# CASUALTY ACTUARIAL SOCIETY
AND THE
# CANADIAN INSTITUTE OF ACTUARIES



# Exam 3

**Actuarial Models**

October 30, 2003



**4 HOURS**

## INSTRUCTIONS TO CANDIDATES

1. This 80 point examination consists of 40 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil. Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Please make your marks dark and fill in the spaces completely. Fill in that it is Fall 2003, and the exam number 3.

   Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. (For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row.) Please write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.

   For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets.

6. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

**CONTINUE TO NEXT PAGE OF INSTRUCTIONS**

7. At the end of the examination, place the short-answer card in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS website.

   All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. An examination survey and postage-paid reply envelope are included with the examination. No postage is necessary for surveys mailed within the United States. Candidates mailing the survey outside the United States should use the courtesy reply envelope distributed by your exam supervisor. This survey is also available on the CAS website in the "Exams" section. Please either complete the survey and leave it with the examination supervisor, take the survey and envelope with you when leaving the examination center, or submit the survey online. Please submit your survey to the CAS Office by November 17, 2003. Please do not enclose the survey in the Examination Envelope.

**END OF INSTRUCTIONS**

## EXAM 3, FALL 2003

1) Given:

i) ${}_{3}p_{40} = 0.990$
ii) ${}_{6}p_{40} = 0.980$
iii) ${}_{9}p_{40} = 0.965$
iv) ${}_{12}p_{40} = 0.945$
v) ${}_{15}p_{40} = 0.920$
vi) ${}_{18}p_{40} = 0.890$

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

A. Less than 0.050
B. At least 0.050, but less than 0.055
C. At least 0.055, but less than 0.060
D. At least 0.060, but less than 0.065
E. At least 0.065

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*Use the following information for questions 2) and 3).*

**2)** For a special fully discrete life insurance on (45), you are given:

i) $i = 6\%$
ii) Mortality follows the Illustrative Life Table.
iii) The death benefit is 1,000 until age 65, and 500 thereafter.
iv) Benefit premiums of 12.51 are payable at the beginning of each year for 20 years.

Calculate the actuarial present value of the benefit payment.

A. Less than 100
B. At least 100, but less than 150
C. At least 150, but less than 200
D. At least 200, but less than 250
E. At least 250

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*The following is repeated for convenience.*

**3)** For a special fully discrete life insurance on (45), you are given:

i) $i = 6\%$
ii) Mortality follows the Illustrative Life Table.
iii) The death benefit is 1,000 until age 65, and 500 thereafter.
iv) Benefit premiums of 12.51 are payable at the beginning of each year for 20 years.

Calculate ${}_{19}V$, the benefit reserve at time t=19, the instant before the premium payment is made.

A. Less than 200
B. At least 200, but less than 210
C. At least 210, but less than 220
D. At least 220, but less than 230
E. At least 230

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**4)** Given:

$$\mu_x = \frac{2}{(100-x)}, \quad \text{for} \quad 0 \le x < 100$$

Calculate ${}_{10|}q_{65}$.

A. $\frac{1}{25}$

B. $\frac{1}{35}$

C. $\frac{1}{45}$

D. $\frac{1}{55}$

E. $\frac{1}{65}$

**CONTINUED ON NEXT PAGE**

5) Given:

i) Mortality follows De Moivre's Law.

ii) $\overset{\circ}{e}_{20} = 30$

Calculate $q_{20}$.

A. $\frac{1}{60}$

B. $\frac{1}{70}$

C. $\frac{1}{80}$

D. $\frac{1}{90}$

E. $\frac{1}{100}$

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**6)** Let $Z_1$ be the present value random variable for an n-year term insurance of 1 on (x), and let $Z_2$ be the present value random variable for an n-year endowment insurance of 1 on (x). Claims are payable at the moment of death.

Given:

i) $v^n = 0.250$
ii) ${}_n p_x = 0.400$
iii) $E[Z_2] = 0.400$
iv) $Var[Z_2] = 0.055$

Calculate $Var[Z_1]$.

A. 0.025
B. 0.100
C. 0.115
D. 0.190
E. 0.215

CONTINUED ON NEXT PAGE

7) Given:

i) $i = 5\%$
ii) The force of mortality is constant.
iii) $\overset{\circ}{e}_x = 16.0$

Calculate ${}_{20|}\bar{A}_x$.

A. Less than 0.050
B. At least 0.050, but less than 0.075
C. At least 0.075, but less than 0.100
D. At least 0.100, but less than 0.125
E. At least 0.125

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**8)** Given:

i) $i = 6\%$
ii) ${}_{10}E_{40} = 0.540$
iii) $1000\,A_{40} = 168$
iv) $1000\,A_{50} = 264$

Calculate $1000\,{}_{10}P(A_{40})$, the benefit premium for a 10-payment fully discrete life insurance of 1,000 on (40).

A. 21.53
B. 21.88
C. 22.19
D. 22.51
E. 22.83

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**9)** $Ia$ and $I\ddot{a}$ represent the standard increasing annuities. A person aged 20 buys a special five-year temporary life annuity-due, with payments of 1, 3, 5, 7, and 9.

Given:

i) $\ddot{a}_{20:\overline{4}|} = 3.41$

ii) $a_{20:\overline{4}|} = 3.04$

iii) $(I\ddot{a})_{20:\overline{4}|} = 8.05$

iv) $(Ia)_{20:\overline{4}|} = 7.17$

Calculate the net single premium.

A. Less than 18.0
B. At least 18.0, but less than 18.5
C. At least 18.5, but less than 19.0
D. At least 19.0, but less than 19.5
E. At least 19.5

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**10)** For a special fully continuous last-survivor life insurance of 1 on (x) and (y), you are given:

i) $\delta = 0.05$
ii) T(x) and T(y) are independent.
iii) $\mu(x+t) = \mu(y+t) = 0.07$, for $t>0$.
iv) Premiums are payable until the first death.

Calculate the level annual benefit premium.

A. Less than 0.050
B. At least 0.050, but less than 0.075
C. At least 0.075, but less than 0.100
D. At least 0.100, but less than 0.125
E. At least 0.125

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**11)** Given:

| | $q_x^{(1)}$ | $q_x^{(2)}$ | $q_x^{(3)}$ | $q_x^{(\tau)}$ |
|---|---|---|---|---|
| $x < 40$ | 0.10 | 0.04 | 0.02 | 0.16 |
| $x \geq 40$ | 0.20 | 0.04 | 0.02 | 0.26 |

Calculate ${}_{5|}q_{38}^{(1)}$.

A. Less than 0.06
B. At least 0.06, but less than 0.07
C. At least 0.07, but less than 0.08
D. At least 0.08, but less than 0.09
E. At least 0.09

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**12)** A driver is selected at random. If the driver is a "good" driver, he is from a Poisson population with a mean of 1 claim per year. If the driver is a "bad" driver, he is from a Poisson population with a mean of 5 claims per year. There is equal probability that the driver is either a "good" driver or a "bad" driver. If the driver had 3 claims last year, calculate the probability that the driver is a "good" driver.

A. Less than 0.325
B. At least 0.325, but less than 0.375
C. At least 0.375, but less than 0.425
D. At least 0.425, but less than 0.475
E. At least 0.475

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**13)** The Allerton Insurance Company insures 3 indistinguishable populations. The claims frequency of each insured follows a Poisson process.

Given:

| Population (class) | Expected time between claims | Probability of being in class | Claim cost |
|---|---|---|---|
| I | 12 months | 1/3 | 1,000 |
| II | 15 months | 1/3 | 1,000 |
| III | 18 months | 1/3 | 1,000 |

Calculate the expected loss in year 2 for an insured that had no claims in year 1.

A. Less than 810
B. At least 810, but less than 910
C. At least 910, but less than 1,010
D. At least 1,010, but less than 1,110
E. At least 1,110

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**14)** The Independent Insurance Company insures 25 risks, each with a 4% probability of loss. The probabilities of loss are independent.

On average, how often would 4 or more risks have losses in the same year?

A. Once in 13 years
B. Once in 17 years
C. Once in 39 years
D. Once in 60 years
E. Once in 72 years

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**15)** Two actuaries are simulating the number of automobile claims for a book of business. For the population that they are studying:

i) The claim frequency for each individual driver has a Poisson distribution.
ii) The means of the Poisson distributions are distributed as a random variable, $\Lambda$.
iii) $\Lambda$ has a gamma distribution.

In the first actuary's simulation, a driver is selected and one year's experience is generated. This process of selecting a driver and simulating one year is repeated N times.

In the second actuary's simulation, a driver is selected and N years of experience are generated for that driver.

Which of the following is/are true?

I. The ratio of the number of claims the first actuary simulates to the number of claims the second actuary simulates should tend towards 1 as N tends to infinity.
II. The ratio of the number of claims the first actuary simulates to the number of claims the second actuary simulates will equal 1, provided that the same uniform random numbers are used.
III. When the variances of the two sequences of claim counts are compared the first actuary's sequence will have a smaller variance because more random numbers are used in computing it.

A. I only
B. I and II only
C. I and III only
D. II and III only
E. None of I, II, or III is true

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**16)** According to the Klugman study note, which of the following is/are true, based on the existence of moments test?

I. The Loglogistic Distribution has a heavier tail than the Gamma Distribution.
II. The Paralogistic Distribution has a heavier tail than the Lognormal Distribution.
III. The Inverse Exponential has a heavier tail than the Exponential Distribution.

