 \end{bmatrix}$$

Initial vector $t_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$t_0 M = t_1 = \begin{bmatrix} 0.01 & 0.09 & 0.90 \end{bmatrix}$
$t_1 M = t_2 = \begin{bmatrix} 0.028 & 0.162 & 0.81 \end{bmatrix}$
$t_2 M = t_3 = \begin{bmatrix} 0.0523 & 0.2187 & 0.729 \end{bmatrix}$

At end of 2 months, probability ruined = 0.028<5%
At end of 3 months, probability ruined = 0.0523>5%

**Question #22**
**Answer: D**

This is equivalent to a compound Poisson surplus process. Water from stream is like premiums. Deer arriving to drink is like claims occurring. Water drunk is like claim size.

$E$[water drunk in a day] = (1.5)(250) = 375

$$(1+q) = \frac{500}{375}$$

Prob(surplus level at time $t$ is less than initial surplus, for some $t$)
$$= \Psi(0) = \frac{1}{1+q} = \frac{375}{500} = 75\%$$

**Question #23**
**Answer: C**

Time of first claim is $T_1 = -1/3\log(0.5) = 0.23$. Size of claim $= 10^{0.23} = 1.7$

Time of second claim is $T_2 = 0.23 - 1/3\log(0.2) = 0.77$. Size of claim $= 10^{0.77} = 5.9$

Time of third claim is $T_3 = 0.77 - 1/3\log(0.1) = 1.54$. Size of claim $= 10^{1.54} = 34.7$

Since initial surplus $= 5 >$ first claim, the first claim does not determine c

Test at $T_2$:    Cumulative assets $= \quad 5 + \int_0^{0.77} ct^4 dt = 5 + 0.054c$

               Cumulative claims $= \quad 1.7 + 5.9 = 7.6$
               Assets $\geq$ claims for $c \geq 48.15$

Test at $T_3$:    Cumulative claims $= \quad 1.7 + 5.9 + 34.7 = 42.3$

               Cumulative assets $= \quad 5 + \int_0^{1.54} ct^4 dt = 5 + 1.732c$
               Assets $\geq$ claims for $c \geq 21.54$

If $c < 48.15$, insolvent at $T_2$.
If $c > 48.15$, solvent throughout.
49 is smallest choice $> 48.15$.

**Question #24**
**Answer: C**

$m_{xy} = m_x + m_y = 0.14$

$\bar{A}_x = \bar{A}_y = \dfrac{m}{m+d} = \dfrac{0.07}{0.07+0.05} = 0.5833$

$\bar{A}_{xy} = \dfrac{m_{xy}}{m_{xy}+d} = \dfrac{0.14}{0.14+0.05} = \dfrac{0.14}{0.19} = 0.7368$ and $\bar{a}_{xy} = \dfrac{1}{m_{xy}+d} = \dfrac{1}{0.14+0.05} = 5.2632$

$P = \dfrac{\bar{A}_{\overline{xy}}}{\bar{a}_{xy}} = \dfrac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_{xy}} = \dfrac{2(0.5833) - 0.7368}{5.2632} = 0.0817$

**Question #25**
**Answer: E**

$$\left(_{20}V_{20} + P_{20}\right)(1+i) - q_{40}\left(1 - _{21}V_{20}\right) = _{21}V_{20}$$

$$(0.49 + 0.01)(1+i) - 0.022(1 - 0.545) = 0.545$$

$$(1+i) = \frac{(0.545)(1 - 0.022) + 0.022}{0.50}$$

$$= 1.11$$

$$\left(_{21}V_{20} + P_{20}\right)(1+i) - q_{41}\left(1 - _{22}V_{20}\right) = _{22}V_{20}$$

$$(0.545 + .01)(1.11) - q_{41}(1 - 0.605) = 0.605$$

$$q_{41} = \frac{0.61605 - 0.605}{0.395}$$

$$= 0.028$$

**Question #26**
**Answer: E**

$$1000 \, P_{60} = 1000 \, A_{60} / \ddot{a}_{60}$$

$$= 1000 \, v\left(q_{60} + p_{60} A_{61}\right) / \left(1 + p_{60} \, v \, \ddot{a}_{61}\right)$$

$$= 1000\left(q_{60} + p_{60} \, A_{61}\right) / \left(1.06 + p_{60} \, \ddot{a}_{61}\right)$$

$$= \left(15 + (0.985)(382.79)\right) / \left(1.06 + (0.985)(10.9041)\right) = 33.22$$

