CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam 3, Segment 3L

Life Contingencies and Statistics

October 27, 2009

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2009 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Four rows are available. If your Candidate ID number is fewer than 4 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 0987, enter a zero on the first row, 9 on the second row, 8 on the third row, and 7 on the fourth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 17, 2009.

END OF INSTRUCTIONS

1.

You are given the following information:

- At Company XYZ, workers may only retire on December 31st.
- The earliest a worker may retire is when he is 56 years old.
- All remaining workers are forced to retire on December 31st of the year in which they turn 65.
- For workers age 55 and older, the number of workers follows a discrete distribution such that

$$l_x = 1000 - (x - 55)^3$$

Calculate the probability that a worker who just turned 55 retires when he is 59, 60 or 61.

A. Less than 16%
B. At least 16%, but less than 18%
C. At least 18%, but less than 20%
D. At least 20%, but less than 22%
E. At least 22%

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2.

You are given the following information about the discrete random variable K(x), the time interval of failure for a life aged (x):

- $K(x) = k$, if $k \leq T_x < k+1$, where T_x is the future lifetime of a life age x.
- ${}_kp_x = r^k$
- ${}_3q_x = 0.657$
- r is a fixed but unknown constant between 0 and 1.

Calculate e_x, the curtate expectation of life.

A. Less than 2.50
B. At least 2.50, but less than 2.75
C. At least 2.75, but less than 3.00
D. At least 3.00, but less than 3.25
E. At least 3.25

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3.

You are given the following information:

- Mortality follows the Illustrative Life Table.
- Uniform distribution of deaths applies for fractional ages.

Calculate ${}_{0.75}q_{65.5}$.

A. Less than 0.0158
B. At least 0.0158, but less than 0.0160
C. At least 0.0160, but less than 0.0162
D. At least 0.0162, but less than 0.0164
E. At least 0.0164

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4.

You are given the following information for two lives, (x) and (y), both aged 4:

- Both lives' survival functions are independent and identically distributed.
- Mortality is uniformly distributed on the interval (4, 14).

Calculate the probability that the second death occurs in the interval (10, 11).

A. Less than 8.0%
B. At least 8.0%, but less than 9.5%
C. At least 9.5%, but less than 11.0%
D. At least 11.0%, but less than 12.5%
E. At least 12.5%

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5.

You are given the following excerpt from a life table:

Age	l_x	$d_x^{(1)}$	$d_x^{(2)}$
60	100	15	8
61		17	15
62		9	10
63		18	8

Calculate ${}_2q_{\overline{60:62}}^{(1)}$, the probability that a person aged 60 and a person aged 62 will both die from decrement (1) during the next two years.

A. Less than 5%
B. At least 5%, but less than 10%
C. At least 10%, but less than 15%
D. At least 15%, but less than 20%
E. At least 20%

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6.

You are given the following information:

- Survival functions for lives (x) and (y) are independent and identically distributed.
- Mortality follows De Moivre's law with $\omega = 80$.
- (x) is aged 20.
- (y) is aged 50.
- A = expected future lifetime of the joint life status of (x) and (y).
- B = expected future lifetime of the last survivor status of (x) and (y).

Calculate B - A.

A. Less than 20
B. At least 20, but less than 25
C. At least 25, but less than 30
D. At least 30, but less than 35
E. At least 35

Continued on Next Page

7.

You are given the following excerpt from a life table:

Age	l_x	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$	d_x^{τ}
70	1000		80		
71					
72				80	330
73		100	0		

- $q_{71}^{(3)} = 0.2439$
- ${}_{1|}q_{70}^{(1)} = 0.0500$
- $q_{70}^{(3)} = q_{70}^{(1)}$
- $d_{72}^{(3)} = 2 \times d_{73}^{(3)}$
- $d_{70}^{(1)} = d_{71}^{(1)}$
- $p_{73} = 0$

Calculate $d_{71}^{(2)}$.