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II, and III

**CONTINUED ON NEXT PAGE**

**17)** Losses have an Inverse Exponential distribution. The mode is 10,000.

Calculate the median.

A. Less than 10,000
B. At least 10,000, but less than 15,000
C. At least 15,000, but less than 20,000
D. At least 20,000, but less than 25,000
E. At least 25,000

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**18)** A new actuarial student analyzed the claim frequencies of a group of drivers and concluded that they were distributed according to a negative binomial distribution and that the two parameters, $r$ and $\beta$, were equal.

An experienced actuary reviewed the analysis and pointed out the following:

> "Yes, it is a negative binomial distribution. The $r$ parameter is fine, but the value of the $\beta$ parameter is wrong. Your parameters indicate that $\frac{1}{9}$ of the drivers should be claim-free, but in fact, $\frac{4}{9}$ of them are claim-free."

Based on this information, calculate the variance of the corrected negative binomial distribution.

A. 0.50
B. 1.00
C. 1.50
D. 2.00
E. 2.50

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**19)** For a loss distribution where $x \geq 2$, you are given:

i) The hazard rate function: $h(x) = \dfrac{z^2}{2x}$ , for $x \geq 2$

ii) A value of the distribution function: $F(5) = 0.84$

Calculate z.

A. 2
B. 3
C. 4
D. 5
E. 6

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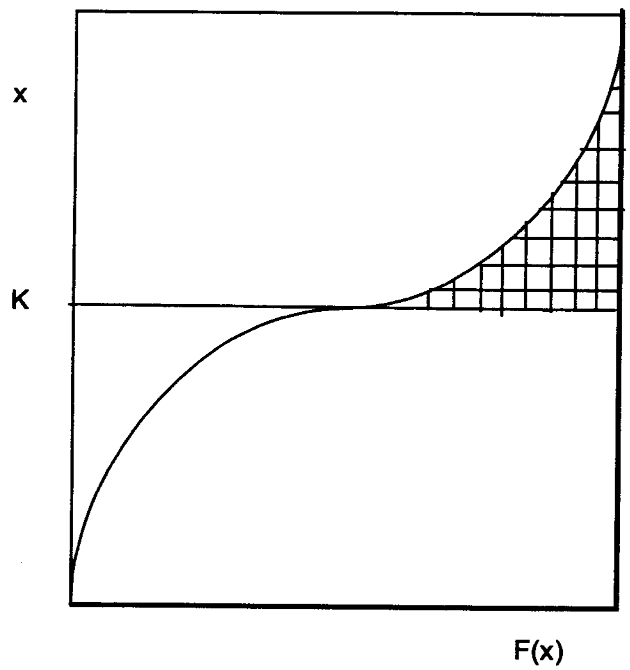
**20)** Let X be the size-of-loss random variable with cumulative distribution function F(x) as shown below:




Which expression(s) below equal(s) the expected loss in the shaded region?

I. $\int_K^{\infty} x\,dF(x)$

II. $E(x)-\int_0^K x\,dF(x)-K[1-F(K)]$

III. $\int_K^{\infty}[1-F(x)]\,dx$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

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**21)** The cumulative loss distribution for a risk is $F(x)=1-\dfrac{10^6}{(x+10^3)^2}$.

Calculate the percent of expected losses within the layer 1,000 to 10,000.

A. 10%
B. 12%
C. 17%
D. 34%
E. 41%

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**22)** The severity distribution function of claims data for automobile property damage coverage for Le Behemoth Insurance Company is given by an exponential distribution, F(x).

$$F(x) = 1 - \exp\left(\frac{-x}{5000}\right)$$

To improve the profitability of this portfolio of policies, Le Behemoth institutes the following policy modifications:

i) It imposes a per-claim deductible of 500.
ii) It imposes a per-claim limit of 25,000.

Previously, there was no deductible and no limit.

Calculate the average savings per (old) claim if the new deductible and policy limit had been in place.

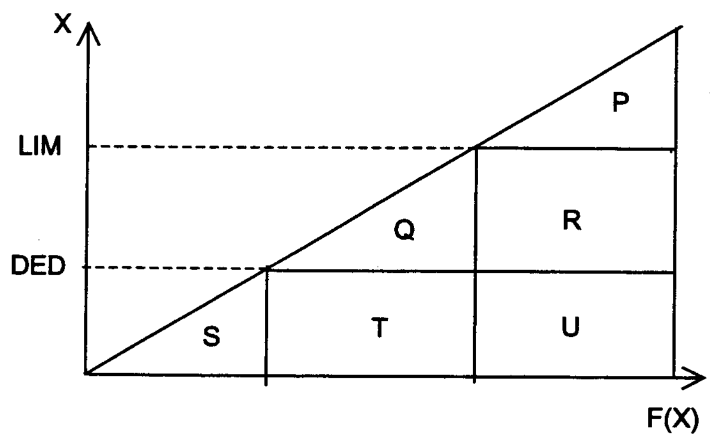
A. 490
B. 500
C. 510
D. 520
E. 530

**CONTINUED ON NEXT PAGE**

**23)** F(x) is the cumulative distribution function for the size-of-loss variable, X.

P, Q, R, S, T, and U represent the areas of the respective regions.

What is the expected value of the insurance payment on a policy with a deductible of "DED" and a limit of "LIM"? (For clarity, that is a policy that pays its first dollar of loss for a loss of DED + 1 and its last dollar of loss for a loss of LIM.)




A. Q
B. Q+R
C. Q+T
D. Q+R+T+U
E. S+T+U

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**24)** Zoom Buy Tire Store, a nationwide chain of retail tire stores, sells 2,000,000 tires per year of various sizes and models. Zoom Buy offers the following road hazard warranty:

"If a tire sold by us is irreparably damaged in the first year after purchase, we'll replace it free, regardless of the cause."

The average annual cost of honoring this warranty is \$10,000,000, with a standard deviation of \$40,000. Individual claim counts follow a binomial distribution, and the average cost to replace a tire is \$100.

All tires are equally likely to fail in the first year, and tire failures are independent.

Calculate the standard deviation of the replacement cost per tire.

A. Less than \$60
B. At least \$60, but less than \$65
C. At least \$65, but less than \$70
D. At least \$70, but less than \$75
E. At least \$75

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**25)** Daily claim counts are modeled by the negative binomial distribution with mean 8 and variance 15. Severities have mean 100 and variance 40,000. Severities are independent of each other and of the number of claims.

Let $\sigma$ be the standard deviation of a day's aggregate losses.

On a certain day, 13 claims occurred, but you have no knowledge of their severities.

Let $\sigma'$ be the standard deviation of that day's aggregate losses, given that 13 claims occurred.

Calculate $\dfrac{\sigma}{\sigma'} - 1$.

A. Less than –7.5%
B. At least –7.5%, but less than 0
C. 0
D. More than 0, but less than 7.5%
E. At least 7.5%

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**26)** A fair coin is flipped by a gambler with 10 chips. If the outcome is "heads," the gambler wins 1 chip; if the outcome is "tails," the gambler loses 1 chip.

The gambler will stop playing when he either has lost all of his chips or he reaches 30 chips.

Of the first ten flips, 4 are "heads" and 6 are "tails."

Calculate the probability that the gambler will lose all of his chips, given the results of the first ten flips.

A. Less than 0.75
B. At least 0.75, but less than 0.80
C. At least 0.80, but less than 0.85
D. At least 0.85, but less than 0.90
E. At least 0.90

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**27)** Not-That-Bad-Burgers employs exactly three types of full-time workers with the following annual salaries:

| Title | Annual Salary |
|---|---|
| Burger Flipper (B) | 6,000 |
| Cashier (C) | 8,400 |
| Manager (M) | 12,000 |

Each month the employees are promoted or demoted according to the following transition probability matrix:

$$\begin{array}{c|ccc|} & B & C & M \\ \hline B & 1/2 & 1/2 & 0 \\ C & 1/4 & 1/2 & 1/4 \\ M & 0 & 1/3 & 2/3 \end{array}$$

Calculate the average long-term annual salary of an employee at Not-That-Bad-Burgers.

A. 8,133
B. 8,533
C. 9,067
D. 9,200
E. 9,467

**CONTINUED ON NEXT PAGE**

**28)** A Markov chain with five states has the following transition probability matrix:

$$\begin{Vmatrix} 0.50 & 0.25 & 0.20 & 0.05 & 0.00 \\ 0.25 & 0.20 & 0.05 & 0.00 & 0.50 \\ 0.20 & 0.05 & 0.00 & 0.50 & 0.25 \\ 0.50 & 0.00 & 0.00 & 0.50 & 0.00 \\ 0.50 & 0.00 & 0.00 & 0.00 & 0.50 \end{Vmatrix}$$

How many classes does this Markov chain have?

A. 1
B. 2
C. 3
D. 4
E. 5

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**29)** A ferry transports fishermen to and from a fishing dock every hour on the hour. The probability of catching a fish in the next hour is a function of the number of fish caught in the preceding hour as follows:

| Number of fish caught in the preceding hour | Probability of 0 fish caught in the next hour | Probability of 1 fish caught in the next hour | Probability of 2+ fish caught in the next hour |
|---|---|---|---|
| 0 | 0.7 | 0.2 | 0.1 |
| 1 | 0.3 | 0.5 | 0.2 |
| 2+ (2 or more) | 0.1 | 0.5 | 0.4 |

If a fisherman has not caught a fish in the hour before the next ferry arrives, he leaves; otherwise he stays and continues to fish. A fisherman arrives at 11:00 AM and catches exactly one fish before noon.

