SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM P PROBABILITY

EXAM P SAMPLE QUESTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

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1. A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A) 24

(B) 36

(C) 41

(D) 52

(E) 60

2. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35% . Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A) 0.05

(B) 0.12

(C) 0.18

(D) 0.25

(E) 0.35

3. You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.

Determine $P[A]$.

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.6

(E) 0.8

4. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44 .

Calculate the number of blue balls in the second urn.

(A) 4

(B) 20

(C) 24

(D) 44

(E) 64

5. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

(i) young or old;

(ii) male or female; and

(iii) married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

How many of the company's policyholders are young, female, and single?

(A) 280

(B) 423

(C) 486

(D) 880

(E) 896

6. A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

(A) 0.115

(B) 0.173

(C) 0.224

(D) 0.327

(E) 0.514

7. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(A) 20

(B) 29

(C) 41

(D) 53

(E) 70

8. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

Determine the probability that a randomly chosen member of this group visits a physical therapist.

(A) 0.26

(B) 0.38

(C) 0.40

(D) 0.48

(E) 0.62

9. An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.

(ii) 70% of the customers insure more than one car.

(iii) 20% of the customers insure a sports car.

(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

(A) 0.13

(B) 0.21

(C) 0.24

(D) 0.25

(E) 0.30

10. An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.

(ii) 64% of the customers insure more than one car.

(iii) 20% of the customers insure a sports car.

(iv) Of those customers who insure more than one car, 15% insure a sports car.

What is the probability that a randomly selected customer insures exactly one car, and that car is not a sports car?

(A) 0.16

(B) 0.19

(C) 0.26

(D) 0.29

(E) 0.31

11. An actuary studying the insurance preferences of automobile owners makes the following conclusions:

(i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.

(ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

(iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15 .

What is the probability that an automobile owner purchases neither collision nor disability coverage?

(A) 0.18

(B) 0.33

(C) 0.48

(D) 0.67

(E) 0.82

12. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.

(ii) 22% have low blood pressure.

(iii) 15% have an irregular heartbeat.

(iv) Of those with an irregular heartbeat, one-third have high blood pressure.

(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure?

(A) 2%

(B) 5%

(C) 8%

(D) 9%

(E) 20%

13. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is $\frac{1}{3}$.

What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

(A) 0.280

(B) 0.311

(C) 0.467

(D) 0.484

(E) 0.700

14. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p_{n+1} = \frac{1}{5} p_n$, where p_n represents the probability that the policyholder files n claims during the period.

Under this assumption, what is the probability that a policyholder files more than one claim during the period?

(A) 0.04

(B) 0.16

(C) 0.20

(D) 0.80

(E) 0.96

15. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively.

Determine the probability that a randomly chosen employee will choose no supplementary coverage.

(A) 0

(B) $\frac{47}{144}$

(C) $\frac{1}{2}$

(D) $\frac{97}{144}$

(E) $\frac{7}{9}$

16. An insurance company determines that N, the number of claims received in a week, is a random variable with P[$N = n$] = $\frac{1}{2^{n+1}}$, where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Determine the probability that exactly seven claims will be received during a given two-week period.

(A) $\frac{1}{256}$

(B) $\frac{1}{128}$

(C) $\frac{7}{512}$

(D) $\frac{1}{64}$

(E) $\frac{1}{32}$

17. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A) 0.10

(B) 0.20

(C) 0.25

(D) 0.40

(E) 0.80

18. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

Assuming the two measurements are independent random variables, what is the probability that their average value is within $0.005h$ of the height of the tower?

(A) 0.38

(B) 0.47

(C) 0.68

(D) 0.84

(E) 0.90

19. An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

(A) 0.13

(B) 0.16

(C) 0.19

(D) 0.23

(E) 0.40

20. An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year.

A policyholder dies in the next year.

What is the probability that the deceased policyholder was ultra-preferred?

(A) 0.0001

(B) 0.0010

(C) 0.0071

(D) 0.0141

(E) 0.2817

21. Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

(i) 10% of the emergency room patients were critical;

(ii) 30% of the emergency room patients were serious;

(iii) the rest of the emergency room patients were stable;

(iv) 40% of the critical patients died;

(vi) 10% of the serious patients died; and

(vii) 1% of the stable patients died.

Given that a patient survived, what is the probability that the patient was categorized as serious upon arrival?

(A) 0.06

(B) 0.29

(C) 0.30

(D) 0.39

(E) 0.64

22. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period.

Calculate the probability that the participant was a heavy smoker.

(A) 0.20

(B) 0.25

(C) 0.35

(D) 0.42

(E) 0.57

23. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

(A) 0.06

(B) 0.16

(C) 0.19

(D) 0.22

(E) 0.25

24. The number of injury claims per month is modeled by a random variable N with

$$P[N = n] = \frac{1}{(n+1)(n+2)}, \text{ where } n \geq 0 .$$

Determine the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{5}$

(E) $\frac{5}{6}$

25. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease.

Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

(A) 0.324

(B) 0.657

(C) 0.945

(D) 0.950

(E) 0.995

26. The probability that a randomly chosen male has a circulation problem is 0.25 . Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem.

What is the conditional probability that a male has a circulation problem, given that he is a smoker?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{1}{2}$

(E) $\frac{2}{3}$

27. A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
1997	0.16	0.05
1998	0.18	0.02
1999	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident.

Determine the probability that the model year of this automobile is 1997 .

(A) 0.22

(B) 0.30

(C) 0.33

(D) 0.45

(E) 0.50

28. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X’s shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective.

What is the probability that this shipment came from Company X?

(A) 0.10

(B) 0.14

(C) 0.37

(D) 0.63

(E) 0.86

29. The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.

What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

(A) 0.15

(B) 0.34

(C) 0.43

(D) 0.57

(E) 0.66

30. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

(A) $\frac{1}{\sqrt{3}}$

(B) 1

(C) $\sqrt{2}$

(D) 2

(E) 4

31. A company establishes a fund of 120 from which it wants to pay an amount, *C*, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year, independent of any other employee.

Determine the maximum value of *C* for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

(A) 24

(B) 30

(C) 40

(D) 60

(E) 120

32. A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool.

This month, the insurance company writes 4 new policies for adults earning their first driver's license.

What is the probability that these 4 will contain at least two more high-risk drivers than low-risk drivers?

(A) 0.006

(B) 0.012

(C) 0.018

(D) 0.049

(E) 0.073

33. The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & \text{for } 0 < x < 20 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, what is the probability that it exceeds 16 ?

(A) $\frac{1}{25}$

(B) $\frac{1}{9}$

(C) $\frac{1}{8}$

(D) $\frac{1}{3}$

(E) $\frac{3}{7}$

34. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function f, where $f(x)$ is proportional to $(10 + x)^{-2}$.

Calculate the probability that the lifetime of the machine part is less than 6.

(A) 0.04

(B) 0.15

(C) 0.47

(D) 0.53

(E) 0.94

35. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function f, where $f(x)$ is proportional to $(10 + x)^{-2}$.

What is the probability that the lifetime of the machine part is less than 5?

(A) 0.03

(B) 0.13

(C) 0.42

(D) 0.58

(E) 0.97

36. A group insurance policy covers the medical claims of the employees of a small company. The value, V, of the claims made in one year is described by

$$V = 100{,}000Y$$

where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4 & \text{for } 0 < y < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

What is the conditional probability that V exceeds 40,000, given that V exceeds 10,000?

(A) 0.08

(B) 0.13

(C) 0.17

(D) 0.20

(E) 0.51

37. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years.

The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year.

If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

(A) 6,321

(B) 7,358

(C) 7,869

(D) 10,256

(E) 12,642

38. An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4} & \text{for } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2 ?

(A) 0.578

(B) 0.684

(C) 0.704

(D) 0.829

(E) 0.875

39. A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.

(ii) In any calendar year, the probability of a hurricane is 0.05 .

(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A) 0.06

(B) 0.19

(C) 0.38

(D) 0.62

(E) 0.92

40. An insurance policy pays for a random loss *X* subject to a deductible of *C*, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Given a random loss *X*, the probability that the insurance payment is less than 0.5 is equal to 0.64 .

Calculate *C*.

(A) 0.1

(B) 0.3

(C) 0.4

(D) 0.6

(E) 0.8

41. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants).

What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

(A) 0.096

(B) 0.192

(C) 0.235

(D) 0.376

(E) 0.469

42. For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000 .

For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000 .

Assume that the total claim amounts of the two companies are independent.

What is the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount?

(A) 0.180

(B) 0.185

(C) 0.217

(D) 0.223

(E) 0.240

43. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is $\frac{3}{5}$. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months.

Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

(A) 0.01

(B) 0.12

(C) 0.23

(D) 0.29

(E) 0.41

44. An insurance policy pays 100 per day for up to 3 days of hospitalization and 50 per day for each day of hospitalization thereafter.

The number of days of hospitalization, *X*, is a discrete random variable with probability function

$$P(X = k) = \begin{cases} \dfrac{6-k}{15} & \text{for } k = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the expected payment for hospitalization under this policy.

(A) 123

(B) 210

(C) 220

(D) 270

(E) 367

45. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{|x|}{10} & \text{for } -2 \le x \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A) $\frac{1}{5}$

(B) $\frac{3}{5}$

(C) 1

(D) $\frac{28}{15}$

(E) $\frac{12}{5}$

46. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$.

Determine $E[X]$.

(A) $2+\frac{1}{3}e^{-6}$

(B) $2-2e^{-2/3}+5e^{-4/3}$

(C) 3

(D) $2+3e^{-2/3}$

(E) 5

47. A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made.

At what level must x be set if the expected payment made under this insurance is to be 1000 ?

(A) 3858

(B) 4449

(C) 5382

(D) 5644

(E) 7235

48. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0 . If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4 .

What is the expected benefit under this policy?

(A) 2234

(B) 2400

(C) 2500

(D) 2667

(E) 2694

49. An insurance policy pays an individual 100 per day for up to 3 days of hospitalization and 25 per day for each day of hospitalization thereafter.

The number of days of hospitalization, *X*, is a discrete random variable with probability function

$$P(X = k) = \begin{cases} \dfrac{6-k}{15} & \text{for } k = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected payment for hospitalization under this policy.

(A) 85

(B) 163

(C) 168

(D) 213

(E) 255

50. A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5 .

What is the expected amount paid to the company under this policy during a one-year period?

(A) 2,769

(B) 5,000

(C) 7,231

(D) 8,347

(E) 10,578

51. A manufacturer's annual losses follow a distribution with density function

$$f(x) = \begin{cases} \dfrac{2.5(0.6)^{2.5}}{x^{3.5}} & \text{for } x > 0.6 \\ 0 & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2.

What is the mean of the manufacturer's annual losses not paid by the insurance policy?

(A) 0.84

(B) 0.88

(C) 0.93

(D) 0.95

(E) 1.00

52. An insurance company sells a one-year automobile policy with a deductible of 2 . The probability that the insured will incur a loss is 0.05 . If there is a loss, the probability of a loss of amount N is $\dfrac{K}{N}$, for $N = 1, \ldots, 5$ and K a constant. These are the only possible loss amounts and no more than one loss can occur.

Determine the net premium for this policy.

(A) 0.031

(B) 0.066

(C) 0.072

(D) 0.110

(E) 0.150

53. An insurance policy reimburses a loss up to a benefit limit of 10 . The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \dfrac{2}{y^3} & \text{for } y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0

(B) 1.3

(C) 1.8

(D) 1.9

(E) 2.0

54. An auto insurance company insures an automobile worth 15,000 for one year under a policy with a 1,000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount X of damage (in thousands) follows a distribution with density function

$$f(x) = \begin{cases} 0.5003\, e^{-x/2} & \text{for } 0 < x < 15 \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected claim payment?

(A) 320

(B) 328

(C) 352

(D) 380

(E) 540

55. An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1 + x)^{-4}$, where $0 < x < \infty$.

Determine the company's expected monthly claims.

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 1

(E) 3

56. An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$.

At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250

(B) 375

(C) 500

(D) 625

(E) 750

57. An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function

$$M_X(t) = \frac{1}{(1-2500t)^4} .$$

Determine the standard deviation of the claim size for this class of accidents.

(A) 1,340

(B) 5,000

(C) 8,660

(D) 10,000

(E) 11,180

58. A company insures homes in three cities, J, K, and L . Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are independent.

The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1 - 2t)^{-3}$$

$$M_K(t) = (1 - 2t)^{-2.5}$$

$$M_L(t) = (1 - 2t)^{-4.5}$$

Let X represent the combined losses from the three cities.

Calculate $E(X^3)$.

(A) 1,320

(B) 2,082

(C) 5,760

(D) 8,000

(E) 10,560

59. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}} & \text{for } x > 200 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X .

(A) 35

(B) 93

(C) 124

(D) 231

(E) 298

60. A recent study indicates that the annual cost of maintaining and repairing a car in a town in Ontario averages 200 with a variance of 260.

If a tax of 20% is introduced on all items associated with the maintenance and repair of cars (*i.e.*, everything is made 20% more expensive), what will be the variance of the annual cost of maintaining and repairing a car?

(A) 208

(B) 260

(C) 270

(D) 312

(E) 374

61. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x)=\begin{cases}\dfrac{2.5(200)^{2.5}}{x^{3.5}} & \text{for } x>200\\ 0 & \text{otherwise.}\end{cases}$$

Calculate the difference between the 25th and 75th percentiles of X .