**Question #27**
**Answer: E**


Method 1:

In each round,
$N$ = result of first roll, to see how many dice you will roll
$X$ = result of for one of the $N$ dice you roll
$S$ = sum of $X$ for the $N$ dice

$$E(X) = E(N) = 3.5$$
$$Var(X) = Var(N) = 2.9167$$

$$E(S) = E(N)*E(X) = 12.25$$
$$Var(S) = E(N)Var(X) + Var(N)E(X)^2$$
$$= (3.5)(2.9167) + (2.9167)(3.5)^2$$
$$= 45.938$$

Let $S_{1000}$ = the sum of the winnings after 1000 rounds

$$E(S_{1000}) = 1000 * 12.25 = 12,250$$
$$Stddev(S_{1000}) = sqrt(1000 * 45.938) = 214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings
of $S_{1000}$.

Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater
than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$Pr(15000 - 12500 + S_{1000} > 14999.5) =$$
$$= Pr((S_{1000} - 12250)/214.33 > (14999.5 - 2500 - 12250)/214.33) =$$
$$= 1 - \Phi(1.17) = 0.12$$

Method 2

Realize that you are going to determine $N$ 1000 times and roll the sum of those 1000 $N$'s dice,
adding the numbers showing.

Let $N_{1000}$ = sum of those $N$'s

$$E(N_{1000}) = 1000E(N) = (1000)(3.5) = 3500$$

$$Var(N_{1000}) = 1000Var(N) = 2916.7$$

$$E(S_{1000}) = E(N_{1000})E(X) = (3500)(3.5) = 12.250$$

$$Var(S_{1000}) = E(N_{1000})Var(X) + Var(N_{1000})E(X)^2$$
$$= (3500)(2.9167) + (2916.7)(35)^2 = 45.938$$

$$Stddev(S_{1000}) = 214.33$$

Now that you have the mean and standard deviation of $S_{1000}$ (same values as method 1), use the normal approximation as shown with method 1.

**Question #28**
**Answer: B**

$$p_k = \left( a + \frac{b}{k} \right) p_{k-1}$$

$$0.25 = (a+b) \times 0.25 \Rightarrow a+b = 1$$

$$0.1875 = \left( a + \frac{b}{2} \right) \times 0.25 \Rightarrow \left( 1 - \frac{b}{2} \right) \times 0.25 = 0.1875$$

$$b = 0.5$$

$$a = 0.5$$

$$p_3 = \left( 0.5 + \frac{0.5}{3} \right) \times 0.1875 = 0.125$$

**Question #29**
**Answer: C**

Limiting probabilities satisfy (where $B$ = Bad = Poor):

$P = 0.95P + 0.15\,S$

$S = 0.04P + 0.80\,S + 0.25B$

$B = 0.01P + 0.05\,S + 0.75B$

$P + S + B = 1.00$

Solving, $P = 0.694$

**Question #30**
**Answer: B**

Transform these scenarios into a four-state Markov chain, where the final disposition of rates in any scenario is that they decrease, rather than if rates increase, as what is given.

| State | from year $t - 3$ to year $t - 2$ | from year $t - 2$ to year $t - 1$ | Probability that year $t$ will decrease from year $t$ - 1 |
|---|---|---|---|
| 0 | Decrease | Decrease | 0.8 |
| 1 | Increase | Decrease | 0.6 |
| 2 | Decrease | Increase | 0.75 |
| 3 | Increase | Increase | 0.9 |

$$\text{Transition matrix is} \begin{bmatrix} 0.80 & 0.00 & 0.20 & 0.00 \\ 0.60 & 0.00 & 0.40 & 0.00 \\ 0.00 & 0.75 & 0.00 & 0.25 \\ 0.00 & 0.90 & 0.00 & 0.10 \end{bmatrix}$$

$P_{00}^2 + P_{01}^2 = 0.8*0.8 + 0.2*0.75 \quad = \quad 0.79$

For this problem, you don't need the full transition matrix. There are two cases to consider. Case 1: decrease in 2003, then decrease in 2004; Case 2: increase in 2003, then decrease in 2004.