Calculate the expected total number of hours that the fisherman spends on the dock.

A. 2 hours
B. 3 hours
C. 4 hours
D. 5 hours
E. 6 hours

**CONTINUED ON NEXT PAGE**

**30)** Speedy Delivery Company makes deliveries 6 days a week. Accidents involving Speedy vehicles occur according to a Poisson process with a rate of 3 per day and are independent. In each accident, damage to the contents of Speedy's vehicles is distributed as follows:

| Amount of damage | Probability |
|---|---|
| $ 0 | ¼ |
| $2,000 | ½ |
| $8,000 | ¼ |

Using the normal approximation, calculate the probability that Speedy's weekly aggregate damages will not exceed $63,000.

A. 0.24
B. 0.31
C. 0.54
D. 0.69
E. 0.76

**CONTINUED ON NEXT PAGE**

**31)** Vehicles arrive at the Bun-and-Run drive-thru at a Poisson rate of 20 per hour. On average, 30% of these vehicles are trucks.

Calculate the probability that at least 3 trucks arrive between noon and 1:00 PM.

A. Less than 0.80
B. At least 0.80, but less than 0.85
C. At least 0.85, but less than 0.90
D. At least 0.90, but less than 0.95
E. At least 0.95

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**32)** Ross, in *Introduction to Probability Models*, identifies four requirements that a counting process N(t) must satisfy.

Which of the following is NOT one of them?

A. N(t) must be greater than or equal to zero.
B. N(t) must be an integer.
C. If s<t, then N(s) must be less than or equal to N(t).
D. The number of events that occur in disjoint time intervals must be independent.
E. For s<t, N(t)-N(s) must equal the number of events that have occurred in the interval (s,t].

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**33)** Stock I and stock II open the trading day at the same price. Let X(t) denote the dollar amount by which stock I's price exceeds stock II's price when 100t percent of the trading day has elapsed. $\{X(t), 0 \le t \le 1\}$ is modeled as a Brownian motion process with variance parameter $\sigma^2 = 0.3695$.

After ¾ of the trading day has elapsed, the price of stock I is 40.25 and the price of stock II is 39.75.

Calculate the probability that $X(1) \ge 0$.

A. Less than 0.935
B. At least 0.935, but less than 0.945
C. At least 0.945, but less than 0.955
D. At least 0.955, but less than 0.965
E. At least 0.965

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34) Given:

i) Initial surplus is 10.
ii) Annual losses are distributed as follows:

| Annual Loss | Probability |
|---|---|
| 0 | 0.60 |
| 10 | 0.25 |
| 20 | 0.10 |
| 30 | 0.05 |

iii) Premium, paid at the beginning of each year, equals expected losses for the year.
iv) If surplus increases in a year, a dividend is paid at the end of that year. The dividend is equal to half of the increase for the year.
v) There are no other cash flows.

Calculate $\psi(10,2)$, the probability of ruin during the first two years.

A. Less than 0.20
B. At least 0.20, but less than 0.25
C. At least 0.25, but less than 0.30
D. At least 0.30, but less than 0.35
E. At least 0.35

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**35)** In *Loss Models — From Data to Decisions*, Klugman et al. discuss survival probabilities and ruin probabilities in terms of discrete time vs. continuous time and finite time vs. infinite time.

Based on that discussion, which of the following are true?

I. $\tilde{\phi}(u,\tau) \geq \phi(u)$

II. $\phi(u,\tau) \geq \tilde{\phi}(u,\tau)$

III. $\phi(u) \geq \psi(u)$

IV. $\tilde{\psi}(u)$ increases as the frequency with which surplus is checked increases.

A. I and IV only.
B. I, II, and III only.
C. I, II, and IV only.
D. I, III, and IV only.
E. II, III, and IV only.

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**36)** A random variable having density function $f(x) = 30x^2(1-x)^2, 0 < x < 1$ is to be generated using the rejection method with $g(x) = 1, \mathit{for}\ 0 < x < 1$.

Using $c = \frac{15}{8}$, which of the following pairs (Y,U) would be rejected?

I. (0.35, 0.80)
II. (0.80, 0.40)
III. (0.15, 0.80)

A. II only
B. III only
C. I and II only
D. I and III only
E. I, II, and III

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**37)** One of the most common methods for generating pseudorandom numbers starts with an initial value, $X_0$, called the seed, and recursively computes successive values, $X_n$, by letting

$$X_n = aX_{n-1} \quad \text{mod}(m).$$

Which of the following is/are criteria that should be satisfied when selecting $a$ and $m$?

I. The number of variables that can be generated before repetition is large.
II. For any $X_0$, generated numbers are independent Normal $(0,1)$ variables.
III. The values can be computed efficiently on a computer.

A. I only
B. II only
C. I and III only
D. II and III only
E. I, II, and III

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**38)** Using the Inverse Transform Method, a Binomial (10,0.20) random variable is generated, with 0.65 from U(0,1) as the initial random number.

Determine the simulated result.

A. 0
B. 1
C. 2
D. 3
E. 4

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**39)** When generating random variables, it is important to consider how much time it takes to complete the process.

Consider a discrete random variable X with the following distribution:

| k | Prob(X=k) |
|---|---|
| 1 | 0.15 |
| 2 | 0.10 |
| 3 | 0.25 |
| 4 | 0.20 |
| 5 | 0.30 |

Of the following algorithms, which is the most efficient way to simulate X?

A. If $U<0.15$ set $X = 1$ and stop.
If $U<0.25$ set $X = 2$ and stop.
If $U<0.50$ set $X = 3$ and stop.
If $U<0.70$ set $X = 4$ and stop.
Otherwise set $X = 5$ and stop.

B. If $U<0.30$ set $X = 5$ and stop.
If $U<0.50$ set $X = 4$ and stop.
If $U<0.75$ set $X = 3$ and stop.
If $U<0.85$ set $X = 2$ and stop.
Otherwise set $X = 1$ and stop.

C. If $U<0.10$ set $X = 2$ and stop.
If $U<0.25$ set $X = 1$ and stop.
If $U<0.45$ set $X = 4$ and stop.
If $U<0.70$ set $X = 3$ and stop.
Otherwise set $X = 5$ and stop.

D. If $U<0.30$ set $X = 5$ and stop.
If $U<0.55$ set $X = 3$ and stop.
If $U<0.75$ set $X = 4$ and stop.
If $U<0.90$ set $X = 1$ and stop.
Otherwise set $X = 2$ and stop.

E. If $U<0.20$ set $X = 4$ and stop.
If $U<0.35$ set $X = 1$ and stop.
If $U<0.45$ set $X = 2$ and stop.
If $U<0.75$ set $X = 5$ and stop.
Otherwise set $X = 3$ and stop.

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**40)** W is a geometric random variable with parameter $\beta = 7/3$.

Use the multiplicative congruential method:

$$X_{n+1} = aX_n \quad \text{mod}(m) \quad \text{where } a = 6,\ m = 25, \text{ and } X_0 = 7$$

and the inverse transform method.

Calculate $W_3$, the third randomly generated value of W.

A. 2
B. 4
C. 8
D. 12
E. 16

**END OF EXAMINATION**

*Please note: On a one-time basis, the CAS is releasing annotated solutions to Fall 2003 Examination 3 as a study aid to candidates. It is anticipated that for future sittings, only the correct multiple-choice answers will be released.*

**1)** Given:

i) ${}_{3}p_{40} = 0.990$
ii) ${}_{6}p_{40} = 0.980$
iii) ${}_{9}p_{40} = 0.965$
iv) ${}_{12}p_{40} = 0.945$
v) ${}_{15}p_{40} = 0.920$
vi) ${}_{18}p_{40} = 0.890$

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

A. Less than 0.050
B. At least 0.050, but less than 0.055
C. At least 0.055, but less than 0.060
D. At least 0.060, but less than 0.065
E. At least 0.065

P(First death after 6 years, but before 12 years)
= P(Neither dies during first 6 and not both alive at 12)
= P(Neither dies during first 6 and (not both alive at 12 given both alive after 6))
= P(Neither dies during first 6) * P(not both alive at 12 given both alive after 6) these are independent

$$= {}_{6}p_{40}^{\,2}(1 - {}_{12}p_{40}^{\,2}\,{}_{6}p_{40}^{\,-2})$$

=0.067375

ANS: E

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*Use the following information for questions 2) and 3).*

**2)** For a special fully discrete life insurance on (45), you are given:

i) $i = 6\%$
ii) Mortality follows the Illustrative Life Table.
iii) The death benefit is 1,000 until age 65, and 500 thereafter.
iv) Benefit premiums of 12.51 are payable at the beginning of each year for 20 years.

Calculate the actuarial present value of the benefit payment.