(A) 124

(B) 148

(C) 167

(D) 224

(E) 298

62. A random variable X has the cumulative distribution function

$$F(x)=\begin{cases}0 & \text{for } x<1\\ \dfrac{x^2-2x+2}{2} & \text{for } 1\le x<2\\ 1 & \text{for } x\ge 2.\end{cases}$$

Calculate the variance of X.

(A) $\dfrac{7}{72}$

(B) $\dfrac{1}{8}$

(C) $\dfrac{5}{36}$

(D) $\dfrac{4}{3}$

(E) $\dfrac{23}{12}$

63. The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine's age at failure, *X*, has density function

$$f(x) = \begin{cases} \frac{1}{5} & \text{for } 0 < x < 5 \\ 0 & \text{otherwise.} \end{cases}$$

Let *Y* be the age of the machine at the time of replacement.

Determine the variance of *Y*.

(A) 1.3

(B) 1.4

(C) 1.7

(D) 2.1

(E) 7.5

64. A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim Size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

What percentage of the claims are within one standard deviation of the mean claim size?

(A) 45%

(B) 55%

(C) 68%

(D) 85%

(E) 100%

65. The owner of an automobile insures it against damage by purchasing an insurance policy with a deductible of 250 . In the event that the automobile is damaged, repair costs can be modeled by a uniform random variable on the interval (0, 1500) .

Determine the standard deviation of the insurance payment in the event that the automobile is damaged.

(A) 361

(B) 403

(C) 433

(D) 464

(E) 521

66. A company agrees to accept the highest of four sealed bids on a property. The four bids are regarded as four independent random variables with common cumulative distribution function

$$F(x) = \frac{1}{2}(1+\sin \pi x) \qquad \text{for } \frac{3}{2} \le x \le \frac{5}{2}.$$

Which of the following represents the expected value of the accepted bid?

(A) $\pi \int_{3/2}^{5/2} x \cos \pi x \, dx$

(B) $\frac{1}{16} \int_{3/2}^{5/2} (1+\sin \pi x)^4 \, dx$

(C) $\frac{1}{16} \int_{3/2}^{5/2} x(1+\sin \pi x)^4 \, dx$

(D) $\frac{1}{4}\pi \int_{3/2}^{5/2} \cos \pi x (1+\sin \pi x)^3 \, dx$

(E) $\frac{1}{4}\pi \int_{3/2}^{5/2} x \cos \pi x (1+\sin \pi x)^3 \, dx$

67. A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay 1000 for each day, up to 2 days, that the opening game is postponed.

The insurance company determines that the number of consecutive days of rain beginning on April 1 is a Poisson random variable with mean 0.6 .

What is the standard deviation of the amount the insurance company will have to pay?

(A) 668

(B) 699

(C) 775

(D) 817

(E) 904

68. An insurance policy reimburses dental expense, *X*, up to a maximum benefit of 250 . The probability density function for *X* is:

$$f(x) = \begin{cases} ce^{-0.004x} & \text{for } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

Calculate the median benefit for this policy.

(A) 161

(B) 165

(C) 173

(D) 182

(E) 250

69. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours.

Calculate the probability that the component will work without failing for at least five hours.

(A) 0.07

(B) 0.29

(C) 0.38

(D) 0.42

(E) 0.57

70. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100 . Losses incurred follow an exponential distribution with mean 300.

What is the 95th percentile of actual losses that exceed the deductible?

(A) 600

(B) 700

(C) 800

(D) 900

(E) 1000

71. The time, *T*, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1-\left(\dfrac{2}{t}\right)^2 & \text{for } t > 2 \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$.

Determine the density function of *Y*, for $y > 4$.

(A) $\dfrac{4}{y^2}$

(B) $\dfrac{8}{y^{3/2}}$

(C) $\dfrac{8}{y^3}$

(D) $\dfrac{16}{y}$

(E) $\dfrac{1024}{y^5}$

72. An investment account earns an annual interest rate R that follows a uniform distribution on the interval $(0.04,\ 0.08)$. The value of a 10,000 initial investment in this account after one year is given by $V = 10{,}000e^R$.

Determine the cumulative distribution function, $F(v)$, of V for values of v that satisfy $0 < F(v) < 1$.

(A) $\dfrac{10{,}000e^{v/10{,}000} - 10{,}408}{425}$

(B) $25e^{v/10{,}000} - 0.04$

(C) $\dfrac{v - 10{,}408}{10{,}833 - 10{,}408}$

(D) $\dfrac{25}{v}$

(E) $25\left[\ln\left(\dfrac{v}{10{,}000}\right) - 0.04\right]$

73. An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$, where X is an exponential random variable with mean 1 year.

Determine the probability density function $f(y)$, for $y > 0$, of the random variable Y.

(A) $10y^{0.8}e^{-8y^{-0.2}}$

(B) $8y^{-0.2}e^{-10y^{0.8}}$

(C) $8y^{-0.2}e^{-(0.1y)^{1.25}}$

(D) $(0.1y)^{1.25}e^{-0.125(0.1y)^{0.25}}$

(E) $0.125(0.1y)^{0.25}e^{-(0.1y)^{1.25}}$

74. Let T denote the time in minutes for a customer service representative to respond to 10 telephone inquiries. T is uniformly distributed on the interval with endpoints 8 minutes and 12 minutes. Let R denote the average rate, in customers per minute, at which the representative responds to inquiries.

Which of the following is the density function of the random variable R on the interval $\left(\frac{10}{12} \le r \le \frac{10}{8}\right)$?

(A) $\frac{12}{5}$

(B) $3 - \frac{5}{2r}$

(C) $3r - \frac{5\ln(r)}{2}$

(D) $\frac{10}{r^2}$

(E) $\frac{5}{2r^2}$

75. The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I.

Determine the probability density function of the monthly profit of Company II.

(A) $\frac{1}{2} f\left(\frac{x}{2}\right)$

(B) $f\left(\frac{x}{2}\right)$

(C) $2f\left(\frac{x}{2}\right)$

(D) $2f(x)$

(E) $2f(2x)$

76. Claim amounts for wind damage to insured homes are independent random variables with common density function

$$f(x) = \begin{cases} \frac{3}{x^4} & \text{for } x > 1 \\ 0 & \text{otherwise} \end{cases}$$

where x is the amount of a claim in thousands.

Suppose 3 such claims will be made.

What is the expected value of the largest of the three claims?

(A) 2025

(B) 2700

(C) 3232

(D) 3375

(E) 4500

77. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x + y}{8} \text{ for } 0 < x < 2 \text{ and } 0 < y < 2 .$$

What is the probability that the device fails during its first hour of operation?

(A) 0.125

(B) 0.141

(C) 0.391

(D) 0.625

(E) 0.875

78. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x,y) = \frac{x+y}{27} \text{ for } 0 < x < 3 \text{ and } 0 < y < 3 .$$

Calculate the probability that the device fails during its first hour of operation.

(A) 0.04

(B) 0.41

(C) 0.44

(D) 0.59

(E) 0.96

79. A device contains two components. The device fails if either component fails. The joint density function of the lifetimes of the components, measured in hours, is $f(s,t)$, where $0 < s < 1$ and $0 < t < 1$.

What is the probability that the device fails during the first half hour of operation?

(A) $\int_0^{0.5} \int_0^{0.5} f(s,t)\, ds\, dt$

(B) $\int_0^{1} \int_0^{0.5} f(s,t)\, ds\, dt$

(C) $\int_{0.5}^{1} \int_{0.5}^{1} f(s,t)\, ds\, dt$

(D) $\int_0^{0.5} \int_0^{1} f(s,t)\, ds\, dt + \int_0^{1} \int_0^{0.5} f(s,t)\, ds\, dt$

(E) $\int_{0}^{0.5}\int_{0.5}^{1} f(s,t)\, ds\, dt + \int_{0}^{1}\int_{0}^{0.5} f(s,t)\, ds\, dt$

80. A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90^{th} percentile for the distribution of the total contributions received.

(A) 6,328,000

(B) 6,338,000

(C) 6,343,000

(D) 6,784,000

(E) 6,977,000

81. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000.

What is the probability that the average of 25 randomly selected claims exceeds 20,000 ?

(A) 0.01

(B) 0.15

(C) 0.27

(D) 0.33

(E) 0.45

82. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by distinct policyholders are independent of one another.

What is the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

(A) 0.68

(B) 0.82

(C) 0.87

(D) 0.95

(E) 1.00

83. A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1 . A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have independent lifetimes.

What is the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability at least 0.9772?

(A) 14

(B) 16

(C) 20

(D) 40

(E) 55

84. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$\begin{aligned}
\mathrm{E}(X) &= 50 \\
\mathrm{E}(Y) &= 20 \\
\mathrm{Var}(X) &= 50 \\
\mathrm{Var}(Y) &= 30 \\
\mathrm{Cov}(X, Y) &= 10
\end{aligned}$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period.

Approximate the value of $P(T < 7100)$.

(A) 0.62

(B) 0.84

(C) 0.87

(D) 0.92

(E) 0.97

85. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{1000} e^{-(x/1000)} \text{ for } x > 0 \text{ .}$$

The premium for the policy is set at 100 over the expected total claim amount. If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

(A) 0.001

(B) 0.159

(C) 0.333

(D) 0.407

(E) 0.460

86. A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

(i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

(ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25 .

(iii) The number of pensions that the city will provide on behalf of each new hire is independent of the number of pensions it will provide on behalf of any other new hire.

Determine the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

(A) 0.60

(B) 0.67

(C) 0.75

(D) 0.93

(E) 0.99

87. In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from -2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people.

What is the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages?

(A) 0.14

(B) 0.38

(C) 0.57

(D) 0.77

(E) 0.88

88. The waiting time for the first claim from a good driver and the waiting time for the first claim from a bad driver are independent and follow exponential distributions with means 6 years and 3 years, respectively.

What is the probability that the first claim from a good driver will be filed within 3 years and the first claim from a bad driver will be filed within 2 years?

(A) $\frac{1}{18}\left(1-e^{-2/3}-e^{-1/2}+e^{-7/6}\right)$

(B) $\frac{1}{18}e^{-7/6}$

(C) $1-e^{-2/3}-e^{-1/2}+e^{-7/6}$

(D) $1-e^{-2/3}-e^{-1/2}+e^{-1/3}$

(E) $1-\frac{1}{3}e^{-2/3}-\frac{1}{6}e^{-1/2}+\frac{1}{18}e^{-7/6}$

89. The future lifetimes (in months) of two components of a machine have the following joint density function:

$$f(x,y)=\begin{cases}\frac{6}{125,000}(50-x-y) & \text{for } 0<x<50-y<50\\ 0 & \text{otherwise.}\end{cases}$$

What is the probability that both components are still functioning 20 months from now?

(A) $\dfrac{6}{125,000}\int_{0}^{20}\int_{0}^{20}(50-x-y)\,dy\,dx$

(B) $\dfrac{6}{125,000}\int_{20}^{30}\int_{20}^{50-x}(50-x-y)\,dy\,dx$

(C) $\dfrac{6}{125,000}\int_{20}^{30}\int_{20}^{50-x-y}(50-x-y)\,dy\,dx$

(D) $\dfrac{6}{125,000}\int_{20}^{50}\int_{20}^{50-x}(50-x-y)\,dy\,dx$

(E) $\dfrac{6}{125,000}\int_{20}^{50}\int_{20}^{50-x-y}(50-x-y)\,dy\,dx$

90. An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic Policy claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim is an independent exponential random variable with mean three days.

What is the probability that the next claim will be a Deluxe Policy claim?

(A) 0.172

(B) 0.223

(C) 0.400

(D) 0.487

(E) 0.500

91. An insurance company insures a large number of drivers. Let X be the random variable representing the company's losses under collision insurance, and let Y represent the company's losses under liability insurance. X and Y have joint density function

$$f(x,y) = \begin{cases} \dfrac{2x+2-y}{4} & \text{for } 0 < x < 1 \text{ and } 0 < y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that the total loss is at least 1 ?

(A) 0.33

(B) 0.38

(C) 0.41

(D) 0.71

(E) 0.75

92. Two insurers provide bids on an insurance policy to a large company. The bids must be between 2000 and 2200 . The company decides to accept the lower bid if the two bids differ by 20 or more. Otherwise, the company will consider the two bids further. Assume that the two bids are independent and are both uniformly distributed on the interval from 2000 to 2200.

Determine the probability that the company considers the two bids further.

(A) 0.10

(B) 0.19

(C) 0.20

(D) 0.41

(E) 0.60

93. A family buys two policies from the same insurance company. Losses under the two policies are independent and have continuous uniform distributions on the interval from 0 to 10. One policy has a deductible of 1 and the other has a deductible of 2. The family experiences exactly one loss under each policy.

Calculate the probability that the total benefit paid to the family does not exceed 5.

(A) 0.13

(B) 0.25

(C) 0.30

(D) 0.32

(E) 0.42

94. Let T_1 be the time between a car accident and reporting a claim to the insurance company. Let T_2 be the time between the report of the claim and payment of the claim. The joint density function of T_1 and T_2, $f(t_1, t_2)$, is constant over the region $0 < t_1 < 6$, $0 < t_2 < 6$, $t_1 + t_2 < 10$, and zero otherwise.

Determine $E[T_1 + T_2]$, the expected time between a car accident and payment of the claim.