For Case 1: decrease in 2003 (following 2 decreases) is 0.8; decrease in 2004 (following 2 decreases is 0.8. Prob(both) = $0.8 \times 0.8 = 0.64$
For Case 2: increase in 2003 (following 2 decreases) is 0.2; decrease in 2004 (following a decrease, then increase) is 0.75. Prob(both) = $0.2 \times 0.75 = 0.15$
Combined probability of Case 1 and Case 2 is $0.64 + 0.15 = 0.79$

**Question #31**
**Answer: B**

$$l_x = w - x = 105 - x$$
$$\Rightarrow {}_tP_{45} = l_{45+t} / l_{45} = 60 - t / 60$$

Let $K$ be the curtate future lifetime of (45). Then the sum of the payments is 0 if $K \leq 19$ and is $K - 19$ if $K \geq 20$.

$$_{20|}\ddot{a}_{45} = \sum_{K=20}^{60} 1 \cdot \left(\frac{60 - K}{60}\right) \cdot 1$$

$$= \frac{(40 + 39 + \ldots + 1)}{60} = \frac{(40)(41)}{2(60)} = 13.6\overline{6}$$

Hence,

$$\text{Prob}(K - 19 > 13.6\overline{6}) = \text{Prob}(K > 32.6\overline{6})$$

$$= \text{Prob}(K \geq 33) \text{ since } K \text{ is an integer}$$

$$= \text{Prob}(T \geq 33)$$

$$= {}_{33}p_{45} = \frac{l_{78}}{l_{45}} = \frac{27}{60}$$

$$= 0.450$$

**Question #32**
**Answer: C**

$$^2\overline{A}_x = \frac{m}{m+2d} = 0.25 \rightarrow m = 0.04$$

$$\overline{A}_x = \frac{m}{m+d} = 0.4$$

$$\left(\overline{IA}\right)_x = \int_0^\infty {}_s\!\overline{A}_x \, ds$$

$$\int_0^\infty {}_s E_x \, \overline{A}_x \, ds$$

$$= \int_0^\infty \left(e^{-0.1s}\right)(0.4)\,ds$$

$$= (0.4)\left(\frac{-e^{-0.1s}}{0.1}\right)\Bigg|_0^\infty = \frac{0.4}{0.1} = 4$$

Alternatively, using a more fundamental formula but requiring more difficult integration.

$$\left(\overline{IA}\right)_x = \int_0^\infty t \;{}_t p_x\, m_x(t)\, e^{-dt}\,dt$$

$$= \int_0^\infty t\, e^{-0.04t}(0.04)e^{-0.06t}\,dt$$

$$= 0.04\int_0^\infty t\, e^{-0.1t}\,dt$$

(integration by parts, not shown)

$$= 0.04\left(\frac{-t}{0.1} - \frac{1}{0.01}\right)e^{-0.1t}\Bigg|_0^\infty$$

$$= \frac{0.04}{0.01} = 4$$

**Question #33**
**Answer: E**


Subscripts A and B here just distinguish between the tools and do not represent ages.