A. Less than 100
B. At least 100, but less than 150
C. At least 150, but less than 200
D. At least 200, but less than 250
E. At least 250

The benefit payments of 1000 until age 65, and 500 thereafter are equivalent to $1000\,A_{45} - 500\;{}_{20|}A_{45}$.

$$\begin{aligned} 1000\,A_{45} - 500\;{}_{20|}A_{45} &= 1000\,A_{45} - 500\;{}_{20}E_{45}\,A_{65} \\ &= 1000(0.20120) - 500\,(0.25634)(0.43980) \\ &= 144.83 \end{aligned}$$

Ans: B

(Since at issue the actuarial present value of the benefit premium equals the actuarial present value of the benefit payment, this could also be done by evaluation the actuarial present value of the given premiums.)

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*The following is repeated for convenience.*

**3)** For a special fully discrete life insurance on (45), you are given:

i) i = 6%
ii) Mortality follows the Illustrative Life Table.
iii) The death benefit is 1,000 until age 65, and 500 thereafter.
iv) Benefit premiums of 12.51 are payable at the beginning of each year for 20 years.

Calculate ${}_{19}V$, the benefit reserve at time t=19, the instant before the premium payment is made.

A. Less than 200
B. At least 200, but less than 210
C. At least 210, but less than 220
D. At least 220, but less than 230
E. At least 230

The desired reserve plus the final premium payment plus one year's investment income needs to be able to pay the year-20 death claims and to purchase a single premium policy of 500 on a 65-year old. That is:

$$({}_{19}V + \text{P})(1 + \text{i}) = 1000\,q_{64} + p_{64}\,500\,A_{65}$$

$$({}_{19}V + 12.51)(1.06) = 1000\,q_{64} + p_{64}\,500\,A_{65}$$

$$({}_{19}V + 12.51)(1.06) = 19.52 + 215.61 = 235.13$$

So: ${}_{19}V = 209.31$

Ans: B

**CONTINUED ON NEXT PAGE**

**4)** Given:

$$m_x = \frac{2}{(100-x)}, \quad \text{for} \quad 0 \le x < 100$$

Calculate ${}_{10|}q_{65}$.

A. $\frac{1}{25}$

B. $\frac{1}{35}$

C. $\frac{1}{45}$

D. $\frac{1}{55}$

E. $\frac{1}{65}$

${}_{10|}q_{65}$ is the probability of survival for 10 years and then death in the eleventh year for a life aged 65. That is survival to age 75, but not to age 76, given survival to age 65.

$m_x = -\frac{S'(x)}{S(x)}$ equivalently, $\int m_x = -\log(S(x)) + C$

So, $S(x) = \frac{(100-x)^2}{100^2}$ The constant is determined by the fact that S(0) = 1.

The probability that we want is $\frac{S(75)-S(76)}{S(65)} = \frac{100^{-2}}{100^{-2}} \frac{25^2 - 24^2}{35^2} = \frac{(25-24)(25+24)}{5^2 7^2} = \frac{1}{25}$

Ans: A

**CONTINUED ON NEXT PAGE**

**5)** Given:

i) Mortality follows De Moivre's Law.

ii) $\overset{\circ}{e}_{20} = 30$

Calculate $q_{20}$.

A. $\dfrac{1}{60}$

B. $\dfrac{1}{70}$

C. $\dfrac{1}{80}$

D. $\dfrac{1}{90}$

E. $\dfrac{1}{100}$

De Moivre's law means that deaths are uniformly distributed from age 0 to age $w$. We must determine $w$.

A twenty year old is expected to live 30 more years, so the average time of death between 20 and $w$ is 20+30. Since deaths are uniformly distributed, we must have $w$ =80.

$q_{20}$ is the probability that a twenty year old dies in the next year. As we saw above, times of death for 20 year olds are uniformly distributed over the next 60 years, so $q_{20} = \dfrac{1}{60}$.

Ans: A

**CONTINUED ON NEXT PAGE**

**6)** Let $Z_1$ be the present value random variable for an n-year term insurance of 1 on (x), and let $Z_2$ be the present value random variable for an n-year endowment insurance of 1 on (x). Claims are payable at the moment of death.

Given:

i) $v^n = 0.250$
ii) ${}_n p_x = 0.400$
iii) $E[Z_2] = 0.400$
iv) $Var[Z_2] = 0.055$

Calculate $Var[Z_1]$.

A. 0.025
B. 0.100
C. 0.115
D. 0.190
E. 0.215

Let M be an n-year pure endowment of 1 on (x). Then:

$E[Z_2] = 0.400 = E[Z_1 + M] = E[Z_1] + (0.25)(0.40)$ So: $E[Z_1] = 0.300$

$Var[Z_2] = 0.055 = E[Z_2^{\,2}] - E[Z_2]^2$ So: $E[Z_2^{\,2}] = 0.055 + 0.160 = 0.215$

$E[Z_2^{\,2}] = E[(Z_1 + M)^2] = E[Z_1^{\,2}] + E[2Z_1M] + E[M^2]$

Now, it is impossible to collect on both the term insurance and the pure endowment, so the middle term is 0.

$E[M^2] = (0.400)(0.250)^2 = 0.025$ So: $E[Z_1^{\,2}] = 0.215 - 0.025 = 0.190$

$E[Z_1]^2 = 0.300^2 = 0.090$

$Var[Z_1] = E[Z_1^{\,2}] - E[Z_1]^2 = 0.190 - 0.090 = 0.100$

Ans: B

**CONTINUED ON NEXT PAGE**

**7)** Given:

i) i = 5%
ii) The force of mortality is constant.
iii) $\overset{\circ}{e}_x = 16.0$

Calculate ${}_{20|}\bar{A}_x$ .

A. Less than 0.050
B. At least 0.050, but less than 0.075
C. At least 0.075, but less than 0.100
D. At least 0.100, but less than 0.125
E. At least 0.125

$$d = \ln(1+0.05) = 0.0488$$

$$m = 1/\overset{o}{e}_x = 0.0625$$

$${}_{20|}\bar{A}_x = \frac{m}{m+d}e^{-20(m+d)} = \frac{0.0625}{0.0625+0.0488}e^{-20(0.0625+0.0488)}$$

$$= 0.0606$$

Ans: B

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**8)** Given:

i) $i = 6\%$
ii) ${}_{10}E_{40} = 0.540$
iii) $1000\,A_{40} = 168$
iv) $1000\,A_{50} = 264$

Calculate $1000\,{}_{10}P(A_{40})$, the benefit premium for a 10-payment fully discrete life insurance of 1,000 on (40).

A. 21.53
B. 21.88
C. 22.19
D. 22.51
E. 22.83

$${}_{10}P(A_{40})\ddot{a}_{40:\overline{10|}} = A_{40}$$

$$\ddot{a}_{40} = \ddot{a}_{40:\overline{10|}} + {}_{10}E_{40}\ddot{a}_{50}$$

i=6% so $d = 1 - \frac{1}{1.06} = 0.0566$

$d\ddot{a}_{40} + A_{40} = 1$ So: $\ddot{a}_{40} = 14.70$

$d\ddot{a}_{50} + A_{50} = 1$ So: $\ddot{a}_{50} = 13.00$

So: $14.70 = \ddot{a}_{40:\overline{10|}} + (0.540)(13.00)$ Hence: $\ddot{a}_{40:\overline{10|}} = 7.68$

And: $1000\,{}_{10}P(A_{40}) = \frac{168}{7.68} = 21.875$

ANS: B

**CONTINUED ON NEXT PAGE**

**9)** $Ia$ and $I\ddot{a}$ represent the standard increasing annuities. A person aged 20 buys a special five-year temporary life annuity-due, with payments of 1, 3, 5, 7, and 9.

Given:

i) $\ddot{a}_{20:\overline{4|}} = 3.41$

ii) $a_{20:\overline{4|}} = 3.04$

iii) $(I\ddot{a})_{20:\overline{4|}} = 8.05$

iv) $(Ia)_{20:\overline{4|}} = 7.17$

Calculate the net single premium.

A. Less than 18.0
B. At least 18.0, but less than 18.5
C. At least 18.5, but less than 19.0
D. At least 19.0, but less than 19.5
E. At least 19.5

The (conditional) payments of 1, 3, 5, 7, and 9 are the same as payments of 1, 1+2(1), 1+2(2), 1+2(3), 1+2(4).

The net single premium is $1 + a_{20:\overline{4|}} + 2(Ia)_{20:\overline{4|}} = 1 + 3.04 + 2(7.17) = 18.38$

Ans: B

**CONTINUED ON NEXT PAGE**

**10)** For a special fully continuous last-survivor life insurance of 1 on (x) and (y), you are given:

i) $d$ = 0.05
ii) T(x) and T(y) are independent.
iii) $m$ (x+t) = $m$ (y+t) = 0.07, for t>0.
iv) Premiums are payable until the first death.

Calculate the level annual benefit premium.

A. Less than 0.050
B. At least 0.050, but less than 0.075
C. At least 0.075, but less than 0.100
D. At least 0.100, but less than 0.125
E. At least 0.125

The force of mortality for (x) and (y) is constant. The lives are independent so the force of mortality for the joint life status is 0.07+0.07 = 0.14.

In the case of constant mortality of $m$ and constant force of interest of $d$ , the actuarial present value of a continuous insurance is $\dfrac{m}{m+d}$ .

Applying this to the joint life status we see that a first-to-die insurance has value $\dfrac{0.14}{0.14+0.05}=0.737$

The corresponding annuity, the first-to-die annuity, has actuarial present value $\dfrac{1-0.737}{0.05}=5.263$

This is the annuity corresponding to the premium payments.