(A) 4.9

(B) 5.0

(C) 5.7

(D) 6.0

(E) 6.7

95. X and Y are independent random variables with common moment generating function $M(t) = e^{t^2/2}$.

Let $W = X + Y$ and $Z = Y - X$.

Determine the joint moment generating function, $M(t_1, t_2)$, of W and Z.

(A) $e^{2t_1^2+2t_2^2}$

(B) $e^{(t_1-t_2)^2}$

(C) $e^{(t_1+t_2)^2}$

(D) $e^{2t_1t_2}$

(E) $e^{t_1^2+t_2^2}$

96. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists.

Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist.

What is the expected revenue of the tour operator?

(A) 935

(B) 950

(C) 967

(D) 976

(E) 985

97. Let T_1 and T_2 represent the lifetimes in hours of two linked components in an electronic device. The joint density function for T_1 and T_2 is uniform over the region defined by $0 \le t_1 \le t_2 \le L$ where L is a positive constant.

Determine the expected value of the sum of the squares of T_1 and T_2 .

(A) $\frac{L^2}{3}$

(B) $\frac{L^2}{2}$

(C) $\frac{2L^2}{3}$

(D) $\frac{3L^2}{4}$

(E) L^2

98. Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} \frac{1}{3} & \text{for } x = 0 \\ \frac{2}{3} & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the moment generating function, M(t), of $Y = X_1X_2X_3$.

(A) $\frac{19}{27} + \frac{8}{27}e^t$

(B) $1 + 2e^t$

(C) $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$

(D) $\frac{1}{27} + \frac{8}{27}e^{3t}$

(E) $\frac{1}{3} + \frac{2}{3}e^{3t}$

99. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5000, the variance of Y is 10,000, and the variance of the total benefit, $X + Y$, is 17,000.

Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim.

Calculate the variance of the total benefit after these revisions have been made.

(A) 18,200

(B) 18,800

(C) 19,300

(D) 19,520

(E) 20,670

100. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let *X* denote the number of luxury cars sold in a given day, and let *Y* denote the number of extended warranties sold.

$$P(X = 0, Y = 0) = \frac{1}{6}$$

$$P(X = 1, Y = 0) = \frac{1}{12}$$

$$P(X = 1, Y = 1) = \frac{1}{6}$$

$$\mathrm{P}(X = 2, Y = 0) = \frac{1}{12}$$

$$P(X = 2, Y = 1) = \frac{1}{3}$$

$$P(X = 2, Y = 2) = \frac{1}{6}$$

What is the variance of *X*?

(A) 0.47

(B) 0.58

(C) 0.83

(D) 1.42

(E) 2.58

101. The profit for a new product is given by $Z = 3X - Y - 5$. X and Y are independent random variables with Var(X) = 1 and Var(Y) = 2.

What is the variance of Z?

(A) 1

(B) 5

(C) 7

(D) 11

(E) 16

102. A company has two electric generators. The time until failure for each generator follows an exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails.

What is the variance of the total time that the generators produce electricity?

(A) 10

(B) 20

(C) 50

(D) 100

(E) 200

103. In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be independent, exponentially distributed random variables with respective means 1.0, 1.5, and 2.4 .

Determine the probability that the maximum of these losses exceeds 3.

(A) 0.002

(B) 0.050

(C) 0.159

(D) 0.287

(E) 0.414

104. A joint density function is given by

$$f(x,y) = \begin{cases} kx & \text{for } 0 < x < 1, \quad 0 < y < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

What is Cov(X,Y) ?

(A) $-\frac{1}{6}$

(B) 0

(C) $\frac{1}{9}$

(D) $\frac{1}{6}$

(E) $\frac{2}{3}$

105. Let X and Y be continuous random variables with joint density function

$$f(x,y) = \begin{cases} \frac{8}{3}xy & \text{for } 0 \le x \le 1, \ x \le y \le 2x \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the covariance of X and Y.

(A) 0.04

(B) 0.25

(C) 0.67

(D) 0.80

(E) 1.24

106. Let X and Y denote the values of two stocks at the end of a five-year period. X is uniformly distributed on the interval $(0, 12)$. Given $X = x$, Y is uniformly distributed on the interval $(0, x)$.

Determine $\mathrm{Cov}(X, Y)$ according to this model.

(A) 0

(B) 4

(C) 6

(D) 12

(E) 24

107. Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which $\mathrm{E}(X) = 5$, $\mathrm{E}(X^2) = 27.4$, $\mathrm{E}(Y) = 7$, $\mathrm{E}(Y^2) = 51.4$, and $\mathrm{Var}(X+Y) = 8$.

Let $C_1 = X+Y$ denote the size of the combined claims before the application of a 20% surcharge on the hospital portion of the claim, and let C_2 denote the size of the combined claims after the application of that surcharge.

Calculate $\mathrm{Cov}(C_1, C_2)$.

(A) 8.80

(B) 9.60

(C) 9.76

(D) 11.52

(E) 12.32

108. A device containing two key components fails when, and only when, both components fail. The lifetimes, T_1 and T_2, of these components are independent with common density function $f(t) = e^{-t}$, $t > 0$. The cost, X, of operating the device until failure is $2T_1 + T_2$.

Which of the following is the density function of X for $x > 0$?

(A) $e^{-x/2} - e^{-x}$

(B) $2\left(e^{-x/2} - e^{-x}\right)$

(C) $\dfrac{x^2 e^{-x}}{2}$

(D) $\dfrac{e^{-x/2}}{2}$

(E) $\dfrac{e^{-x/3}}{3}$

109. A company offers earthquake insurance. Annual premiums are modeled by an exponential random variable with mean 2. Annual claims are modeled by an exponential random variable with mean 1. Premiums and claims are independent. Let *X* denote the ratio of claims to premiums.

What is the density function of *X*?

(A) $\dfrac{1}{2x+1}$

(B) $\dfrac{2}{(2x+1)^2}$

(C) e^{-x}

(D) $2e^{-2x}$

(E) xe^{-x}

110. Let *X* and *Y* be continuous random variables with joint density function

$$f\left(x,y\right)=\begin{cases}24xy & \text{for } 0<x<1 \text{ and } 0<y<1-x\\ 0 & \text{otherwise.}\end{cases}$$

Calculate $P\left[Y < X \middle| X = \frac{1}{3}\right]$.

(A) $\frac{1}{27}$

(B) $\frac{2}{27}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{4}{9}$

111. Once a fire is reported to a fire insurance company, the company makes an initial estimate, *X*, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, *Y*, to the claimant. The company has determined that *X* and *Y* have the joint density function

$$f(x, y) = \frac{2}{x^2(x-1)} y^{-(2x-1)/(x-1)} \quad x > 1, \ y > 1.$$

Given that the initial claim estimated by the company is 2, determine the probability that the final settlement amount is between 1 and 3 .

(A) $\frac{1}{9}$

(B) $\frac{2}{9}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

(E) $\frac{8}{9}$

112. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let X denote the proportion of employees who purchase the basic policy, and Y the proportion of employees who purchase the supplemental policy. Let X and Y have the joint density function $f(x,y) = 2(x + y)$ on the region where the density is positive.

Given that 10% of the employees buy the basic policy, what is the probability that fewer than 5% buy the supplemental policy?

(A) 0.010

(B) 0.013

(C) 0.108

(D) 0.417

(E) 0.500

113. Two life insurance policies, each with a death benefit of 10,000 and a one-time premium of 500, are sold to a couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96 .

What is the expected excess of premiums over claims, given that the husband survives at least ten years?

(A) 350

(B) 385

(C) 397

(D) 870

(E) 897

114. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by:

$$P(X = 0, Y = 0) = 0.800$$

$$P(X = 1, Y = 0) = 0.050$$

$$P(X = 0, Y = 1) = 0.025$$

$$P(X = 1, Y = 1) = 0.125$$

Calculate $\mathrm{Var}(Y \mid X = 1)$.

(A) 0.13

(B) 0.15

(C) 0.20

(D) 0.51

(E) 0.71

115. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x, y) = \begin{cases} 2x & \text{for } 0 < x < 1, \ \ x < y < x+1 \\ 0 & \text{otherwise.} \end{cases}$$

What is the conditional variance of Y given that $X = x$?

(A) $\frac{1}{12}$

(B) $\frac{7}{6}$

(C) $x + \frac{1}{2}$

(D) $x^2 - \frac{1}{6}$

(E) $x^2 + x + \frac{1}{3}$

116. An actuary determines that the annual numbers of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number of tornadoes in county P	0	0.12	0.06	0.05	0.02
	1	0.13	0.15	0.12	0.03
	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

(A) 0.51

(B) 0.84

(C) 0.88

(D) 0.99

(E) 1.76

117. A company is reviewing tornado damage claims under a farm insurance policy. Let *X* be the portion of a claim representing damage to the house and let *Y* be the portion of the same claim representing damage to the rest of the property. The joint density function of *X* and *Y* is

$$f(x, y) = \begin{cases} 6\left[1-(x+y)\right] & \text{for } x>0,\ y>0,\ x+y<1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the probability that the portion of a claim representing damage to the house is less than 0.2 .

(A) 0.360

(B) 0.480

(C) 0.488

(D) 0.512

(E) 0.520

118. Let X and Y be continuous random variables with joint density function

$$f(x,y)=\begin{cases}15y & \text{for } x^2\le y\le x\\ 0 & \text{otherwise.}\end{cases}$$

Let g be the marginal density function of Y.

Which of the following represents g?

(A) $g(y)=\begin{cases}15y & \text{for } 0<y<1\\ 0 & \text{otherwise}\end{cases}$

(B) $g(y)=\begin{cases}\dfrac{15y^2}{2} & \text{for } x^2<y<x\\ 0 & \text{otherwise}\end{cases}$

(C) $g(y)=\begin{cases}\dfrac{15y^2}{2} & \text{for } 0<y<1\\ 0 & \text{otherwise}\end{cases}$

(D) $g(y)=\begin{cases}15y^{3/2}\left(1-y^{1/2}\right) & \text{for } x^2<y<x\\ 0 & \text{otherwise}\end{cases}$

(E) $g(y)=\begin{cases}15y^{3/2}\left(1-y^{1/2}\right) & \text{for } 0<y<1\\ 0 & \text{otherwise}\end{cases}$

119. An auto insurance policy will pay for damage to both the policyholder's car and the other driver's car in the event that the policyholder is responsible for an accident. The size of the payment for damage to the policyholder's car, X, has a marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver's car, Y, has conditional density of 1 for $x < y < x + 1$.

If the policyholder is responsible for an accident, what is the probability that the payment for damage to the other driver's car will be greater than 0.500 ?

(A) $\frac{3}{8}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{7}{8}$

(E) $\frac{15}{16}$

120. An insurance policy is written to cover a loss X where X has density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$.

Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A) 0.17

(B) 0.25

(C) 0.32

(D) 0.58

(E) 0.83

121. Let X represent the age of an insured automobile involved in an accident. Let Y represent the length of time the owner has insured the automobile at the time of the accident.

X and Y have joint probability density function

$$f(x,y)=\begin{cases}\frac{1}{64}\left(10-xy^2\right) & \text{for } 2\le x\le 10 \text{ and } 0\le y\le 1\\ 0 & \text{otherwise.}\end{cases}$$

Calculate the expected age of an insured automobile involved in an accident.

(A) 4.9

(B) 5.2

(C) 5.8

(D) 6.0

(E) 6.4

122. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails.

Let X and Y be the times at which the first and second circuits fail, respectively. X and Y have joint probability density function

$$f(x,y)=\begin{cases}6e^{-x}e^{-2y} & \text{for } 0<x<y<\infty\\ 0 & \text{otherwise.}\end{cases}$$

What is the expected time at which the device fails?

(A) 0.33

(B) 0.50

(C) 0.67

(D) 0.83

(E) 1.50

123. You are given the following information about *N*, the annual number of claims for a randomly selected insured:

$$P(N=0)=\frac{1}{2}$$
$$P(N=1)=\frac{1}{3}$$
$$P(N>1)=\frac{1}{6}$$

Let *S* denote the total annual claim amount for an insured. When $N = 1$, *S* is exponentially distributed with mean 5 . When $N > 1$, *S* is exponentially distributed with mean 8 .

Determine $P(4 < S < 8)$.

(A) 0.04

(B) 0.08

(C) 0.12

(D) 0.24

(E) 0.25

124. The joint probability density for X and Y is

$$f(x,y)=\begin{cases}2e^{-(x+2y)}, & \text{for } x>0, y>0\\ 0, & \text{otherwise.}\end{cases}$$

Calculate the variance of Y given that $X > 3$ and $Y > 3$.

(A) 0.25

(B) 0.50

(C) 1.00

(D) 3.25

(E) 3.50

125. The distribution of Y, given X, is uniform on the interval $[0, X]$. The marginal density of X is

$$f(x)=\begin{cases}2x, & \text{for } 0<x<1\\ 0, & \text{otherwise.}\end{cases}$$

Determine the conditional density of X, given $Y = y$, where positive.

(A) 1

(B) 2

(C) $2x$

(D) $\dfrac{1}{y}$

(E) $\dfrac{1}{1-y}$

126. Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let p_n be the probability that a policyholder files *n* claims during a given year, where $n = 0, 1, 2, 3, 4, 5$. An actuary makes the following observations:

i) $p_n \geq p_{n+1}$ for $n = 0, 1, 2, 3, 4$.

ii) The difference between p_n and p_{n+1} is the same for $n = 0, 1, 2, 3, 4$.

iii) Exactly 40% of policyholders file fewer than two claims during a given year.