A. Less than 50
B. At least 50, but less than 70
C. At least 70, but less than 90
D. At least 90, but less than 110
E. At least 110

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8.

You are given the following information about a multiple decrement model:

- Decrement (1) affects the population with constant force $\mu(x) = 0.06$.
- Decrement (2) affects the population only between ages 45 and 50.
- For each year between ages 45 and 50, fifty additional lives die due to decrement (2).
- Assume all deaths from decrement (2) occur at the end of the year.
- $l_0 = 10{,}000$

Calculate ${}_5q_{45}^{(1)}$

A. Less than 0.20
B. At least 0.20, but less than 0.25
C. At least 0.25, but less than 0.30
D. At least 0.30, but less than 0.35
E. At least 0.35

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9.

You are using a Markov Chain to calculate a subject's transition between two states and have been given four matrices that could be used for that calculation as shown below.

- Assume standard definitions of d, v, i, a_x, A_x where d = 0.03
- All insurance policies and annuities have a unit benefit.

Which of the following matrices is an invalid Markov Chain transition matrix?

$$A = \begin{pmatrix} {}_tp_x & {}_tq_x \\ 0.67 & 0.33 \end{pmatrix} \qquad B = \begin{pmatrix} d\,\ddot{a}_x & A_x \\ 0.5 & 0.5 \end{pmatrix}$$

$$C = \begin{pmatrix} d^2 & 2v - v^2 \\ 1 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 0.45 & 0.55 \\ d & \frac{1}{1+i} \end{pmatrix}$$

A. A only
B. B only
C. C only
D. D only
E. None of the above (all are valid Markov Chain transition matrices)

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10.

You are given the following information:

- An insurance company processes policy endorsements at a Poisson rate of λ=200 per day.
- 45% of these endorsements result in a premium increase.
- Policy endorsements are observed for one day.

Using the normal approximation with no continuity correction, calculate the probability that there are more endorsements resulting in a premium increase than endorsements not resulting in a premium increase.

A. Less than 5.0%
B. At least 5.0%, but less than 7.5%
C. At least 7.5%, but less than 10.0%
D. At least 10.0%, but less than 12.5%
E. At least 12.5%

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11.

You are given the following information:

- Claims follow a compound Poisson process.
- Claims occur at the rate of $\lambda=10$ per day.
- Claim severity follows an exponential distribution with $\theta=15{,}000$.
- A claim is considered a large loss if its severity is greater than 50,000.

What is the probability that there are exactly 9 large losses in a 30-day period?

A. Less than 5%
B. At least 5%, but less than 7.5%
C. At least 7.5%, but less than 10%
D. At least 10%, but less than 12.5%
E. At least 12.5%

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12.

You are given the following excerpt from a life table:

Age	l_x
40	500
41	495
42	485
43	470
44	450
45	425
46	395
47	360
48	320
49	285
50	235

- $a_{40:\overline{10}|} = 10$
- $i = 0.05$

Calculate $s_{40:\overline{10}|}$, the actuarial accumulated value of the annuity shown above in ten years.

A. Less than 12
B. At least 12, but less than 18
C. At least 18, but less than 24
D. At least 24, but less than 30
E. At least 30

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13.

You are given the following information:

- ${}^{2}A_{26} - {}^{2}A_{25} = 0.011$
- $A_{26} = 1.39 \times {}^{2}A_{26}$
- $i = 0.03$
- $p_{25} = 0.97$
- Z_x is defined as the present value, at policy issue, of the benefit payment on a whole life policy for a life aged x.

Calculate the variance of Z_{26}.

A. Less than 0.06
B. At least 0.06, but less than 0.07
C. At least 0.07, but less than 0.08
D. At least 0.08, but less than 0.09
E. At least 0.09

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14.