To compute the actuarial present value of the last-to-die insurance, we observe that this insurance is identical to insurance on each of the two lives minus the first-to-die insurance.

Insurance on each of the two lives has value $\dfrac{0.07}{0.07+0.05}=0.583$

So, the net single premium for the last-to-die insurance is 2(0.583) – 0.737 = 0.430

And the level annual benefit premium is $\dfrac{0.430}{5.263}=0.082$

Ans: C

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**11)** Given:

| | $q_x^{(1)}$ | $q_x^{(2)}$ | $q_x^{(3)}$ | $q_x^{(t)}$ |
|---|---|---|---|---|
| $x < 40$ | 0.10 | 0.04 | 0.02 | 0.16 |
| $x \geq 40$ | 0.20 | 0.04 | 0.02 | 0.26 |

Calculate ${}_{5|}q_{38}^{(1)}$.

A. Less than 0.06
B. At least 0.06, but less than 0.07
C. At least 0.07, but less than 0.08
D. At least 0.08, but less than 0.09
E. At least 0.09

We need to compute the probability of surviving all perils for five years and the succumbing to peril 1.

That is: (1-0.16)(1-0.16)(1-0.26)(1-0.26)(1-0.26)(0.20) = 0.0572

Ans: A

**CONTINUED ON NEXT PAGE**

**12)** A driver is selected at random. If the driver is a "good" driver, he is from a Poisson population with a mean of 1 claim per year. If the driver is a "bad" driver, he is from a Poisson population with a mean of 5 claims per year. There is equal probability that the driver is either a "good" driver or a "bad" driver. If the driver had 3 claims last year, calculate the probability that the driver is a "good" driver.

A. Less than 0.325
B. At least 0.325, but less than 0.375
C. At least 0.375, but less than 0.425
D. At least 0.425, but less than 0.475
E. At least 0.475

This can easily be done using Bayes' Theorem. Here is an alternative solution using life table techniques. Suppose we had, say, 20,000 drivers. Since "good" and "bad" are equally likely we may assume that we have 10,000 of each. The 10,000 "good" drivers each have probability = $\frac{1^3}{3!}e^{-1}$ = 0.0613 of having exactly 3 accidents --- we expect 613 of them to have 3 accidents.

The 10,000 "bad" drivers have probability = $\frac{5^3}{3!}e^{-5}$ =0.1404 of having exactly 3 accidents --- we expect 1404 of them to have 3 accidents.

So, we expect 613+1404 = 2017 drivers to have 3 accidents and 613 of them are "good" drivers, so the probability of a driver with 3 accidents being a good driver is $\frac{613}{2017} = 0.304$ .

Ans: A

**CONTINUED ON NEXT PAGE**

**13)** The Allerton Insurance Company insures 3 indistinguishable populations. The claims frequency of each insured follows a Poisson process.

Given:

| Population (class) | Expected time between claims | Probability of being in class | Claim cost |
|---|---|---|---|
| I | 12 months | 1/3 | 1,000 |
| II | 15 months | 1/3 | 1,000 |
| III | 18 months | 1/3 | 1,000 |

Calculate the expected loss in year 2 for an insured that had no claims in year 1.

A. Less than 810
B. At least 810, but less than 910
C. At least 910, but less than 1,010
D. At least 1,010, but less than 1,110
E. At least 1,110

Populations I, II, and III are Poisson with annual expected frequency 12/12, 12/15, and 12/18 respectively.

$P(0) = P(0|I)P(I) + P(0|II)P(II) + P(0|III)P(III)$ where 0 is the event 0 claims and I, II, and III are the events the insured is in the respective population.

$$P(0) = e^{-\frac{12}{12}}\frac{1}{3} + e^{-\frac{12}{15}}\frac{1}{3} + e^{-\frac{12}{18}}\frac{1}{3} = 0.4435$$

$$P(I|0) = \frac{P(I)}{P(0)}P(0|I) = \frac{1/3}{0.4435}e^{-\frac{12}{12}} = 0.2765$$

$$P(II|0) = \frac{P(II)}{P(0)}P(0|II) = \frac{1/3}{0.4435}e^{-\frac{12}{15}} = 0.3377$$

$$P(III|0) = \frac{P(III)}{P(0)}P(0|III) = \frac{1/3}{0.4435}e^{-\frac{12}{18}} = 0.3858$$ Note that these sum to 1 as they should.

So, the expected number of claims is $\frac{12}{12}(0.2765) + \frac{12}{15}(0.3377) + \frac{12}{18}(0.3858) = 0.8038$

So, expected loss next year is 804.

Ans: A

**CONTINUED ON NEXT PAGE**

**14)** The Independent Insurance Company insures 25 risks, each with a 4% probability of loss. The probabilities of loss are independent.

On average, how often would 4 or more risks have losses in the same year?

A. Once in 13 years
B. Once in 17 years
C. Once in 39 years
D. Once in 60 years
E. Once in 72 years

$$P(0\ losses) = \binom{25}{0} 0.96^{25} 0.04^{25-25} = 0.3604$$
$$P(1\ loss) = \binom{25}{1} 0.96^{24} 0.04^{25-24} = 0.3754$$
$$P(2\ losses) = \binom{25}{2} 0.96^{23} 0.04^{25-23} = 0.1877$$
$$P(3\ losses) = \binom{25}{3} 0.96^{22} 0.04^{25-22} = 0.0600$$

So, probability of 4 or more = 1-0.3604-0.3754-0.1877-0.600 = 0.0165

We would expect such an event to occur once every 1/0.0165 years = 60.61 years

Ans: D

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**15)** Two actuaries are simulating the number of automobile claims for a book of business. For the population that they are studying:

i) The claim frequency for each individual driver has a Poisson distribution.
ii) The means of the Poisson distributions are distributed as a random variable, $\Lambda$ .
iii) $\Lambda$ has a gamma distribution.

In the first actuary's simulation, a driver is selected and one year's experience is generated. This process of selecting a driver and simulating one year is repeated N times.

In the second actuary's simulation, a driver is selected and N years of experience are generated for that driver.

Which of the following is/are true?

I. The ratio of the number of claims the first actuary simulates to the number of claims the second actuary simulates should tend towards 1 as N tends to infinity.
II. The ratio of the number of claims the first actuary simulates to the number of claims the second actuary simulates will equal 1, provided that the same uniform random numbers are used.
III. When the variances of the two sequences of claim counts are compared the first actuary's sequence will have a smaller variance because more random numbers are used in computing it.

A. I only
B. I and II only
C. I and III only
D. II and III only
E. None of I, II, or III is true

The first actuary is generating the negative binomial random variable for the population. The second actuary is generating the Poisson random variable for the driver that he picked.

There is no reason that the first two should be true, the second actuary's result depends on which driver he picked. The variances they observe could be higher or lower again depending on which driver the second actuary picked; the number of random numbers used is irrelevant.

None of these statements is true.

Ans: E

**CONTINUED ON NEXT PAGE**

**16)** According to the Klugman study note, which of the following is/are true, based on the existence of moments test?

I. The Loglogistic Distribution has a heavier tail than the Gamma Distribution.
II. The Paralogistic Distribution has a heavier tail than the Lognormal Distribution.
III. The Inverse Exponential has a heavier tail than the Exponential Distribution.

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II, and III

According to the existence of moments test, distribution A has a heavier tail than distribution B if A has fewer moments than B. The moments for the various distributions are in the supplied tables, in all three cases the distribution on the left has only finitely many moments and the distribution on the left has all moments, so all three statements are true.

Ans: E

**CONTINUED ON NEXT PAGE**

**17)** Losses have an Inverse Exponential distribution. The mode is 10,000.

Calculate the median.

A. Less than 10,000
B. At least 10,000, but less than 15,000
C. At least 15,000, but less than 20,000
D. At least 20,000, but less than 25,000
E. At least 25,000

From the supplied tables for an Inverse Exponential, mode = 10,000 ➔ $q$ = 20,000.

The median of a continuous distribution is that value x so that F(x) = ½.

$F(x) = e^{-x/q}$ for the Inverse Exponential (again from the table).

We want to find x so that: $\frac{1}{2} = e^{-20000/x}$. So, $x = -20000/\ln(1/2) = 28{,}854$

Ans: E

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**18)** A new actuarial student analyzed the claim frequencies of a group of drivers and concluded that they were distributed according to a negative binomial distribution and that the two parameters, $r$ and $b$, were equal.

An experienced actuary reviewed the analysis and pointed out the following:

> "Yes, it is a negative binomial distribution. The $r$ parameter is fine, but the value of the $b$ parameter is wrong. Your parameters indicate that $\frac{1}{9}$ of the drivers should be claim-free, but in fact, $\frac{4}{9}$ of them are claim-free."

Based on this information, calculate the variance of the corrected negative binomial distribution.

A. 0.50
B. 1.00
C. 1.50
D. 2.00
E. 2.50

The probability of 0 claims from a negative binomial with parameters $r$ and $b$ is $(1+b)^{-r}$. We are told that originally these two parameters were equal and that the result was $\frac{1}{9}$. By inspection, $r = b = 2$ works.

So, the corrected $r$ value is 2, as we were told it didn't change.
The corrected $b$ value needs to satisfy $(1+b)^{-2} = \frac{4}{9}$. So, $b = \frac{1}{2}$.