Calculate the probability that a random policyholder will file more than three claims during a given year.

(A) 0.14

(B) 0.16

(C) 0.27

(D) 0.29

(E) 0.33

127. Automobile losses reported to an insurance company are independent and uniformly distributed between 0 and 20,000. The company covers each such loss subject to a deductible of 5,000.

Calculate the probability that the total payout on 200 reported losses is between 1,000,000 and 1,200,000.

(A) 0.0803

(B) 0.1051

(C) 0.1799

(D) 0.8201

(E) 0.8575

SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM P PROBABILITY

EXAM P SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

P-09-07 **PRINTED IN U.S.A.**

1. Solution: D

Let

$G =$ event that a viewer watched gymnastics

$B =$ event that a viewer watched baseball

$S =$ event that a viewer watched soccer

Then we want to find

$$\Pr\left[\left(G \cup B \cup S\right)^c\right] = 1 - \Pr\left(G \cup B \cup S\right)$$

$$= 1 - \left[\Pr(G) + \Pr(B) + \Pr(S) - \Pr(G \cap B) - \Pr(G \cap S) - \Pr(B \cap S) + \Pr(G \cap B \cap S)\right]$$

$$= 1 - (0.28 + 0.29 + 0.19 - 0.14 - 0.10 - 0.12 + 0.08) = 1 - 0.48 = 0.52$$

----

2. Solution: A

Let R = event of referral to a specialist

L = event of lab work

We want to find

P[R∩L] = P[R] + P[L] – P[R∪L] = P[R] + P[L] – 1 + P[~(R∪L)]

= P[R] + P[L] – 1 + P[~R∩~L] = 0.30 + 0.40 – 1 + 0.35 = 0.05 .

----

3. Solution: D

First note

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$P[A \cup B'] = P[A] + P[B'] - P[A \cap B']$$

Then add these two equations to get

$$P[A \cup B] + P[A \cup B'] = 2P[A] + \left(P[B] + P[B']\right) - \left(P[A \cap B] + P[A \cap B']\right)$$

$$0.7 + 0.9 = 2P[A] + 1 - P\left[(A \cap B) \cup (A \cap B')\right]$$

$$1.6 = 2P[A] + 1 - P[A]$$

$$P[A] = 0.6$$

4. Solution: A

For $i = 1, 2$, let

R_i = event that a red ball is drawn form urn i

B_i = event that a blue ball is drawn from urn i .

Then if x is the number of blue balls in urn 2,

$$
\begin{aligned}
0.44 &= \Pr[(R_1 \cap R_2) \cup (B_1 \cap B_2)] = \Pr[R_1 \cap R_2] + \Pr[B_1 \cap B_2] \\
&= \Pr[R_1]\Pr[R_2] + \Pr[B_1]\Pr[B_2] \\
&= \frac{4}{10}\left(\frac{16}{x+16}\right) + \frac{6}{10}\left(\frac{x}{x+16}\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
2.2 &= \frac{32}{x+16} + \frac{3x}{x+16} = \frac{3x+32}{x+16} \\
2.2x + 35.2 &= 3x + 32 \\
0.8x &= 3.2 \\
x &= 4
\end{aligned}
$$

----

5. Solution: D

Let N(C) denote the number of policyholders in classification C . Then
N(Young ∩ Female ∩ Single) = N(Young ∩ Female) – N(Young ∩ Female ∩ Married)
= N(Young) – N(Young ∩ Male) – [N(Young ∩ Married) – N(Young ∩ Married ∩ Male)] = 3000 – 1320 – (1400 – 600) = 880 .

----

6. Solution: B

Let

H = event that a death is due to heart disease

F = event that at least one parent suffered from heart disease

Then based on the medical records,

$$P\left[H \cap F^c\right] = \frac{210-102}{937} = \frac{108}{937}$$

$$P\left[F^c\right] = \frac{937-312}{937} = \frac{625}{937}$$

and $P\left[H \mid F^c\right] = \dfrac{P\left[H \cap F^c\right]}{P\left[F^c\right]} = \dfrac{108}{937} \Big/ \dfrac{625}{937} = \dfrac{108}{625} = 0.173$

7. Solution: D

Let

A = event that a policyholder has an auto policy

H = event that a policyholder has a homeowners policy

Then based on the information given,

$$\Pr(A \cap H) = 0.15$$

$$\Pr(A \cap H^c) = \Pr(A) - \Pr(A \cap H) = 0.65 - 0.15 = 0.50$$

$$\Pr(A^c \cap H) = \Pr(H) - \Pr(A \cap H) = 0.50 - 0.15 = 0.35$$

and the portion of policyholders that will renew at least one policy is given by

$$0.4 \Pr(A \cap H^c) + 0.6 \Pr(A^c \cap H) + 0.8 \Pr(A \cap H)$$

$$= (0.4)(0.5) + (0.6)(0.35) + (0.8)(0.15) = 0.53 \qquad (= 53\%)$$

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8. Solution: D

Let

C = event that patient visits a chiropractor

T = event that patient visits a physical therapist

We are given that

$$\Pr[C] = \Pr[T] + 0.14$$

$$\Pr(C \cap T) = 0.22$$

$$\Pr(C^c \cap T^c) = 0.12$$

Therefore,

$$0.88 = 1 - \Pr[C^c \cap T^c] = \Pr[C \cup T] = \Pr[C] + \Pr[T] - \Pr[C \cap T]$$

$$= \Pr[T] + 0.14 + \Pr[T] - 0.22$$

$$= 2\Pr[T] - 0.08$$

or

$$\Pr[T] = (0.88 + 0.08)/2 = 0.48$$

9. Solution: B

Let

M = event that customer insures more than one car

S = event that customer insures a sports car

Then applying DeMorgan's Law, we may compute the desired probability as follows:

$$\Pr\left(M^c \cap S^c\right) = \Pr\left[\left(M \cup S\right)^c\right] = 1 - \Pr\left(M \cup S\right) = 1 - \left[\Pr\left(M\right) + \Pr\left(S\right) - \Pr\left(M \cap S\right)\right]$$

$$= 1 - \Pr\left(M\right) - \Pr\left(S\right) + \Pr\left(S \middle| M\right)\Pr\left(M\right) = 1 - 0.70 - 0.20 + \left(0.15\right)\left(0.70\right) = 0.205$$

----

10. Solution: C

Consider the following events about a randomly selected auto insurance customer:

A = customer insures more than one car

B = customer insures a sports car

We want to find the probability of the complement of A intersecting the complement of B (exactly one car, non-sports). But P ($A^c \cap B^c$) = 1 – P (A $\cup$ B)

And, by the Additive Law, P (A $\cup$ B) = P (A) + P (B) – P (A $\cap$ B).

By the Multiplicative Law, P (A $\cap$ B) = P (B | A) P (A) = 0.15 * 0.64 = 0.096

It follows that P (A $\cup$ B) = 0.64 + 0.20 – 0.096 = 0.744 and P ($A^c \cap B^c$) = 0.744 = 0.256

----

11. Solution: B

Let

C = Event that a policyholder buys collision coverage

D = Event that a policyholder buys disability coverage

Then we are given that P[C] = 2P[D] and P[C $\cap$ D] = 0.15 .

By the independence of C and D, it therefore follows that

0.15 = P[C $\cap$ D] = P[C] P[D] = 2P[D] P[D] = $2(P[D])^2$

$(P[D])^2$ = 0.15/2 = 0.075

P[D] = $\sqrt{0.075}$ and P[C] = 2P[D] = $2\sqrt{0.075}$

Now the independence of C and D also implies the independence of C^C and D^C . As a result, we see that $P[C^C \cap D^C] = P[C^C]\,P[D^C]$ = (1 – P[C]) (1 – P[D])

= $(1 - 2\sqrt{0.075})\,(1 - \sqrt{0.075})$ = 0.33 .

12. Solution: E

"Boxed" numbers in the table below were computed.

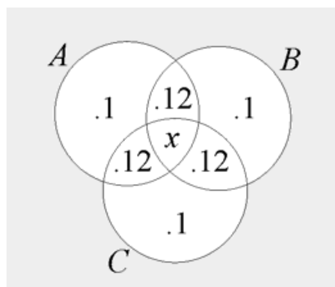
	High BP	Low BP	Norm BP	Total
Regular heartbeat	0.09	0.20	0.56	0.85
Irregular heartbeat	0.05	0.02	0.08	0.15
Total	0.14	0.22	0.64	1.00

From the table, we can see that 20% of patients have a regular heartbeat and low blood pressure.

----

13. Solution: C

The Venn diagram below summarizes the unconditional probabilities described in the problem.



In addition, we are told that

$$\frac{1}{3} = P[A \cap B \cap C \mid A \cap B] = \frac{P[A \cap B \cap C]}{P[A \cap B]} = \frac{x}{x + 0.12}$$

It follows that

$$x = \frac{1}{3}(x + 0.12) = \frac{1}{3}x + 0.04$$

$$\frac{2}{3}x = 0.04$$

$$x = 0.06$$

Now we want to find

$$\begin{aligned} P\left[(A \cup B \cup C)^c \mid A^c\right] &= \frac{P\left[(A \cup B \cup C)^c\right]}{P\left[A^c\right]} \\ &= \frac{1 - P[A \cup B \cup C]}{1 - P[A]} \\ &= \frac{1 - 3(0.10) - 3(0.12) - 0.06}{1 - 0.10 - 2(0.12) - 0.06} \\ &= \frac{0.28}{0.60} = 0.467 \end{aligned}$$

14. Solution: A

$$p_k = \frac{1}{5}p_{k-1} = \frac{1}{5}\frac{1}{5}p_{k-2} = \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}p_{k-3} = \ldots = \left(\frac{1}{5}\right)^k p_0 \quad k \ge 0$$

$$1 = \sum_{k=0}^{\infty} p_k = \sum_{k=0}^{\infty}\left(\frac{1}{5}\right)^k p_0 = \frac{p_0}{1-\frac{1}{5}} = \frac{5}{4}p_0$$

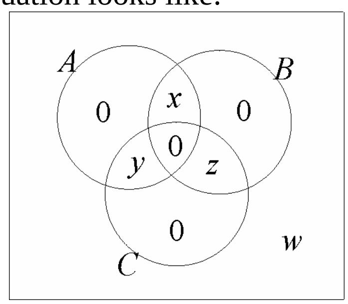
$p_0 = 4/5$.

Therefore, P[N > 1] = 1 – P[N ≤1] = 1 – (4/5 + 4/5 · 1/5) = 1 – 24/25 = 1/25 = 0.04 .

----

15. Solution: C

A Venn diagram for this situation looks like:



We want to find $w = 1-(x+y+z)$

We have $x+y=\frac{1}{4},\quad x+z=\frac{1}{3},\quad y+z=\frac{5}{12}$

Adding these three equations gives

$$(x+y)+(x+z)+(y+z) = \frac{1}{4}+\frac{1}{3}+\frac{5}{12}$$

$$2(x+y+z)=1$$

$$x+y+z=\frac{1}{2}$$

$$w = 1-(x+y+z) = 1-\frac{1}{2} = \frac{1}{2}$$

Alternatively the three equations can be solved to give x = 1/12, y = 1/6, z =1/4

again leading to $w = 1-\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{4}\right) = \frac{1}{2}$

16. Solution: D

Let N_1 and N_2 denote the number of claims during weeks one and two, respectively. Then since N_1 and N_2 are independent,

$$\Pr[N_1 + N_2 = 7] = \sum_{n=0}^{7} \Pr[N_1 = n]\Pr[N_2 = 7-n]$$

$$= \sum_{n=0}^{7}\left(\frac{1}{2^{n+1}}\right)\left(\frac{1}{2^{8-n}}\right)$$

$$= \sum_{n=0}^{7}\frac{1}{2^9}$$

$$= \frac{8}{2^9} = \frac{1}{2^6} = \frac{1}{64}$$

----

17. Solution: D

Let

O = Event of operating room charges

E = Event of emergency room charges

Then

$$0.85 = \Pr(O \cup E) = \Pr(O) + \Pr(E) - \Pr(O \cap E)$$

$$= \Pr(O) + \Pr(E) - \Pr(O)\Pr(E) \quad (\text{Independence})$$

Since $\Pr(E^c) = 0.25 = 1 - \Pr(E)$, it follows $\Pr(E) = 0.75$.

So $0.85 = \Pr(O) + 0.75 - \Pr(O)(0.75)$

$$\Pr(O)(1-0.75) = 0.10$$

$$\Pr(O) = 0.40$$

----

18. Solution: D

Let X_1 and X_2 denote the measurement errors of the less and more accurate instruments, respectively. If $N(\mu,\sigma)$ denotes a normal random variable with mean μ and standard deviation σ, then we are given X_1 is N(0, 0.0056h), X_2 is N(0, 0.0044h) and X_1, X_2 are independent. It follows that Y = $\frac{X_1 + X_2}{2}$ is N (0, $\sqrt{\frac{0.0056^2h^2 + 0.0044^2h^2}{4}}$) = N(0, 0.00356h) . Therefore, P[–0.005h ≤ Y ≤ 0.005h] = P[Y ≤ 0.005h] – P[Y ≤ –0.005h] = P[Y ≤ 0.005h] – P[Y ≥ 0.005h]

= 2P[Y ≤ 0.005h] – 1 = 2P$\left[Z \le \frac{0.005h}{0.00356h}\right]$ – 1 = 2P[Z ≤ 1.4] – 1 = 2(0.9192) – 1 = 0.84.