We have to find $\overset{o}{e}_{\overline{AB}}$

$$\overset{o}{e}_A = \int_0^{10}\left(1-\frac{t}{10}\right)dt = t - \frac{t^2}{20}\Big|_0^{10} = 10 - 5 = 5$$

$$\overset{o}{e}_B = \int_0^{7}\left(1-\frac{t}{7}\right)dt = t - \frac{t^2}{14}\Big|_0^{7} = 49 - \frac{49}{14} = 3.5$$

$$\overset{o}{e}_{AB} = \int_0^{7}\left(1-\frac{t}{7}\right)\left(1-\frac{t}{10}\right)dt = \int_0^{7}\left(1-\frac{t}{10}-\frac{t}{7}+\frac{t^2}{70}\right)dt$$

$$= t - \frac{t^2}{20} - \frac{t^2}{14} + \frac{t^3}{210}\Big|_0^{7}$$

$$= 7 - \frac{49}{20} - \frac{49}{14} + \frac{343}{210} = 2.683$$

$$\overset{o}{e}_{\overline{AB}} = \overset{o}{e}_A + \overset{o}{e}_B - \overset{o}{e}_{AB}$$

$$= 5 + 3.5 - 2.683 = 5.817$$

**Question #34**
**Answer: A**

$$m_x^{(\tau)}(t) = 0.100 + 0.004 = 0.104$$

$$_t p_x^{(\tau)} = e^{-0.104t}$$

Actuarial present value (APV) = APV for cause 1 + APV for cause 2.

$$2000 \int_0^5 e^{-0.04t} e^{-0.104t} (0.100) dt + 500{,}000 \int_0^5 e^{-0.04t} e^{-0.104t} (0.400) dt$$

$$= \left(2000(0.10) + 500{,}000(0.004)\right) \int_0^5 e^{-0.144t} dt$$

$$= \frac{2200}{0.144} \left(1 - e^{-0.144(5)}\right) = 7841$$

**Question #35**
**Answer: A**

$$R = 1 - p_x = q_x$$

$$S = 1 - p_x \times e^{(-k)} \text{ since } e^{-\int_0^1 (m_x(t) + k) dt} = e^{-\int_0^1 m_x(t) dt - \int_0^1 k \, dt}$$

$$= e^{-\int_0^1 m_x(t) dt} e^{-\int_0^1 k \, dt}$$

So $S = 0.75R \Rightarrow 1 - p_x \times e^{-k} = 0.75 q_x$

$$e^{-k} = \frac{1 - 0.75 q_x}{p_x}$$

$$e^k = \frac{p_x}{1 - 0.75 q_x} = \frac{1 - q_x}{1 - 0.75 q_x}$$

$$k = \ln\left[\frac{1 - q_x}{1 - 0.75 q_x}\right]$$

**Question #36**
**Answer: E**

$b =$ mean $= 4$;   $p_k = b^k / (1 + b)^{k+1}$

| $n$ | $P(N = n)$ |
|-----|------------|
| 0 | 0.2 |
| 1 | 0.16 |
| 2 | 0.128 |
| 3 | 0.1024 |

| $x$ | $f^{(1)}(x)$ | $f^{(2)}(x)$ | $f^{(3)}(x)$ |
|-----|--------------|--------------|--------------|
| 0 | 0 | 0 | 0 |
| 1 | 0.25 | 0 | 0 |
| 2 | 0.25 | 0.0625 | 0 |
| 3 | 0.25 | 0.125 | 0.0156 |

$f^{(k)}(x) =$ probability that, given exactly $k$ claims occur, that the aggregate amount is $x$.

$f^{(1)}(x) = f(x)$;  the claim amount distribution for a single claim

$$f^{(k)}(x) = \sum_{j=0}^{x} \left( f^{(k-1)}(j) \right) x f(x - j)$$

$$f_s(x) = \sum_{k=0}^{x} P(N = k) \times f^{(k)}(x);$$ upper limit of sum is really $\infty$, but here with smallest possible

claim size 1, $f^{(k)}(x) = 0$ for $k > x$

$f_s(0) = 0.2$
$f_s(1) = 0.16 * 0.25 = 0.04$
$f_s(2) = 0.16 * 0.25 + 0.128 * 0.0625 = 0.048$
$f_s(3) = 0.16 * 0.25 + 0.128 * 0.125 + 0.1024 * 0.0156 = 0.0576$