The variance of a negative binomial with parameters $r$ and $b$ is $r\,b\,(1+b) = 2\,\frac{1}{2}\,(\frac{3}{2}) = 1.5$

Ans: C

**CONTINUED ON NEXT PAGE**

**19)** For a loss distribution where $x \geq 2$, you are given:

i) The hazard rate function: $h(x) = \dfrac{z^2}{2x}$ , for $x \geq 2$

ii) A value of the distribution function: F(5) = 0.84

Calculate z.

A. 2
B. 3
C. 4
D. 5
E. 6

$S(5) = 1 - F(5) = 0.16$

$S(x) = e^{-\int_{-\infty}^{\infty} h(x)dx}$

$S(5) = 0.16 = e^{-\int_2^5 z^2 / 2x \, dx}$

$= e^{-(z^2/2)(\ln 5 - \ln 2)}$

$\ln 0.16 = -(z^2/2)(\ln 5 - \ln 2)$

$z^2 = 4$

$z = 2$

Ans: A

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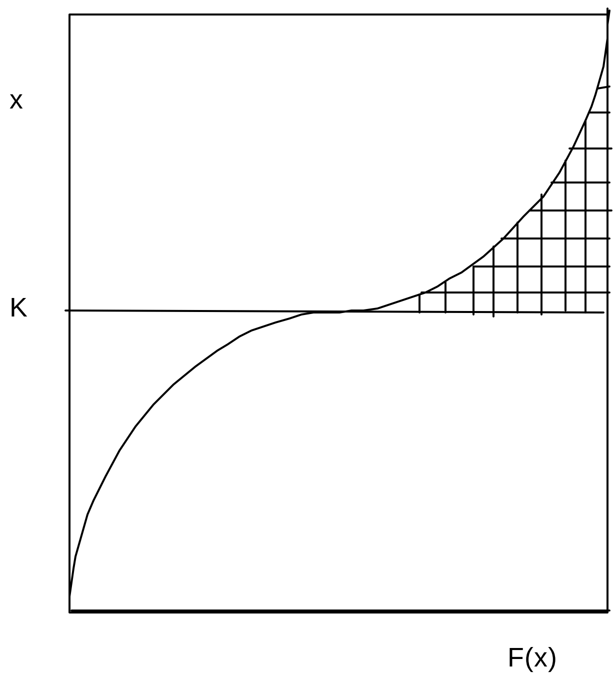
**20)** Let X be the size-of-loss random variable with cumulative distribution function F(x) as shown below:




Which expression(s) below equal(s) the expected loss in the shaded region?

I. $\int_K^{\infty} x dF(x)$

II. $E(x) - \int_0^K x dF(x) - K[1 - F(K)]$

III. $\int_K^{\infty} [1 - F(x)] dx$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

I is false, since the correct representation would be $\int_K^{\infty} x dF(x) - KG(K)$
II and III are true.

Ans: E

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**21)** The cumulative loss distribution for a risk is $F(x)=1-\dfrac{10^6}{(x+10^3)^2}$ .

Calculate the percent of expected losses within the layer 1,000 to 10,000.

A. 10%
B. 12%
C. 17%
D. 34%
E. 41%

Total losses:

$$\int_0^\infty G(x)dx = \int_0^\infty \frac{10^6}{(x+1000)^2}dx$$

$$= 10^6 \int_{1000}^\infty \frac{1}{u^2}du = 1000$$

Layer losses:

$$\int_{1000}^{10000} G(x)dx = \int_{1000}^{10000} \frac{10^6}{(x+1000)^2}dx$$

$$= 10^6 \int_{2000}^{11000} \frac{1}{u^2}dx = 409$$

Percentage = 409/1000 = 41%

Ans: E

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**22)** The severity distribution function of claims data for automobile property damage coverage for Le Behemoth Insurance Company is given by an exponential distribution, F(x).

$$F(x) = 1 - \exp\left(\frac{-x}{5000}\right)$$

To improve the profitability of this portfolio of policies, Le Behemoth institutes the following policy modifications:

i) It imposes a per-claim deductible of 500.
ii) It imposes a per-claim limit of 25,000.

Previously, there was no deductible and no limit.

Calculate the average savings per (old) claim if the new deductible and policy limit had been in place.

A. 490
B. 500
C. 510
D. 520
E. 530

Unlimited coverage:

$$E(X) = \int_0^\infty G(x)dx$$
$$= \int_0^\infty e^{-x/5,000} dx = 5,000$$

where G(x) = 1 – F(x)

Limited coverage:

$$E(X;d,L) = \int_d^L G(x)dx$$
$$= \int_{500}^{25,000} e^{-x/5,000} dx = -5,000(e^{-5} - e^{-0.1}) = 4,490$$

Savings = 5,000 – 4,490 = 510

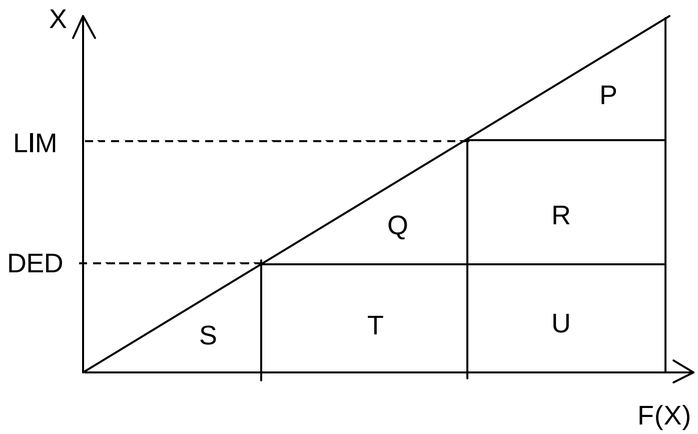
Ans: C

**CONTINUED ON NEXT PAGE**

**23)** F(x) is the cumulative distribution function for the size-of-loss variable, X.

P, Q, R, S, T, and U represent the areas of the respective regions.

What is the expected value of the insurance payment on a policy with a deductible of "DED" and a limit of "LIM"? (For clarity, that is a policy that pays its first dollar of loss for a loss of DED + 1 and its last dollar of loss for a loss of LIM.)




A. Q
B. Q+R
C. Q+T
D. Q+R+T+U
E. S+T+U

Lee integrates horizontally: G(x) = 1 – F(x)

$$\int_{DED}^{LIM} G(x)dx = Q + R$$

Ans: B

**CONTINUED ON NEXT PAGE**

**24)** Zoom Buy Tire Store, a nationwide chain of retail tire stores, sells 2,000,000 tires per year of various sizes and models. Zoom Buy offers the following road hazard warranty:

"If a tire sold by us is irreparably damaged in the first year after purchase, we'll replace it free, regardless of the cause."

The average annual cost of honoring this warranty is $10,000,000, with a standard deviation of $40,000. Individual claim counts follow a binomial distribution, and the average cost to replace a tire is $100.

All tires are equally likely to fail in the first year, and tire failures are independent.

Calculate the standard deviation of the replacement cost per tire.

A. Less than $60
B. At least $60, but less than $65
C. At least $65, but less than $70
D. At least $70, but less than $75
E. At least $75

$E(X) = 100$; $E(S) = 10{,}000{,}000$; $Var(S) = 40{,}000^2$
$m$ = tires sold = 2 million
$E(N) = E(S)/E(X) = 100{,}000$
Frequency of loss = $E(N)/m = 0.05 = q$
$Var(N) = mq(1-q) = (2{,}000{,}000)(0.05)(1-0.05) = 95{,}000$

$Var(S) = E(N)Var(X) + Var(N)E(X)^2$
$40{,}000^2 = (100{,}000)Var(X) + (95{,}000)(100^2)$

$Var(X) = 6{,}500$
Standard Deviation = 80.62

Ans: E

**CONTINUED ON NEXT PAGE**

**25)** Daily claim counts are modeled by the negative binomial distribution with mean 8 and variance 15. Severities have mean 100 and variance 40,000. Severities are independent of each other and of the number of claims.

Let $s$ be the standard deviation of a day's aggregate losses.

On a certain day, 13 claims occurred, but you have no knowledge of their severities.

Let $s'$ be the standard deviation of that day's aggregate losses, given that 13 claims occurred.

Calculate $\frac{s}{s'} - 1$.

A. Less than –7.5%
B. At least –7.5%, but less than 0
C. 0
D. More than 0, but less than 7.5%
E. At least 7.5%

Beginning of day:

$$\begin{aligned}\text{Var}(S) &= E(N)\text{Var}(X) + \text{Var}(N)E(X)^2 \\ &= (8)(40{,}000) + (15)(100^2) \\ &= 470{,}000\end{aligned}$$

Std Dev (S) = 685.56

End of day:

$$\begin{aligned}\text{Var}(S) &= N * \text{Var}(X) \\ &= (13)(40{,}000) \\ &= 520{,}000\end{aligned}$$

Std Dev (S) = 721.11

Increase = 685.56 / 721.11 – 1 = – 4.9%

Ans: B

**CONTINUED ON NEXT PAGE**

**26)** A fair coin is flipped by a gambler with 10 chips. If the outcome is “heads,” the gambler wins 1 chip; if the outcome is “tails,” the gambler loses 1 chip.

The gambler will stop playing when he either has lost all of his chips or he reaches 30 chips.

Of the first ten flips, 4 are “heads” and 6 are “tails.”