19. Solution: B

Apply Bayes' Formula. Let

$A =$ Event of an accident

$B_1 =$ Event the driver's age is in the range 16-20

$B_2 =$ Event the driver's age is in the range 21-30

$B_3 =$ Event the driver's age is in the range 30-65

$B_4 =$ Event the driver's age is in the range 66-99

Then

$$\Pr\left(B_1 \middle| A\right) = \frac{\Pr\left(A \middle| B_1\right)\Pr\left(B_1\right)}{\Pr\left(A \middle| B_1\right)\Pr\left(B_1\right) + \Pr\left(A \middle| B_2\right)\Pr\left(B_2\right) + \Pr\left(A \middle| B_3\right)\Pr\left(B_3\right) + \Pr\left(A \middle| B_4\right)\Pr\left(B_4\right)}$$

$$= \frac{(0.06)(0.08)}{(0.06)(0.08) + (0.03)(0.15) + (0.02)(0.49) + (0.04)(0.28)} = 0.1584$$

----

20. Solution: D

Let

S = Event of a standard policy

F = Event of a preferred policy

U = Event of an ultra-preferred policy

D = Event that a policyholder dies

Then

$$P[U \mid D] = \frac{P[D \mid U]P[U]}{P[D \mid S]P[S] + P[D \mid F]P[F] + P[D \mid U]P[U]}$$

$$= \frac{(0.001)(0.10)}{(0.01)(0.50) + (0.005)(0.40) + (0.001)(0.10)}$$

$$= 0.0141$$

----

21. Solution: B

Apply Baye's Formula:

$$\Pr\left[\text{Seri.} \middle| \text{Surv.}\right]$$

$$= \frac{\Pr\left[\text{Surv.} \middle| \text{Seri.}\right]\Pr\left[\text{Seri.}\right]}{\Pr\left[\text{Surv.} \middle| \text{Crit.}\right]\Pr\left[\text{Crit.}\right] + \Pr\left[\text{Surv.} \middle| \text{Seri.}\right]\Pr\left[\text{Seri.}\right] + \Pr\left[\text{Surv.} \middle| \text{Stab.}\right]\Pr\left[\text{Stab.}\right]}$$

$$= \frac{(0.9)(0.3)}{(0.6)(0.1) + (0.9)(0.3) + (0.99)(0.6)} = 0.29$$

22. Solution: D

Let

H = Event of a heavy smoker

L = Event of a light smoker

N = Event of a non-smoker

D = Event of a death within five-year period

Now we are given that $\Pr\left[D|L\right] = 2\ \Pr\left[D|N\right]$ and $\Pr\left[D|L\right] = \frac{1}{2}\Pr\left[D|H\right]$

Therefore, upon applying Bayes' Formula, we find that

$$\Pr\left[H|D\right] = \frac{\Pr\left[D|H\right]\Pr\left[H\right]}{\Pr\left[D|N\right]\Pr\left[N\right] + \Pr\left[D|L\right]\Pr\left[L\right] + \Pr\left[D|H\right]\Pr\left[H\right]}$$

$$= \frac{2\Pr\left[D|L\right](0.2)}{\frac{1}{2}\Pr\left[D|L\right](0.5) + \Pr\left[D|L\right](0.3) + 2\Pr\left[D|L\right](0.2)} = \frac{0.4}{0.25 + 0.3 + 0.4} = 0.42$$

---

23. Solution: D

Let

C = Event of a collision

T = Event of a teen driver

Y = Event of a young adult driver

M = Event of a midlife driver

S = Event of a senior driver

Then using Bayes' Theorem, we see that

$$P[Y|C] = \frac{P[C|Y]P[Y]}{P[C|T]P[T] + P[C|Y]P[Y] + P[C|M]P[M] + P[C|S]P[S]}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08) + (0.08)(0.16) + (0.04)(0.45) + (0.05)(0.31)} = 0.22\ .$$

---

24. Solution: B

Observe

$$\Pr\left[N \ge 1|N \le 4\right] = \frac{\Pr\left[1 \le N \le 4\right]}{\Pr\left[N \le 4\right]} = \left[\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right] \Big/ \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}\right]$$

$$= \frac{10 + 5 + 3 + 2}{30 + 10 + 5 + 3 + 2} = \frac{20}{50} = \frac{2}{5}$$

25. Solution: B

Let Y = positive test result

D = disease is present (and ~D = not D)

Using Baye's theorem:

$$P[D|Y] = \frac{P[Y \mid D]P[D]}{P[Y \mid D]P[D] + P[Y \mid \sim D]P[\sim D]} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.005)(0.99)} = 0.657 .$$

---

26. Solution: C

Let:

S = Event of a smoker

C = Event of a circulation problem

Then we are given that P[C] = 0.25 and $P[S \mid C] = 2\,P[S \mid C^C]$

Now applying Bayes' Theorem, we find that $P[C \mid S] = \dfrac{P[S \mid C]P[C]}{P[S \mid C]P[C] + P[S \mid C^C](P[C^C])}$

$$= \frac{2P[S \mid C^C]P[C]}{2P[S \mid C^C]P[C] + P[S \mid C^C](1 - P[C])} = \frac{2(0.25)}{2(0.25) + 0.75} = \frac{2}{2+3} = \frac{2}{5} .$$

---

27. Solution: D

Use Baye's Theorem with A = the event of an accident in one of the years 1997, 1998 or 1999.

$$P[1997|A] = \frac{P[A \mid 1997]P[1997]}{P[A \mid 1997][P[1997] + P[A \mid 1998]P[1998] + P[A \mid 1999]P[1999]}$$

$$= \frac{(0.05)(0.16)}{(0.05)(0.16) + (0.02)(0.18) + (0.03)(0.20)} = 0.45 .$$

---

28. Solution: A

Let

C = Event that shipment came from Company X

I_1 = Event that one of the vaccine vials tested is ineffective

Then by Bayes' Formula, $P[C \mid I_1] = \dfrac{P[I_1 \mid C]P[C]}{P[I_1 \mid C]P[C] + P[I_1 \mid C^c]P[C^c]}$

Now

$$P[C] = \frac{1}{5}$$

$$P[C^c] = 1 - P[C] = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P[I_1 \mid C] = \binom{30}{1}(0.10)(0.90)^{29} = 0.141$$

$$P[I_1 \mid C^c] = \binom{30}{1}(0.02)(0.98)^{29} = 0.334$$

Therefore,

$$P[C \mid I_1] = \frac{(0.141)(1/5)}{(0.141)(1/5) + (0.334)(4/5)} = 0.096$$

---

29. Solution: C

Let T denote the number of days that elapse before a high-risk driver is involved in an accident. Then T is exponentially distributed with unknown parameter λ . Now we are given that

$0.3 = P[T \le 50] = \int_0^{50} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_0^{50} = 1 - e^{-50\lambda}$

Therefore, $e^{-50\lambda} = 0.7$ or $\lambda = -(1/50)\ln(0.7)$

It follows that $P[T \le 80] = \int_0^{80} \lambda e^{-\lambda t} dt = -e^{-\lambda t} \Big|_0^{80} = 1 - e^{-80\lambda}$

$= 1 - e^{(80/50)\ln(0.7)} = 1 - (0.7)^{80/50} = 0.435$.

---

30. Solution: D

Let N be the number of claims filed. We are given $P[N = 2] = \dfrac{e^{-\lambda}\lambda^2}{2!} = 3\dfrac{e^{-\lambda}\lambda^4}{4!} = 3 \cdot P[N$

$= 4]24\,\lambda^2 = 6\,\lambda^4$

$\lambda^2 = 4 \Rightarrow \lambda = 2$

Therefore, $Var[N] = \lambda = 2$.

31. Solution: D

Let X denote the number of employees that achieve the high performance level. Then X follows a binomial distribution with parameters $n = 20$ and $p = 0.02$. Now we want to determine x such that

$$\Pr[X > x] \le 0.01$$

or, equivalently,

$$0.99 \le \Pr[X \le x] = \sum_{k=0}^{x} \binom{20}{k} (0.02)^k (0.98)^{20-k}$$

The following table summarizes the selection process for x:

x	$\Pr[X = x]$	$\Pr[X \le x]$
0	$(0.98)^{20} = 0.668$	0.668
1	$20(0.02)(0.98)^{19} = 0.272$	0.940
2	$190(0.02)^2(0.98)^{18} = 0.053$	0.993

Consequently, there is less than a 1% chance that more than two employees will achieve the high performance level. We conclude that we should choose the payment amount C such that

$$2C = 120,000$$

or

$$C = 60,000$$

----

32. Solution: D

Let

X = number of low-risk drivers insured
Y = number of moderate-risk drivers insured
Z = number of high-risk drivers insured
$f(x, y, z)$ = probability function of X, Y, and Z

Then f is a trinomial probability function, so

$$\Pr[z \ge x + 2] = f(0,0,4) + f(1,0,3) + f(0,1,3) + f(0,2,2)$$

$$= (0.20)^4 + 4(0.50)(0.20)^3 + 4(0.30)(0.20)^3 + \frac{4!}{2!2!}(0.30)^2(0.20)^2$$

$$= 0.0488$$

33. Solution: B

Note that

$$\Pr\left[X > x\right] = \int_x^{20} 0.005\left(20 - t\right) dt = 0.005\left(20t - \frac{1}{2}t^2\right)\Big|_x^{20}$$

$$= 0.005\left(400 - 200 - 20x + \frac{1}{2}x^2\right) = 0.005\left(200 - 20x + \frac{1}{2}x^2\right)$$

where $0 < x < 20$. Therefore,

$$\Pr\left[X > 16 \middle| X > 8\right] = \frac{\Pr\left[X > 16\right]}{\Pr\left[X > 8\right]} = \frac{200 - 20(16) + \frac{1}{2}(16)^2}{200 - 20(8) + \frac{1}{2}(8)^2} = \frac{8}{72} = \frac{1}{9}$$

----

34. Solution: C

We know the density has the form $C\left(10 + x\right)^{-2}$ for $0 < x < 40$ (equals zero otherwise).

First, determine the proportionality constant C from the condition $\int_0^{40} f\,(x)dx = 1$:

$$1 = \int_0^{40} C\left(10 + x\right)^{-2} dx = -C(10 + x)^{-1}\Big|_0^{40} = \frac{C}{10} - \frac{C}{50} = \frac{2}{25}C$$

so $C = 25/2$, or 12.5 . Then, calculate the probability over the interval (0, 6):

$$12.5\int_0^6 \left(10 + x\right)^{-2} dx = -\left(10 + x\right)^{-1}\Big|_0^6 = \left(\frac{1}{10} - \frac{1}{16}\right)\left(12.5\right) = 0.47 \ .$$

----

35. Solution: C

Let the random variable T be the future lifetime of a 30-year-old. We know that the density of T has the form $f\,(x) = C(10 + x)^{-2}$ for $0 < x < 40$ (and it is equal to zero otherwise). First, determine the proportionality constant C from the condition $\int_0^{40} f\,(x)dx = 1$:

$$1 = \int_0^{40} f\,(x)dx = -C(10 + x)^{-1}\,\Big|_0^{40} = \frac{2}{25}C$$

so that $C = \dfrac{25}{2} = 12.5$. Then, calculate $P(T < 5)$ by integrating $f\,(x) = 12.5\,(10 + x)^{-2}$ over the interval (0.5).

36. Solution: B

To determine k, note that

$$1 = \int_0^1 k(1-y)^4\,dy = -\frac{k}{5}(1-y)^5\Big|_0^1 = \frac{k}{5}$$

k = 5

We next need to find P[V > 10,000] = P[100,000 Y > 10,000] = P[Y > 0.1]

$$= \int_{0.1}^1 5(1-y)^4 dy = -(1-y)^5\Big|_{0.1}^1 = (0.9)^5 = 0.59$$ and P[V > 40,000]

$$= P[100{,}000\ Y > 40{,}000] = P[Y > 0.4] = \int_{0.4}^1 5(1-y)^4 dy = -(1-y)^5\Big|_{0.4}^1 = (0.6)^5 = 0.078\ .$$

It now follows that P[V > 40,000 | V > 10,000]

$$= \frac{P[V > 40{,}000 \cap V > 10{,}000]}{P[V > 10{,}000]} = \frac{P[V > 40{,}000]}{P[V > 10{,}000]} = \frac{0.078}{0.590} = 0.132\ .$$

---

37. Solution: D

Let T denote printer lifetime. Then f(t) = ½ $e^{-t/2}$, 0 ≤ t ≤

Note that

$$P[T \le 1] = \int_0^1 \frac{1}{2}e^{-t/2}dt = e^{-t/2}\Big|_0^1 = 1 - e^{-1/2} = 0.393$$

$$P[1 \le T \le 2] = \int_1^2 \frac{1}{2}e^{-t/2}dt = e^{-t/2}\Big|_1^2 = e^{-1/2} - e^{-1} = 0.239$$

Next, denote refunds for the 100 printers sold by independent and identically distributed random variables $Y_1, \ldots, Y_{100}$ where

$$Y_i = \begin{cases} 200 & \text{with probability } 0.393 \\ 100 & \text{with probability } 0.239 \\ 0 & \text{with probability } 0.368 \end{cases} \qquad i = 1, \ldots, 100$$

Now $E[Y_i]$ = 200(0.393) + 100(0.239) = 102.56

Therefore, Expected Refunds = $\sum_{i=1}^{100} E[Y_i]$ = 100(102.56) = 10,256 .