$F_s(3) = 0.2 + 0.04 + 0.048 + 0.0576 = 0.346$

**Question #37**
**Answer: E**

Let $L$ = incurred losses; $P$ = earned premium = 800,000

$$
\begin{aligned}
\text{Bonus} \;=\; & 0.15 \times \left( 0.60 - \frac{L}{P} \right) \times P \;\; \text{if positive}\\
=\; & 0.15 \times \left( 0.60\,P - L \right) \;\; \text{if positive}\\
=\; & 0.15 \times \left( 480{,}000 - L \right) \;\; \text{if positive}\\
=\; & 0.15 \,\check{}\, \left( 480{,}000 - \left( L \, \grave{\mathbf{U}} \, 480{,}000 \right) \right)
\end{aligned}
$$

$E\,(\text{Bonus}) = 0.15\,(480{,}000 - E\left( L \, \grave{\mathbf{U}} \, 480{,}000 \right))$

From Appendix A.2.3.1

$= 0.15\{480{,}000 - [500{,}000 \times (1 - (500{,}000\,/\,(480{,}000 + 500{,}000)))]\}$
$= 35{,}265$

**Question #38**
**Answer: D**

$$
\begin{aligned}
\overline{A}^{\,1}_{28:\overline{2}|} \;=\; & \int_0^2 e^{-d\,t}\, \tfrac{1}{72}\,dt \\
=\; & \frac{1}{72d}\left( 1 - e^{-2d} \right) = 0.02622 \;\; \text{since } d = \ln(1.06) = 0.05827
\end{aligned}
$$

$\ddot{a}_{28:\overline{2}|} = 1 + v\left( \dfrac{71}{72} \right) = 1.9303$

$_3V = 500{,}000\,\overline{A}^{\,1}_{28:\overline{2}|} - 6643\,\ddot{a}_{28:\overline{2}|}$

$\quad = 287$

**Question #39**
**Answer: D**


Let $\bar{A}_x$ and $\bar{a}_x$ be calculated with $m_x(t)$ and $d = 0.06$

Let $\bar{A}_x^*$ and $\bar{a}_x^*$ be the corresponding values with $m_x(t)$
increased by 0.03 and $d$ decreased by 0.03

$$\bar{a}_x = \frac{1 - \bar{A}_x}{d} = \frac{0.4}{0.06} = 6.667$$

$$\bar{a}_x^* = \bar{a}_x$$

$$\left[ \text{Proof: } \bar{a}_x^* = \int_0^{\textbf{¥}} e^{-\int_0^t \left( m_x(s) + 0.03 \right) ds} e^{-0.03t} dt \right.$$

$$= \int_0^{\textbf{¥}} e^{-\int_0^t m_x(s) ds} e^{-0.03t} e^{-0.03t} dt$$

$$= \int_0^{\textbf{¥}} e^{-\int_0^t m_x(s) ds} e^{-0.06t} dt$$

$$\left. = \bar{a}_x \right]$$


$$\bar{A}_x^* = 1 - 0.03 \bar{a}_x^* = 1 - 0.03 \bar{a}_x$$
$$= 1 - (0.03)(6.667)$$
$$= 0.8$$

**Question #40**
**Answer: A**

| year | bulb ages 0 | 1 | 2 | 3 | # replaced |
|------|------|------|------|------|------|
| 0 | 10000 | 0 | 0 | 0 | - |
| 1 | 1000 | 9000 | 0 | 0 | 1000 |
| 2 | 100+2700 | 900 | 6300 | 0 | 2800 |
| 3 | 280+270+3150 | | | | 3700 |

The diagonals represent bulbs that don't burn out.
E.g., of the initial 10,000, (10,000) (1-0.1) = 9000 reach year 1.
(9000) (1-0.3) = 6300 of those reach year 2.

Replacement bulbs are new, so they start at age 0.
At the end of year 1, that's (10,000) (0.1) = 1000
At the end of 2, it's (9000) (0.3) + (1000) (0.1) = 2700 + 100
At the end of 3, it's (2800) (0.1) + (900) (0.3) + (6300) (0.5) = 3700

$$\text{Actuarial present value } = \frac{1000}{1.05} + \frac{2800}{1.05^2} + \frac{3700}{1.05^3}$$
$$= 6688$$