Calculate the probability that the gambler will lose all of his chips, given the results of the first ten flips.

A. Less than 0.75
B. At least 0.75, but less than 0.80
C. At least 0.80, but less than 0.85
D. At least 0.85, but less than 0.90
E. At least 0.90

After 10 flips, the gambler is down 2 and has 8 chips left.
He will quit if he loses his remaining 8 chips or reaches 30 chips (22 more than 8).

Pr(down 8 before up 22) = 22/(8+22) = 0.733

Ans: A

**CONTINUED ON NEXT PAGE**

**27)** Not-That-Bad-Burgers employs exactly three types of full-time workers with the following annual salaries:

| Title | Annual Salary |
|---|---|
| Burger Flipper (B) | 6,000 |
| Cashier (C) | 8,400 |
| Manager (M) | 12,000 |

Each month the employees are promoted or demoted according to the following transition probability matrix:

$$\begin{array}{c|ccc|} & B & C & M \\ B & 1/2 & 1/2 & 0 \\ C & 1/4 & 1/2 & 1/4 \\ M & 0 & 1/3 & 2/3 \end{array}$$

Calculate the average long-term annual salary of an employee at Not-That-Bad-Burgers.

A. 8,133
B. 8,533
C. 9,067
D. 9,200
E. 9,467

$$\sum_j r(j)p_j$$

$$(1) p_B = \tfrac{1}{2}p_B + \tfrac{1}{4}p_C$$

$$(2) p_C = \tfrac{1}{2}p_B + \tfrac{1}{2}p_C + \tfrac{1}{3}p_M$$

$$(3) p_M = \tfrac{1}{4}p_C + \tfrac{2}{3}p_M$$

$$(4) 1 = p_B + p_C + p_M$$

$$(1) \rightarrow \tfrac{1}{2}p_B = \tfrac{1}{4}p_C \rightarrow p_B = \tfrac{1}{2}p_C$$

$$(3) \rightarrow \tfrac{1}{3}p_M = \tfrac{1}{4}p_C \rightarrow p_M = \tfrac{3}{4}p_C$$

$$(4) \rightarrow 1 = \tfrac{1}{2}p_C + p_C + \tfrac{3}{4}p_C$$

$$p_B = \tfrac{2}{9}, p_C = \tfrac{4}{9}, p_M = \tfrac{3}{9}$$

(2/9)(6,000)+(4/9)(8,400)+(3/9)(12,000)=9,067

Ans: C

**CONTINUED ON NEXT PAGE**

**28)** A Markov chain with five states has the following transition probability matrix:

$$\begin{Vmatrix} 0.50 & 0.25 & 0.20 & 0.05 & 0.00 \\ 0.25 & 0.20 & 0.05 & 0.00 & 0.50 \\ 0.20 & 0.05 & 0.00 & 0.50 & 0.25 \\ 0.50 & 0.00 & 0.00 & 0.50 & 0.00 \\ 0.50 & 0.00 & 0.00 & 0.00 & 0.50 \end{Vmatrix}$$

How many classes does this Markov chain have?

A. 1
B. 2
C. 3
D. 4
E. 5

State 0 communicates with state 1.
State 1 communicates with state 2.
State 0 communicates with state 3.
State 4 is accessible from state 1. State 1 is accessible from state 4 (4-0-1).
Therefore, state 4 and state 1 communicate.

Therefore, all states communicate, so there is one class.

Ans: A

CONTINUED ON NEXT PAGE

**29)** A ferry transports fishermen to and from a fishing dock every hour on the hour. The probability of catching a fish in the next hour is a function of the number of fish caught in the preceding hour as follows:

| Number of fish caught in the preceding hour | Probability of 0 fish caught in the next hour | Probability of 1 fish caught in the next hour | Probability of 2+ fish caught in the next hour |
|---|---|---|---|
| 0 | 0.7 | 0.2 | 0.1 |
| 1 | 0.3 | 0.5 | 0.2 |
| 2+ (2 or more) | 0.1 | 0.5 | 0.4 |

If a fisherman has not caught a fish in the hour before the next ferry arrives, he leaves; otherwise he stays and continues to fish. A fisherman arrives at 11:00 AM and catches exactly one fish before noon.

Calculate the expected total number of hours that the fisherman spends on the dock.

A. 2 hours
B. 3 hours
C. 4 hours
D. 5 hours
E. 6 hours

Since the fisherman only stays if he catches a fish, the probabilities associated with 0 fish caught are not relevant. Therefore,

$$P^T = \begin{pmatrix} 0.5 & 0.2 \\ 0.5 & 0.4 \end{pmatrix}$$

$$I - P^T = \begin{pmatrix} 0.5 & -0.2 \\ -0.5 & 0.6 \end{pmatrix}$$

$$(I - P^T)^{-1} = \begin{pmatrix} 0.6/0.2 & 0.2/0.2 \\ 0.5/0.2 & 0.5/0.2 \end{pmatrix}$$

E(Time in 1 | Started in 1) = 3
E(Time in 2 | Started in 1) = 1
E(Time in 0) = 1

Total expected time on the dock = 3 + 1 + 1 = 5

Ans: D

**CONTINUED ON NEXT PAGE**

**30)** Speedy Delivery Company makes deliveries 6 days a week. Accidents involving Speedy vehicles occur according to a Poisson process with a rate of 3 per day and are independent. In each accident, damage to the contents of Speedy's vehicles is distributed as follows:

| Amount of damage | Probability |
|---|---|
| $ 0 | ¼ |
| $2,000 | ½ |
| $8,000 | ¼ |

Using the normal approximation, calculate the probability that Speedy's weekly aggregate damages will not exceed $63,000.

A. 0.24
B. 0.31
C. 0.54
D. 0.69
E. 0.76

Let $Y_i$ represent damages for accident *i*, and X(6) represent weekly damages.

$$E[Y_i] = 0(1/4) + 2{,}000(1/2) + 8{,}000(1/4) = 3{,}000$$

$$E[Y_i^2] = 0(1/4) + 2{,}000^2(1/2) + 8{,}000^2(1/4) = 18\,million$$

$$E[X(6)] = (6)(3)(3{,}000) = 54{,}000$$

$$Var[X(6)] = (6)(3)E[Y_i^2] = 324\,million$$

$$\Pr(X(6) \le 63{,}000) \cong \Pr\left(\frac{X(6) - 54{,}000}{\sqrt{324mil}} \le \frac{63{,}000 - 54{,}000}{\sqrt{324mil}}\right)$$

$$= \Phi(0.5) = 0.6915$$

Ans: D

**CONTINUED ON NEXT PAGE**

**31)** Vehicles arrive at the Bun-and-Run drive-thru at a Poisson rate of 20 per hour. On average, 30% of these vehicles are trucks.

Calculate the probability that at least 3 trucks arrive between noon and 1:00 PM.

A. Less than 0.80
B. At least 0.80, but less than 0.85
C. At least 0.85, but less than 0.90
D. At least 0.90, but less than 0.95
E. At least 0.95

$$l\,tp = (20)(1)(0.30) = 6$$

$$P(n) = \frac{e^{-ltp}(l\,tp)^n}{n!}$$

$$P(0) = e^{-6} = 0.0025$$

$$P(1) = \frac{e^{-6}(6)^1}{1!} = 0.0149$$

$$P(2) = \frac{e^{-6}(6)^2}{2!} = 0.0446$$

So the probability of *at least* 3 trucks = 1 – P(0) – P(1) – P(2)
= 1 – 0.0025 – 0.0149 – 0.0446 = 0.938

Ans: D

**CONTINUED ON NEXT PAGE**

**32)** Ross, in *Introduction to Probability Models*, identifies four requirements that a counting process N(t) must satisfy.

Which of the following is NOT one of them?

A. N(t) must be greater than or equal to zero.
B. N(t) must be an integer.
C. If s<t, then N(s) must be less than or equal to N(t).
D. The number of events that occur in disjoint time intervals must be independent.
E. For s<t, N(t)-N(s) must equal the number of events that have occurred in the interval (s,t].

D only applies if the counting process possesses *independent increments*.

Ans: D

**CONTINUED ON NEXT PAGE**

**33)** Stock I and stock II open the trading day at the same price. Let X(t) denote the dollar amount by which stock I's price exceeds stock II's price when 100t percent of the trading day has elapsed. $\{X(t), 0 \le t \le 1\}$ is modeled as a Brownian motion process with variance parameter $s^2 = 0.3695$.

After ¾ of the trading day has elapsed, the price of stock I is 40.25 and the price of stock II is 39.75.

Calculate the probability that $X(1) \ge 0$.

A. Less than 0.935
B. At least 0.935, but less than 0.945
C. At least 0.945, but less than 0.955
D. At least 0.955, but less than 0.965
E. At least 0.965

$$\Pr\{X(1) > 0 \mid X(3/4) = 0.50\}$$
$$= \Pr\{[X(1) - X(3/4)] > -0.50 \mid X(3/4) = 0.50\}$$
$$= \Pr\{[X(1) - X(3/4)] > -0.50\}$$
$$= \Pr\{X(1/4) > -0.50\}$$
$$= \Pr\{\frac{X(1/4)}{\sqrt{s^2/4}} > \frac{-0.50}{\sqrt{s^2/4}}\}$$
$$= \Pr\{\frac{X(1/4)}{\sqrt{s^2/4}} > -1/s\}$$
$$= \Phi(1/s) = \Phi(1.645) = 0.95$$

Ans: C

**CONTINUED ON NEXT PAGE**

**34)** Given:

i) Initial surplus is 10.
ii) Annual losses are distributed as follows:

| **Annual Loss** | **Probability** |
|---|---|
| 0 | 0.60 |
| 10 | 0.25 |
| 20 | 0.10 |
| 30 | 0.05 |

iii) Premium, paid at the beginning of each year, equals expected losses for the year.
iv) If surplus increases in a year, a dividend is paid at the end of that year. The dividend is equal to half of the increase for the year.
v) There are no other cash flows.