38. Solution: A

Let F denote the distribution function of f. Then

$$F(x) = \Pr[X \le x] = \int_1^x 3t^{-4} dt = -t^{-3}\Big|_1^x = 1 - x^{-3}$$

Using this result, we see

$$\Pr[X < 2 | X \ge 1.5] = \frac{\Pr[(X < 2) \cap (X \ge 1.5)]}{\Pr[X \ge 1.5]} = \frac{\Pr[X < 2] - \Pr[X \le 1.5]}{\Pr[X \ge 1.5]}$$

$$= \frac{F(2) - F(1.5)}{1 - F(1.5)} = \frac{(1.5)^{-3} - (2)^{-3}}{(1.5)^{-3}} = 1 - \left(\frac{3}{4}\right)^3 = 0.578$$

---

39. Solution: E

Let X be the number of hurricanes over the 20-year period. The conditions of the problem give x is a binomial distribution with n = 20 and p = 0.05 . It follows that

$P[X < 2] = (0.95)^{20}(0.05)^0 + 20(0.95)^{19}(0.05) + 190(0.95)^{18}(0.05)^2$

$= 0.358 + 0.377 + 0.189 = 0.925$.

---

40. Solution: B

Denote the insurance payment by the random variable Y. Then

$$Y = \begin{cases} 0 & \text{if} \quad 0 < X \le C \\ X - C & \text{if} \quad C < X < 1 \end{cases}$$

Now we are given that

$$0.64 = \Pr(Y < 0.5) = \Pr(0 < X < 0.5 + C) = \int_0^{0.5+C} 2x \, dx = x^2\Big|_0^{0.5+C} = (0.5 + C)^2$$

Therefore, solving for C, we find $C = \pm 0.8 - 0.5$

Finally, since $0 < C < 1$, we conclude that $C = 0.3$

41. Solution: E

Let

X = number of group 1 participants that complete the study.

Y = number of group 2 participants that complete the study.

Now we are given that X and Y are independent.

Therefore,

$$P\left\{\left[(X \geq 9)\cap(Y<9)\right]\cup\left[(X<9)\cap(Y \geq 9)\right]\right\}$$
$$= P\left[(X \geq 9)\cap(Y<9)\right] + P\left[(X<9)\cap(Y\geq 9)\right]$$
$$= 2P\left[(X \geq 9)\cap(Y<9)\right] \qquad \text{(due to symmetry)}$$
$$= 2P\left[X \geq 9\right]P\left[Y<9\right]$$
$$= 2P\left[X \geq 9\right]P\left[X<9\right] \qquad \text{(again due to symmetry)}$$
$$= 2P\left[X \geq 9\right]\left(1-P\left[X \geq 9\right]\right)$$
$$= 2\left[\binom{10}{9}(0.2)(0.8)^9 + \binom{10}{10}(0.8)^{10}\right]\left[1-\binom{10}{9}(0.2)(0.8)^9 - \binom{10}{10}(0.8)^{10}\right]$$
$$= 2\left[0.376\right]\left[1-0.376\right] = 0.469$$

----

42. Solution: D

Let

I_A = Event that Company A makes a claim

I_B = Event that Company B makes a claim

X_A = Expense paid to Company A if claims are made

X_B = Expense paid to Company B if claims are made

Then we want to find

$$\Pr\left\{\left[I_A^C \cap I_B\right]\cup\left[(I_A\cap I_B)\cap(X_A < X_B)\right]\right\}$$
$$= \Pr\left[I_A^C \cap I_B\right] + \Pr\left[(I_A\cap I_B)\cap(X_A<X_B)\right]$$
$$= \Pr\left[I_A^C\right]\Pr\left[I_B\right] + \Pr\left[I_A\right]\Pr\left[I_B\right]\Pr\left[X_A<X_B\right] \qquad \text{(independence)}$$
$$= (0.60)(0.30) + (0.40)(0.30)\Pr\left[X_B - X_A \geq 0\right]$$
$$= 0.18 + 0.12\Pr\left[X_B - X_A \geq 0\right]$$

Now $X_B - X_A$ is a linear combination of independent normal random variables.

Therefore, $X_B - X_A$ is also a normal random variable with mean

$$M = E\left[X_B - X_A\right] = E\left[X_B\right] - E\left[X_A\right] = 9,000 - 10,000 = -1,000$$

and standard deviation $\sigma = \sqrt{\operatorname{Var}(X_B) + \operatorname{Var}(X_A)} = \sqrt{(2000)^2 + (2000)^2} = 2000\sqrt{2}$

It follows that

$$
\begin{aligned}
\Pr\left[X_B - X_A \ge 0\right] &= \Pr\left[Z \ge \frac{1000}{2000\sqrt{2}}\right] \qquad (Z \text{ is standard normal}) \\
&= \Pr\left[Z \ge \frac{1}{2\sqrt{2}}\right] \\
&= 1 - \Pr\left[Z < \frac{1}{2\sqrt{2}}\right] \\
&= 1 - \Pr\left[Z < 0.354\right] \\
&= 1 - 0.638 = 0.362
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\Pr\left\{\left[I_A^C \cap I_B\right] \cup \left[\left(I_A \cap I_B\right) \cap \left(X_A < X_B\right)\right]\right\} &= 0.18 + (0.12)(0.362) \\
&= 0.223
\end{aligned}
$$

----

43. Solution: D

If a month with one or more accidents is regarded as success and k = the number of failures before the fourth success, then k follows a negative binomial distribution and the requested probability is

$$
\begin{aligned}
\Pr\left[k \ge 4\right] = 1 - \Pr\left[k \le 3\right] &= 1 - \sum_{k=0}^{3} \binom{3+k}{k} \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^k \\
&= 1 - \left(\frac{3}{5}\right)^4 \left[\binom{3}{0}\left(\frac{2}{5}\right)^0 + \binom{4}{1}\left(\frac{2}{5}\right)^1 + \binom{5}{2}\left(\frac{2}{5}\right)^2 + \binom{6}{3}\left(\frac{2}{5}\right)^3\right] \\
&= 1 - \left(\frac{3}{5}\right)^4 \left[1 + \frac{8}{5} + \frac{8}{5} + \frac{32}{25}\right] \\
&= 0.2898
\end{aligned}
$$

Alternatively the solution is

$$
\left(\frac{2}{5}\right)^4 + \binom{4}{1}\left(\frac{2}{5}\right)^4 \frac{3}{5} + \binom{5}{2}\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + \binom{6}{3}\left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^3 = 0.2898
$$

which can be derived directly or by regarding the problem as a negative binomial distribution with

i) success taken as a month with no accidents
ii) k = the number of failures before the fourth success, and
iii) calculating $\Pr\left[k \le 3\right]$

44. Solution: C

If k is the number of days of hospitalization, then the insurance payment $g(k)$ is

$$g(k) = \begin{cases} 100k & \text{for } k = 1,\ 2,\ 3 \\ 300 + 50(k-3) & \text{for } k = 4,\ 5. \end{cases}$$

Thus, the expected payment is $\sum_{k=1}^{5} g(k)\, p_k = 100\, p_1 + 200\, p_2 + 300\, p_3 + 350\, p_4 + 400\, p_5$ =

$\frac{1}{15}\left(100\times 5 + 200\times 4 + 300\times 3 + 350\times 2 + 400\times 1\right) = 220$

----

45. Solution: D

Note that $E\left(X\right) = \int_{-2}^{0} -\frac{x^2}{10}dx + \int_{0}^{4} \frac{x^2}{10}dx = -\left.\frac{x^3}{30}\right|_{-2}^{0} + \left.\frac{x^3}{30}\right|_{0}^{4} = -\frac{8}{30} + \frac{64}{30} = \frac{56}{30} = \frac{28}{15}$

----

46. Solution: D

The density function of T is

$$f\left(t\right) = \frac{1}{3}e^{-t/3} \quad , \quad 0 < t < \infty$$

Therefore,

$$\begin{aligned} E\left[X\right] &= E\left[\max\left(T, 2\right)\right] \\ &= \int_0^2 \frac{2}{3}e^{-t/3}dt + \int_2^\infty \frac{t}{3}e^{-t/3}dt \\ &= -2e^{-t/3}\Big|_0^2 - te^{-t/3}\Big|_2^\infty + \int_2^\infty e^{-t/3}dt \\ &= -2e^{-2/3} + 2 + 2e^{-2/3} - 3e^{-t/3}\Big|_2^\infty \\ &= 2 + 3e^{-2/3} \end{aligned}$$

47. Solution: D

Let T be the time from purchase until failure of the equipment. We are given that T is exponentially distributed with parameter λ = 10 since 10 = E[T] = λ . Next define the payment

P under the insurance contract by $P = \begin{cases} x & \text{for } 0 \le T \le 1 \\ \frac{x}{2} & \text{for } 1 < T \le 3 \\ 0 & \text{for } T > 3 \end{cases}$

We want to find x such that

$1000 = E[P] = \int_0^1 \frac{x}{10} e^{-t/10}\, dt + \int_1^3 \frac{x}{2}\frac{1}{10} e^{-t/10}\, dt = -xe^{-t/10}\Big|_0^1 - \frac{x}{2}e^{-t/10}\Big|_1^3$

$= -x e^{-1/10} + x - (x/2) e^{-3/10} + (x/2) e^{-1/10} = x(1 - ½ e^{-1/10} - ½ e^{-3/10}) = 0.1772x$.

We conclude that x = 5644 .

----

48. Solution: E

Let X and Y denote the year the device fails and the benefit amount, respectively. Then the density function of X is given by

$$f(x) = (0.6)^{x-1}(0.4) \quad , \quad x = 1, 2, 3...$$

and

$$y = \begin{cases} 1000(5-x) & \text{if} \quad x = 1, 2, 3, 4 \\ 0 & \text{if} \quad x > 4 \end{cases}$$

It follows that

$$E[Y] = 4000(0.4) + 3000(0.6)(0.4) + 2000(0.6)^2(0.4) + 1000(0.6)^3(0.4)$$
$$= 2694$$

----

49. Solution: D

Define $f(X)$ to be hospitalization payments made by the insurance policy. Then

$$f(X) = \begin{cases} 100X & \text{if} \quad X = 1, 2, 3 \\ 300 + 25(X-3) & \text{if} \quad X = 4, 5 \end{cases}$$

and

$$E\left[f(X)\right]=\sum_{k=1}^{5} f(k)\Pr\left[X=k\right]$$
$$=100\left(\frac{5}{15}\right)+200\left(\frac{4}{15}\right)+300\left(\frac{3}{15}\right)+325\left(\frac{2}{15}\right)+350\left(\frac{1}{15}\right)$$
$$=\frac{1}{3}\left[100+160+180+130+70\right]=\frac{640}{3}=213.33$$

---

50. Solution: C

Let N be the number of major snowstorms per year, and let P be the amount paid to the company under the policy. Then Pr[N = n] = $\frac{(3/2)^n e^{-3/2}}{n!}$, n = 0, 1, 2, . . . and

$$P=\begin{cases}0 & \text{for } N=0\\ 10,000(N-1) & \text{for } N\geq 1\end{cases}.$$

Now observe that E[P] = $\sum_{n=1}^{\infty}10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}$

= 10,000 $e^{-3/2}$ + $\sum_{n=0}^{\infty}10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}$ = 10,000 $e^{-3/2}$ + E[10,000 (N – 1)]

= 10,000 $e^{-3/2}$ + E[10,000N] – E[10,000] = 10,000 $e^{-3/2}$ + 10,000 (3/2) – 10,000 = 7,231 .

---

51. Solution: C

Let Y denote the manufacturer's retained annual losses.

Then $Y=\begin{cases}x & \text{for } 0.6<x\leq 2\\ 2 & \text{for } x>2\end{cases}$

and E[Y] = $\int_{0.6}^{2} x\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx+\int_{2}^{\infty}2\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx=\int_{0.6}^{2}\frac{2.5(0.6)^{2.5}}{x^{2.5}}dx-\frac{2(0.6)^{2.5}}{x^{2.5}}\Big|_{2}^{\infty}$

= $-\frac{2.5(0.6)^{2.5}}{1.5x^{1.5}}\Big|_{0.6}^{2}+\frac{2(0.6)^{2.5}}{(2)^{2.5}}=-\frac{2.5(0.6)^{2.5}}{1.5(2)^{1.5}}+\frac{2.5(0.6)^{2.5}}{1.5(0.6)^{1.5}}+\frac{(0.6)^{2.5}}{2^{1.5}}$ = 0.9343 .

52. Solution: A

Let us first determine K. Observe that

$$1 = K\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = K\left(\frac{60+30+20+15+12}{60}\right) = K\left(\frac{137}{60}\right)$$

$$K = \frac{60}{137}$$

It then follows that

$$\Pr[N=n] = \Pr[N=n \mid \text{Insured Suffers a Loss}]\Pr[\text{Insured Suffers a Loss}]$$

$$= \frac{60}{137N}(0.05) = \frac{3}{137N} \quad , \quad N = 1,\ldots,5$$

Now because of the deductible of 2, the net annual premium $P = E[X]$ where

$$X = \begin{cases} 0 & , \text{ if } \quad N \le 2 \\ N-2 & , \text{ if } \quad N > 2 \end{cases}$$

Then,

$$P = E[X] = \sum_{N=3}^{5}(N-2)\frac{3}{137N} = (1)\left(\frac{1}{137}\right) + 2\left[\frac{3}{137(4)}\right] + 3\left[\frac{3}{137(5)}\right] = 0.0314$$

----

53. Solution: D

Let W denote claim payments. Then $W = \begin{cases} y & \text{for } 1 < y \le 10 \\ 10 & \text{for } y \ge 10 \end{cases}$

It follows that E[W] = $\int_1^{10} y\frac{2}{y^3}dy + \int_{10}^{\infty} 10\frac{2}{y^3}dy = -\frac{2}{y}\Big|_1^{10} - \frac{10}{y^2}\Big|_{10}^{\infty}$ = 2 – 2/10 + 1/10 = 1.9 .