For a fully continuous 10-year term life insurance policy, you are given the following information:

- Benefits are paid at the moment of death and premiums are paid continuously.
- Survival follows the exponential distribution with $\mu = 0.16$.
- $\delta = 0.04$

Calculate ${}_5\overline{V}^{1}_{x:\overline{10|}}$, the benefit reserve at $t = 5$.

A. Less than -0.5
B. At least -0.5, but less than -0.3
C. At least -0.3, but less than -0.1
D. At least -0.1, but less than 0.1
E. At least 0.1

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15.

You are given the following information:

- Mortality follows De Moivre's law with $\omega = 100$.
- $i = 0.03$
- A deferred life insurance policy is purchased when (x) is age 20 and will pay the benefit at the end of the year of death, beginning after (x) reaches age 65.
- The benefit will increase by a factor of 1.03 every year, with the first benefit = 1.

Calculate the actuarial present value of this policy.

A. Less than 0.1
B. At least 0.1, but less than 0.2
C. At least 0.2, but less than 0.3
D. At least 0.3, but less than 0.4
E. At least 0.4

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16.

You are given the following information:

- An insurance company has 64 medical malpractice claims in litigation.
- Each claim in litigation costs $5,000 per year, as long as it is in litigation.
- If the insurance company loses the litigation, then it pays $50,000 in damages.
- If the insurance company wins, then it pays $0 in damages.
- The insurance company has a 50% chance of winning each claim in litigation.
- Each year, claims in litigation have a 25% chance of being decided and a 75% chance of staying in litigation.
- Claims that are decided are never re-opened.
- $i = 4\%$
- Payments are made in the middle of the year.
- Transitions occur at the end of the year.

Calculate the expected present value of payments over the next 4 years.

A. Less than $1.5 million
B. At least $1.5 million, but less than $1.7 million
C. At least $1.7 million, but less than $1.9 million
D. At least $1.9 million, but less than $2.1 million
E. At least $2.1 million

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17.

You are creating a model to describe exam progress. You are given the following information:

- Let X be the number of exams passed in a given year.
- The probability mass function is defined as follows:

  $P(X=0) = 1-p-q$

  $P(X=1) = p$

  $P(X=2) = q$
- Over the last 5 years, you observe the following values of X:

  0 0 1 2 2

Calculate the method of moments estimate of p.

A. Less than 0.15
B. At least 0.15, but less than 0.21
C. At least 0.21, but less than 0.27
D. At least 0.27, but less than 0.33
E. At least 0.33

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18.

You are given the following five observations from an inverse exponential distribution:

3 9 13 33 51

The probability density function of the inverse exponential distribution is:

$$f(x) = \frac{\theta e^{-\theta / x}}{x^2}$$

Calculate the maximum likelihood estimate for θ.

A. Less than 10
B. At least 10, but less than 15
C. At least 15, but less than 20
D. At least 20, but less than 25
E. At least 25

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19.

You are given the following information:

- The number of trials before success follows a geometric distribution.
- A random sample of size 10 from that process is:
  0 1 2 3 4 4 5 6 7 8

Calculate the maximum likelihood estimate of the variance for the underlying geometric distribution.

A. Less than 10
B. At least 10, but less than 12
C. At least 12, but less than 14
D. At least 14, but less than 16
E. At least 16

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20.

You are given the following information:

- A sample of size 14 is drawn from a normally distributed population.
- The sample mean is 8.
- The unbiased sample variance is S^2.
- A two-sided 95% confidence interval for σ^2 is determined. The significance level in each tail is equal.
- The lower limit for the confidence interval is 53.77.

Note: The 95% confidence interval is defined as the complement of the critical region with 5% significance.

Calculate S^2.

A. Less than 25
B. At least 25, but less than 50
C. At least 50, but less than 75
D. At least 75, but less than 100
E. At least 100

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21.

You have a coin with heads on one side and tails on the other. You believe that the probability, p, of flipping the coin and getting heads is greater than 50%. You perform the following test by flipping the coin n times:

- H_0: p=0.5
- H_1: p>0.5
- Reject H_0 if 51% or more of the flips result in heads.