Calculate $\psi(10,2)$, the probability of ruin during the first two years.

A. Less than 0.20
B. At least 0.20, but less than 0.25
C. At least 0.25, but less than 0.30
D. At least 0.30, but less than 0.35
E. At least 0.35

Year 1:

| U0 | P | L | Rebate | U1 | Prob |
|---|---|---|---|---|---|
| 10 | 6 | 0 | 3 | 13 | |
| 10 | 6 | 10 | 0 | 6 | |
| 10 | 6 | 20 | 0 | -4 | 0.10 |
| 10 | 6 | 30 | 0 | -14 | 0.05 |

Year 2:

| U1 | P | L | Rebate | U2 | Prob |
|---|---|---|---|---|---|
| 13 | 6 | 0 | 3 | 16 | |
| 13 | 6 | 10 | 0 | 9 | |
| 13 | 6 | 20 | 0 | -1 | 0.60*0.10=0.06 |
| 13 | 6 | 30 | 0 | -11 | 0.60*0.05=0.03 |
| OR | | | | | |
| 6 | 6 | 0 | 3 | 9 | |
| 6 | 6 | 10 | 0 | 2 | |
| 6 | 6 | 20 | 0 | -8 | 0.25*0.10=0.025 |
| 6 | 6 | 30 | 0 | -18 | 0.25*0.05=0.0125 |

0.10+0.05+0.06+0.03+0.025+0.0125=0.2775

Ans: C

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**35)** In *Loss Models --- From Data to Decisions*, Klugman et al. discuss survival probabilities and ruin probabilities in terms of discrete time vs. continuous time and finite time vs. infinite time.

Based on that discussion, which of the following are true?

I. $\tilde{f}(u,t) \geq f(u)$

II. $f(u,t) \geq \tilde{f}(u,t)$

III. $f(u) \geq y\,(u)$

IV. $\tilde{y}\,(u)$ increases as the frequency with which surplus is checked increases.

A. I and IV only.
B. I, II, and III only.
C. I, II, and IV only.
D. I, III, and IV only.
E. II, III, and IV only.

The discrete time survival probability with finite time horizon is greater than the continuous time survival probability with the same finite horizon (short term ruins during the gaps aren't seen) which in turn is greater than the infinite time horizon survival probability (if ruin occurs, it occurs at some finite time), so I is true.

Short term ruins during the time gaps are caught in the continuous time, but might not be in discrete time, so II is false (the inequality goes the other way).

The continuous time, infinite horizon survival probability and continuous time, infinite horizon ruin probability add to one, but neither is necessarily larger than the other, so III is false.

The more often you look, the more likely you are to catch a short term ruin, so IV is true.

Ans: A

**CONTINUED ON NEXT PAGE**

**36)** A random variable having density function $f(x) = 30x^2(1-x)^2,\ 0 < x < 1$ is to be generated using the rejection method with $g(x) = 1,\ for\ \ 0 < x < 1$.

Using c=$\frac{15}{8}$, which of the following pairs (Y,U) would be <u>rejected</u>?

I. (0.35, 0.80)
II. (0.80, 0.40)
III. (0.15, 0.80)

A. II only
B. III only
C. I and II only
D. I and III only
E. I, II, and III

A pair (Y,U) is rejected if $U > \frac{f(Y)}{cg(Y)}$.
g(Y) is identically 1.

It is easily verified that only III is rejected.

Ans: B

**CONTINUED ON NEXT PAGE**

**37)** One of the most common methods for generating pseudorandom numbers starts with an initial value, $X_0$, called the seed, and recursively computes successive values, $X_n$, by letting

$$X_n = aX_{n-1} \quad \mathrm{mod}(m).$$

Which of the following is/are criteria that should be satisfied when selecting $a$ and $m$?

I. The number of variables that can be generated before repetition is large.
II. For any $X_0$, generated numbers are independent Normal (0,1) variables.
III. The values can be computed efficiently on a computer.

A. I only
B. II only
C. I and III only
D. II and III only
E. I, II, and III

Once the pseudorandom numbers start to repeat, they really stop looking like random numbers, we want this to occur as late as possible, so I is true.

The given procedure generates a sequence of residue classes mod(m), these are then associated with the integers from 0 to m-1 (these are the X's). The generated X's are then divided by m to get number in the interval [0,1]. Neither the X's nor their rescaled counterparts are normally distributed, so II is false.

Typically, the pseudorandom numbers will be used by a computer program, so being able to generate them efficiently on a computer is a good feature, thus III is true.

Ans: C

**CONTINUED ON NEXT PAGE**

**38)** Using the Inverse Transform Method, a Binomial (10,0.20) random variable is generated, with 0.65 from U(0,1) as the initial random number.

Determine the simulated result.

A. 0
B. 1
C. 2
D. 3
E. 4

Prob(0) = $0.8^{10}$ = 0.1074 < 0.65 keep going

Prob(0 or 1) = $10(0.8)^9(0.2)^1$ + 0.1074 = 0.2684 + 0.1074 < 0.65 keep going

Prob(0 or 1 or 2) = $\frac{10 \cdot 9}{2}(0.8)^8(0.2)^2$ + 0.2684 + 0.1074 = 0.6778 > 0.65 Success.

So, generated value is 2.

Ans: C

**CONTINUED ON NEXT PAGE**

**39)** When generating random variables, it is important to consider how much time it takes to complete the process.

Consider a discrete random variable X with the following distribution:

| k | Prob(X=k) |
|---|---|
| 1 | 0.15 |
| 2 | 0.10 |
| 3 | 0.25 |
| 4 | 0.20 |
| 5 | 0.30 |

Of the following algorithms, which is the most efficient way to simulate X?

A. If $U<0.15$ set $X = 1$ and stop.
If $U<0.25$ set $X = 2$ and stop.
If $U<0.50$ set $X = 3$ and stop.
If $U<0.70$ set $X = 4$ and stop.
Otherwise set $X = 5$ and stop.

Expected number of tests = 1(.15) + 2(.10) + 3(.25) +4(.50)

B. If $U<0.30$ set $X = 5$ and stop.
If $U<0.50$ set $X = 4$ and stop.
If $U<0.75$ set $X = 3$ and stop.
If $U<0.85$ set $X = 2$ and stop.
Otherwise set $X = 1$ and stop.

Expected number of tests = 1(.30) + 2(.20) + 3(.25) +4(.25)

C. If $U<0.10$ set $X = 2$ and stop.
If $U<0.25$ set $X = 1$ and stop.
If $U<0.45$ set $X = 4$ and stop.
If $U<0.70$ set $X = 3$ and stop.
Otherwise set $X = 5$ and stop.

Expected number of tests = 1(.10) + 2(.15) + 3(.20) +4(.55)

D. If $U<0.30$ set $X = 5$ and stop.
If $U<0.55$ set $X = 3$ and stop.
If $U<0.75$ set $X = 4$ and stop.
If $U<0.90$ set $X = 1$ and stop.
Otherwise set $X = 2$ and stop.

Expected number of tests = 1(.30) + 2(.25) + 3(.20) +4(.25)

E. If $U<0.20$ set $X = 4$ and stop.
If $U<0.35$ set $X = 1$ and stop.
If $U<0.45$ set $X = 2$ and stop.
If $U<0.75$ set $X = 5$ and stop.
Otherwise set $X = 3$ and stop.

Expected number of tests = 1(.20) + 2(.15) + 3(.20) +4(.55)

Ans: D D has the smallest expected number of tests, so it is the most efficient of these algorithms.

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**40)** W is a geometric random variable with parameter $b = {}^{7}\!/_{3}$ .

Use the multiplicative congruential method:

$$X_{n+1} = aX_n \quad \text{mod}(m) \quad \text{where } a = 6,\ m = 25, \text{ and } X_0 = 7$$

and the inverse transform method.

Calculate $W_3$, the third randomly generated value of W.

A. 2
B. 4
C. 8
D. 12
E. 16

$X_1 = 6(7) = 42 = 17 \mod(25)$
$X_2 = 6(17) = 102 = 2 \mod(25)$
$X_3 = 6(2) = 12 = 12 \mod(25)$

The associated uniform pseudo-random variable is $\dfrac{12}{25} = 0.48$

The geometric random variable with parameter 7/3 counts the number of Bernoulli trials (each with success probability 0.3) needed to observe a success.

Using the inverse transform method, we have:

Probability of success on first try = 0.3 < 0.48 (didn't happen on first trial) keep going

Probability of success on first or second try = 0.3 + (0.7)(0.3) = 0.51 > 0.48 Stop.

So, the first success occurred on the second trial.

Ans: A

(*A grading adjustment was made for question 40 to account for an alternative approach for solving this problem.*)

**END OF EXAMINATION**