54. Solution: B

Let Y denote the claim payment made by the insurance company.

Then

$$Y = \begin{cases} 0 & \text{with probability } 0.94 \\ \text{Max } (0, x-1) & \text{with probability } 0.04 \\ 14 & \text{with probability } 0.02 \end{cases}$$

and

$$\begin{aligned} E[Y] &= (0.94)(0)+(0.04)(0.5003)\int_1^{15}(x-1)e^{-x/2}dx+(0.02)(14) \\ &= (0.020012)\left[\int_1^{15} xe^{-x/2}dx - \int_1^{15} e^{-x/2}dx\right]+0.28 \\ &= 0.28+(0.020012)\left[-2xe^{-x/2}\Big|_1^{15} +2\int_1^{15} e^{-x/2}dx - \int_1^{15} e^{-x/2}dx\right] \\ &= 0.28+(0.020012)\left[-30e^{-7.5}+2e^{-0.5}+\int_1^{15} e^{-x/2}dx\right] \\ &= 0.28+(0.020012)\left[-30e^{-7.5}+2e^{-0.5}-2e^{-x/2}\Big|_1^{15}\right] \\ &= 0.28+(0.020012)\left(-30e^{-7.5}+2e^{-0.5}-2e^{-7.5}+2e^{-0.5}\right) \\ &= 0.28+(0.020012)\left(-32e^{-7.5}+4e^{-0.5}\right) \\ &= 0.28+(0.020012)(2.408) \\ &= 0.328 \quad \text{(in thousands)} \end{aligned}$$

It follows that the expected claim payment is 328 .

---

55. Solution: C

The pdf of x is given by f(x) = $\frac{k}{(1+x)^4}$, 0 < x < ∞ . To find k, note

$$1 = \int_0^\infty \frac{k}{(1+x)^4}dx = -\frac{k}{3}\frac{1}{(1+x)^3}\Big|_0^\infty = \frac{k}{3}$$

k = 3

It then follows that E[x] = $\int_0^\infty \frac{3x}{(1+x)^4}dx$ and substituting u = 1 + x, du = dx, we see

$$\text{E[x]} = \int_1^\infty \frac{3(u-1)}{u^4}du = 3\int_1^\infty (u^{-3}-u^{-4})du = 3\left[\frac{u^{-2}}{-2}-\frac{u^{-3}}{-3}\right]_1^\infty = 3\left[\frac{1}{2}-\frac{1}{3}\right] = 3/2 - 1 = ½ .$$

56. Solution: C

Let Y represent the payment made to the policyholder for a loss subject to a deductible D.

That is $Y = \begin{cases} 0 & \text{for } 0 \le X \le D \\ x - D & \text{for } D < X \le 1 \end{cases}$

Then since E[X] = 500, we want to choose D so that

$$\frac{1}{4}500 = \int_D^{1000} \frac{1}{1000}(x-D)dx = \frac{1}{1000}\frac{(x-D)^2}{2}\Bigg|_D^{1000} = \frac{(1000-D)^2}{2000}$$

$(1000 - D)^2 = 2000/4 \cdot 500 = 500^2$

$1000 - D = \pm 500$

D = 500 (or D = 1500 which is extraneous).

----

57. Solution: B

We are given that $M_x(t) = \dfrac{1}{(1-2500t)^4}$ for the claim size X in a certain class of accidents.

First, compute $M_x'(t) = \dfrac{(-4)(-2500)}{(1-2500t)^5} = \dfrac{10,000}{(1-2500t)^5}$

$$M_x''(t) = \frac{(10,000)(-5)(-2500)}{(1-2500t)^6} = \frac{125,000,000}{(1-2500t)^6}$$

Then $E[X] = M_x'(0) = 10,000$

$E[X^2] = M_x''(0) = 125,000,000$

$Var[X] = E[X^2] - \{E[X]\}^2 = 125,000,000 - (10,000)^2 = 25,000,000$

$\sqrt{Var[X]} = 5,000$.

----

58. Solution: E

Let X_J, X_K, and X_L represent annual losses for cities J, K, and L, respectively. Then $X = X_J + X_K + X_L$ and due to independence

$M(t) = E\left[e^{xt}\right] = E\left[e^{(x_J + x_K + x_L)t}\right] = E\left[e^{x_J t}\right]E\left[e^{x_K t}\right]E\left[e^{x_L t}\right]$

$= M_J(t)\, M_K(t)\, M_L(t) = (1 - 2t)^{-3}\,(1 - 2t)^{-2.5}\,(1 - 2t)^{-4.5} = (1 - 2t)^{-10}$

Therefore,

$M'(t) = 20(1 - 2t)^{-11}$

$M''(t) = 440(1 - 2t)^{-12}$

$M'''(t) = 10,560(1 - 2t)^{-13}$

$E[X^3] = M'''(0) = 10,560$

59. Solution: B

The distribution function of X is given by

$$F(x) = \int_{200}^{x} \frac{2.5(200)^{2.5}}{t^{3.5}}\,dt = \left.\frac{-(200)^{2.5}}{t^{2.5}}\right|_{200}^{x} = 1 - \frac{(200)^{2.5}}{x^{2.5}} \quad , \quad x > 200$$

Therefore, the p^{th} percentile x_p of X is given by

$$\frac{p}{100} = F(x_p) = 1 - \frac{(200)^{2.5}}{x_p^{\ 2.5}}$$

$$1 - 0.01p = \frac{(200)^{2.5}}{x_p^{\ 2.5}}$$

$$(1-0.01p)^{2/5} = \frac{200}{x_p}$$

$$x_p = \frac{200}{(1-0.01p)^{2/5}}$$

It follows that $x_{70} - x_{30} = \dfrac{200}{(0.30)^{2/5}} - \dfrac{200}{(0.70)^{2/5}} = 93.06$

----

60. Solution: E

Let X and Y denote the annual cost of maintaining and repairing a car before and after the 20% tax, respectively. Then Y = 1.2X and Var[Y] = Var[1.2X] = $(1.2)^2$ Var[X] = $(1.2)^2(260) = 374$.

----

61. Solution: A

The first quartile, Q1, is found by ¾ = $\int_{Q1}^{\infty}$ f(x) dx . That is, ¾ = $(200/Q1)^{2.5}$ or Q1 = 200 $(4/3)^{0.4}$ = 224.4 . Similarly, the third quartile, Q3, is given by Q3 = 200 $(4)^{0.4}$ = 348.2 . The interquartile range is the difference Q3 – Q1 .

62. Solution: C

First note that the density function of X is given by

$$f(x)=\begin{cases}\frac{1}{2} & \text{if} \quad x=1 \\ x-1 & \text{if} \quad 1<x<2 \\ 0 & \text{otherwise}\end{cases}$$

Then

$$E(X)=\frac{1}{2}+\int_1^2 x(x-1)\,dx=\frac{1}{2}+\int_1^2\left(x^2-x\right)dx=\frac{1}{2}+\left(\frac{1}{3}x^3-\frac{1}{2}x^2\right)\Bigg|_1^2$$

$$=\frac{1}{2}+\frac{8}{3}-\frac{4}{2}-\frac{1}{3}+\frac{1}{2}=\frac{7}{3}-1=\frac{4}{3}$$

$$E\left(X^2\right)=\frac{1}{2}+\int_1^2 x^2(x-1)\,dx=\frac{1}{2}+\int_1^2\left(x^3-x^2\right)dx=\frac{1}{2}+\left(\frac{1}{4}x^4-\frac{1}{3}x^3\right)\Bigg|_1^2$$

$$=\frac{1}{2}+\frac{16}{4}-\frac{8}{3}-\frac{1}{4}+\frac{1}{3}=\frac{17}{4}-\frac{7}{3}=\frac{23}{12}$$

$$Var(X)=E\left(X^2\right)-\left[E(X)\right]^2=\frac{23}{12}-\left(\frac{4}{3}\right)^2=\frac{23}{12}-\frac{16}{9}=\frac{5}{36}$$

----

63. Solution: C

Note $Y=\begin{cases}X & \text{if} \quad 0\le X\le 4 \\ 4 & \text{if} \quad 4< X\le 5\end{cases}$

Therefore,

$$E[Y]=\int_0^4\frac{1}{5}x\,dx+\int_4^5\frac{4}{5}dx=\frac{1}{10}x^2\Big|_0^4+\frac{4}{5}x\Big|_4^5$$

$$=\frac{16}{10}+\frac{20}{5}-\frac{16}{5}=\frac{8}{5}+\frac{4}{5}=\frac{12}{5}$$

$$E\left[Y^2\right]=\int_0^4\frac{1}{5}x^2dx+\int_4^5\frac{16}{5}dx=\frac{1}{15}x^3\Big|_0^4+\frac{16}{5}x\Big|_4^5$$

$$=\frac{64}{15}+\frac{80}{5}-\frac{64}{5}=\frac{64}{15}+\frac{16}{5}=\frac{64}{15}+\frac{48}{15}=\frac{112}{15}$$

$$\text{Var}[Y]=E\left[Y^2\right]-\left(E[Y]\right)^2=\frac{112}{15}-\left(\frac{12}{5}\right)^2=1.71$$

64. Solution: A

Let X denote claim size. Then E[X] = [20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30)] = (3 + 3 + 2 + 10 + 6 + 7 + 24) = 55

$E[X^2]$ = 400(0.15) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30) = 60 + 90 + 80 + 500 + 360 + 490 + 1920 = 3500

$\text{Var}[X] = E[X^2] – (E[X])^2 = 3500 – 3025 = 475$ and $\sqrt{Var[X]} = 21.79$.

Now the range of claims within one standard deviation of the mean is given by [55.00 – 21.79, 55.00 + 21.79] = [33.21, 76.79]

Therefore, the proportion of claims within one standard deviation is 0.05 + 0.20 + 0.10 + 0.10 = 0.45 .

----

65. Solution: B

Let X and Y denote repair cost and insurance payment, respectively, in the event the auto is damaged. Then

$$Y = \begin{cases} 0 & \text{if} \quad x \le 250 \\ x-250 & \text{if} \quad x > 250 \end{cases}$$

and

$$E[Y] = \int_{250}^{1500} \frac{1}{1500}(x-250)dx = \frac{1}{3000}(x-250)^2 \Big|_{250}^{1500} = \frac{1250^2}{3000} = 521$$

$$E[Y^2] = \int_{250}^{1500} \frac{1}{1500}(x-250)^2\,dx = \frac{1}{4500}(x-250)^3 \Big|_{250}^{1500} = \frac{1250^3}{4500} = 434,028$$

$$\text{Var}[Y] = E[Y^2] - \{E[Y]\}^2 = 434,028 - (521)^2$$

$$\sqrt{\text{Var}[Y]} = 403$$

----

66. Solution: E

Let X_1, X_2, X_3, and X_4 denote the four independent bids with common distribution function F. Then if we define $Y = \max(X_1, X_2, X_3, X_4)$, the distribution function G of Y is given by

$$\begin{aligned} G(y) &= \Pr[Y \le y] \\ &= \Pr[(X_1 \le y) \cap (X_2 \le y) \cap (X_3 \le y) \cap (X_4 \le y)] \\ &= \Pr[X_1 \le y]\Pr[X_2 \le y]\Pr[X_3 \le y]\Pr[X_4 \le y] \\ &= [F(y)]^4 \\ &= \frac{1}{16}(1+\sin \pi y)^4 \quad , \quad \frac{3}{2} \le y \le \frac{5}{2} \end{aligned}$$

It then follows that the density function g of Y is given by

$$g(y) = G'(y)$$
$$= \frac{1}{4}(1+\sin\pi y)^3(\pi\cos\pi y)$$
$$= \frac{\pi}{4}\cos\pi y(1+\sin\pi y)^3 \quad , \quad \frac{3}{2} \le y \le \frac{5}{2}$$

Finally,

$$E[Y] = \int_{3/2}^{5/2} yg(y)\,dy$$
$$= \int_{3/2}^{5/2} \frac{\pi}{4} y\cos\pi y(1+\sin\pi y)^3\,dy$$

----

67. Solution: B

The amount of money the insurance company will have to pay is defined by the random variable

$$Y = \begin{cases} 1000x & \text{if} \quad x < 2 \\ 2000 & \text{if} \quad x \ge 2 \end{cases}$$

where x is a Poisson random variable with mean 0.6 . The probability function for X is

$$p(x) = \frac{e^{-0.6}(0.6)^k}{k!} \qquad k = 0,1,2,3\text{L} \quad \text{and}$$

$$E[Y] = 0 + 1000(0.6)e^{-0.6} + 2000e^{-0.6}\sum_{k=2}^{\infty}\frac{0.6^k}{k!}$$
$$= 1000(0.6)e^{-0.6} + 2000\left(e^{-0.6}\sum_{k=0}^{\infty}\frac{0.6^k}{k!} - e^{-0.6} - (0.6)e^{-0.6}\right)$$
$$= 2000e^{-0.6}\sum_{k=0}^{\infty}\frac{(0.6)^k}{k!} - 2000e^{-0.6} - 1000(0.6)e^{-0.6} = 2000 - 2000e^{-0.6} - 600e^{-0.6}$$
$$= 573$$

$$E[Y^2] = (1000)^2(0.6)e^{-0.6} + (2000)^2 e^{-0.6}\sum_{k=2}^{\infty}\frac{0.6^k}{k!}$$
$$= (2000)^2 e^{-0.6}\sum_{k=0}^{\infty}\frac{0.6^k}{k!} - (2000)^2 e^{-0.6} - \left[(2000)^2 - (1000)^2\right](0.6)e^{-0.6}$$
$$= (2000)^2 - (2000)^2 e^{-0.6} - \left[(2000)^2 - (1000)^2\right](0.6)e^{-0.6}$$
$$= 816{,}893$$

$$\text{Var}[Y] = E[Y^2] - \{E[Y]\}^2 = 816{,}893 - (573)^2 = 488{,}564$$
$$\sqrt{\text{Var}[Y]} = 699$$

68. Solution: C

Note that X has an exponential distribution. Therefore, c = 0.004 . Now let Y denote the claim benefits paid. Then $Y = \begin{cases} x & \text{for } x < 250 \\ 250 & \text{for } x \ge 250 \end{cases}$ and we want to find m such that 0.50

$= \int_0^m 0.004e^{-0.004x}dx = -e^{-0.004x}\Big|_0^m = 1 - e^{-0.004m}$

This condition implies $e^{-0.004m} = 0.5 \Rightarrow m = 250 \ln 2 = 173.29$.