Using the normal approximation with no continuity correction, calculate the smallest value of n that will ensure that the probability of Type I error is less than 1%.

A. Less than 5,000
B. At least 5,000, but less than 7,500
C. At least 7,500, but less than 10,000
D. At least 10,000, but less than 12,500
E. At least 12,500

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22.

You are given the following information:

- Claim severities are exponentially distributed with mean θ.
- You sample 70 claims.
- The sample mean and the maximum likelihood estimate for θ are both 135.
- H_0: $\theta = 100$
- H_1: $\theta \neq 100$
- $-2\ln(\Lambda)$, where Λ is the likelihood ratio with the null hypothesis in the numerator, has an approximate chi-square distribution with 1 degree of freedom under the null hypothesis.

Using the likelihood ratio test, which is the correct conclusion?

A. Reject the null hypothesis at the 0.5% significance level
B. Reject the null hypothesis at the 1% significance level, but not the 0.5% significance level
C. Reject the null hypothesis at the 2.5% significance level, but not the 1% significance level
D. Reject the null hypothesis at the 5% significance level, but not the 2.5% significance level
E. Do not reject the null hypothesis at the 5% significance level

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23.

An auto insurer is studying the relationship between gender and the groupings given by its proprietary credit model. You are given the following information:

- The distribution of policy counts by gender and credit group:

	Female	Male
Worst Credit	50	100
Average Credit	100	100
Best Credit	25	40

- H_0: The attributes of credit and gender are independent
- H_1: The attributes of credit and gender are not independent

Using the chi-square test for independence, which is the correct conclusion?

A. Reject the null hypothesis at the 0.5% significance level
B. Reject the null hypothesis at the 1% significance level, but not the 0.5% significance level
C. Reject the null hypothesis at the 2.5% significance level, but not the 1% significance level
D. Reject the null hypothesis at the 5% significance level, but not the 2.5% significance level
E. Do not reject the null hypothesis at the 5% significance level

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24.

You are given the following information:

- Claim severity follows a uniform distribution over the interval (0,100).
- Three claim severities are observed.
- The probability density function of the order statistic Y_k with a sample size n is:
  $\frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k} f(y).$
- The sample median is an unbiased estimator of the mean of the distribution.

Calculate the standard deviation of the sample median.

A. Less than 15
B. At least 15, but less than 25
C. At least 25, but less than 35
D. At least 35, but less than 45
E. At least 45

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25.

You are given the following:

- S_1^2 is the unbiased sample variance from a random sample of size 12 drawn from a normal distribution with unknown mean, μ_1, and unknown variance, σ_1^2.
- S_2^2 is the unbiased sample variance from a random sample of size 13 drawn from a normal distribution with unknown mean, μ_2, and unknown variance, σ_2^2.
- The samples from which S_1^2 and S_2^2 are drawn are independent.
- $H_0 : \sigma_1^2 = \sigma_2^2 / k$
- $H_1 : \sigma_1^2 > \sigma_2^2 / k$
- The critical value for testing the ratio of S_1^2 / S_2^2 at size $\alpha = .05$ is 5.2.

Calculate the smallest value for the ratio S_1^2 / S_2^2 at which one would reject the null hypothesis at the $\alpha = .01$ level of significance.

A. Less than 6.5
B. At least 6.5, but less than 7.5
C. At least 7.5, but less than 8.5
D. At least 8.5, but less than 9.5
E. At least 9.5

End of Examination

Exam 3L, October 2009
Answer Key

Question	Answer
1	E
2	A
3	E
4	E
5	D
6	B
7	D
8	B
9	E
10	C
11	D
12	E
13	A
14	D
15	B
16	B
17	B
18	A
19	E
20	E
21	E
22	B
23	B
24	B
25	C