----

69. Solution: D

The distribution function of an exponential random variable

T with parameter θ is given by $F(t) = 1 - e^{-t/\theta}$, $t > 0$

Since we are told that T has a median of four hours, we may determine θ as follows:

$$\frac{1}{2} = F(4) = 1 - e^{-4/\theta}$$

$$\frac{1}{2} = e^{-4/\theta}$$

$$-\ln(2) = -\frac{4}{\theta}$$

$$\theta = \frac{4}{\ln(2)}$$

Therefore, $\Pr(T \ge 5) = 1 - F(5) = e^{-5/\theta} = e^{-\frac{5\ln(2)}{4}} = 2^{-5/4} = 0.42$

----

70. Solution: E

Let X denote actual losses incurred. We are given that X follows an exponential distribution with mean 300, and we are asked to find the 95th percentile of all claims that exceed 100 . Consequently, we want to find p_{95} such that

$0.95 = \dfrac{\Pr[100 < x < p_{95}]}{P[X > 100]} = \dfrac{F(p_{95}) - F(100)}{1 - F(100)}$ where F(x) is the distribution function of X .

Now $F(x) = 1 - e^{-x/300}$.

Therefore, $0.95 = \dfrac{1 - e^{-p_{95}/300} - (1 - e^{-100/300})}{1 - (1 - e^{-100/300})} = \dfrac{e^{-1/3} - e^{-p_{95}/300}}{e^{-1/3}} = 1 - e^{1/3}e^{-p_{95}/300}$

$e^{-p_{95}/300} = 0.05\, e^{-1/3}$

$p_{95} = -300 \ln(0.05\, e^{-1/3}) = 999$

71. Solution: A

The distribution function of Y is given by

$$G(y) = \Pr\left(T^2 \le y\right) = \Pr\left(T \le \sqrt{y}\right) = F\left(\sqrt{y}\right) = 1 - 4/y$$

for $y > 4$. Differentiate to obtain the density function $g(y) = 4y^{-2}$

Alternate solution:

Differentiate $F(t)$ to obtain $f(t) = 8t^{-3}$ and set $y = t^2$. Then $t = \sqrt{y}$ and

$$g(y) = f(t(y))\left|dt/dy\right| = f\left(\sqrt{y}\right)\left|\frac{d}{dt}\left(\sqrt{y}\right)\right| = 8y^{-3/2}\left(\frac{1}{2}y^{-1/2}\right) = 4y^{-2}$$

---

72. Solution: E

We are given that R is uniform on the interval $(0.04, 0.08)$ and $V = 10,000e^R$

Therefore, the distribution function of V is given by

$$F(v) = \Pr[V \le v] = \Pr\left[10,000e^R \le v\right] = \Pr\left[R \le \ln(v) - \ln(10,000)\right]$$

$$= \frac{1}{0.04}\int_{0.04}^{\ln(v)-\ln(10,000)} dr = \frac{1}{0.04}\, r\Big|_{0.04}^{\ln(v)-\ln(10,000)} = 25\ln(v) - 25\ln(10,000) - 1$$

$$= 25\left[\ln\left(\frac{v}{10,000}\right) - 0.04\right]$$

---

73. Solution: E

$$F(y) = \Pr[Y \le y] = \Pr\left[10X^{0.8} \le y\right] = \Pr\left[X \le \left(Y/10\right)^{10/8}\right] = 1 - e^{-\left(Y/10\right)^{10/8}}$$

Therefore, $f(y) = F'(y) = \frac{1}{8}\left(\frac{Y}{10}\right)^{1/4} e^{-(Y/10)^{5/4}}$

74. Solution: E

First note R = 10/T . Then

$F_R(r) = P[R \le r] = P\left[\frac{10}{T} \le r\right] = P\left[T \ge \frac{10}{r}\right] = 1 - F_T\left(\frac{10}{r}\right)$. Differentiating with respect to

r $f_R(r) = F'_R(r) = d/dr \left(1 - F_T\left(\frac{10}{r}\right)\right) = -\left(\frac{d}{dt}F_T(t)\right)\left(\frac{-10}{r^2}\right)$

$\frac{d}{dt}F_T(t) = f_T(t) = \frac{1}{4}$ since T is uniformly distributed on [8, 12] .

Therefore $f_R(r) = \frac{-1}{4}\left(\frac{-10}{r^2}\right) = \frac{5}{2r^2}$.

----

75. Solution: A

Let X and Y be the monthly profits of Company I and Company II, respectively. We are given that the pdf of X is f . Let us also take g to be the pdf of Y and take F and G to be the distribution functions corresponding to f and g . Then G(y) = Pr[Y ≤ y] = P[2X ≤ y] = P[X ≤ y/2] = F(y/2) and g(y) = G′(y) = d/dy F(y/2) = ½ F′(y/2) = ½ f(y/2) .

----

76. Solution: A

First, observe that the distribution function of X is given by

$$F(x) = \int_1^x \frac{3}{t^4}\,dt = -\frac{1}{t^3}\Big|_1^x = 1 - \frac{1}{x^3} \quad , \quad x > 1$$

Next, let X_1, X_2, and X_3 denote the three claims made that have this distribution. Then if Y denotes the largest of these three claims, it follows that the distribution function of Y is given by

$$G(y) = \Pr[X_1 \le y]\Pr[X_2 \le y]\Pr[X_3 \le y]$$

$$= \left(1 - \frac{1}{y^3}\right)^3 \quad , \quad y > 1$$

while the density function of Y is given by

$$g(y) = G'(y) = 3\left(1 - \frac{1}{y^3}\right)^2\left(\frac{3}{y^4}\right) = \left(\frac{9}{y^4}\right)\left(1 - \frac{1}{y^3}\right)^2 \quad , \quad y > 1$$

Therefore,

$$E[Y] = \int_1^\infty \frac{9}{y^3}\left(1-\frac{1}{y^3}\right)^2 dy = \int_1^\infty \frac{9}{y^3}\left(1-\frac{2}{y^3}+\frac{1}{y^6}\right)dy$$

$$= \int_1^\infty \left(\frac{9}{y^3}-\frac{18}{y^6}+\frac{9}{y^9}\right)dy = \left[-\frac{9}{2y^2}+\frac{18}{5y^5}-\frac{9}{8y^8}\right]_1^\infty$$

$$= 9\left[\frac{1}{2}-\frac{2}{5}+\frac{1}{8}\right] = 2.025 \text{ (in thousands)}$$

---

77. Solution: D

Prob. = $1 - \int_1^2 \int_1^2 \frac{1}{8}(x+y)\,dxdy$ = 0.625

Note

$\Pr\left[(X \le 1) \cup (Y \le 1)\right] = \Pr\left\{\left[(X > 1) \cap (Y > 1)\right]^c\right\}$ (De Morgan's Law)

$= 1 - \Pr\left[(X>1) \cap (Y>1)\right]$ $= 1 - \int_1^2 \int_1^2 \frac{1}{8}(x+y)\,dxdy$ $= 1 - \frac{1}{8}\int_1^2 \frac{1}{2}(x+y)^2 \Big|_1^2 dy$

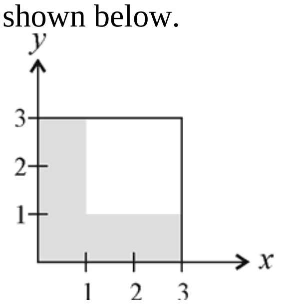
$= 1 - \frac{1}{16}\int_1^2 \left[(y+2)^2 - (y+1)^2\right]dy$ $= 1 - \frac{1}{48}\left[(y+2)^3 - (y+1)^3\right]\Big|_1^2 = 1 - \frac{1}{48}(64-27-27+8)$

$= 1 - \frac{18}{48} = \frac{30}{48} = 0.625$

---

78. Solution: B

That the device fails within the first hour means the joint density function must be integrated over the shaded region shown below.



This evaluation is more easily performed by integrating over the unshaded region and subtracting from 1.

$$\Pr\left[(X<1)\cup(Y<1)\right]$$

$$=1-\int_1^3\int_1^3 \frac{x+y}{27}\,dx\,dy=1-\int_1^3 \left.\frac{x^2+2xy}{54}\right|_1^3 dy=1-\frac{1}{54}\int_1^3 (9+6y-1-2y)\,dy$$

$$=1-\frac{1}{54}\int_1^3 (8+4y)\,dy=1-\frac{1}{54}\left.\left(8y+2y^2\right)\right|_1^3=1-\frac{1}{54}(24+18-8-2)=1-\frac{32}{54}=\frac{11}{27}=0.41$$

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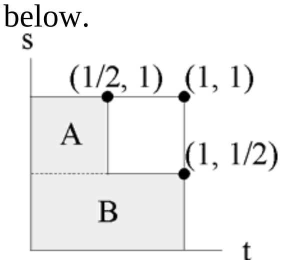
79. Solution: E

The domain of *s* and *t* is pictured below.



Note that the shaded region is the portion of the domain of s and t over which the device fails sometime during the first half hour. Therefore,

$$\Pr\left[\left(S\le\frac{1}{2}\right)\cup\left(T\le\frac{1}{2}\right)\right]=\int_0^{1/2}\int_{1/2}^1 f(s,t)\,ds\,dt+\int_0^1\int_0^{1/2} f(s,t)\,ds\,dt$$

(where the first integral covers A and the second integral covers B).

----

80. Solution: C

By the central limit theorem, the total contributions are approximately normally distributed with mean $n\mu=(2025)(3125)=6,328,125$ and standard deviation $\sigma\sqrt{n}=250\sqrt{2025}=11,250$. From the tables, the 90th percentile for a standard normal random variable is 1.282 . Letting *p* be the 90th percentile for total contributions, $\frac{p-n\mu}{\sigma\sqrt{n}}=1.282,$ and so $p=n\mu+1.282\sigma\sqrt{n}=6,328,125+(1.282)(11,250)=6,342,548$.

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81. Solution: C

Let $X_1, \ldots, X_{25}$ denote the 25 collision claims, and let $\bar{X} = \frac{1}{25}(X_1 + \ldots + X_{25})$. We are given that each X_i ($i = 1, \ldots, 25$) follows a normal distribution with mean 19,400 and standard deviation 5000. As a result $\bar{X}$ also follows a normal distribution with mean 19,400 and standard deviation $\frac{1}{\sqrt{25}}(5000) = 1000$. We conclude that $P[\bar{X} > 20{,}000]$

$$= P\left[\frac{\bar{X} - 19{,}400}{1000} > \frac{20{,}000 - 19{,}400}{1000}\right] = P\left[\frac{\bar{X} - 19{,}400}{1000} > 0.6\right] = 1 - \Phi(0.6) = 1 - 0.7257$$

$= 0.2743$.

---

82. Solution: B

Let $X_1, \ldots, X_{1250}$ be the number of claims filed by each of the 1250 policyholders. We are given that each X_i follows a Poisson distribution with mean 2. It follows that $E[X_i] = Var[X_i] = 2$. Now we are interested in the random variable $S = X_1 + \ldots + X_{1250}$. Assuming that the random variables are independent, we may conclude that S has an approximate normal distribution with $E[S] = Var[S] = (2)(1250) = 2500$.

Therefore $P[2450 < S < 2600] =$

$$P\left[\frac{2450 - 2500}{\sqrt{2500}} < \frac{S - 2500}{\sqrt{2500}} < \frac{2600 - 2500}{\sqrt{2500}}\right] = P\left[-1 < \frac{S - 2500}{50} < 2\right]$$

$$= P\left[\frac{S - 2500}{50} < 2\right] - P\left[\frac{S - 2500}{50} < -1\right]$$

Then using the normal approximation with $Z = \frac{S - 2500}{50}$, we have $P[2450 < S < 2600]$

$\approx P[Z < 2] - P[Z > 1] = P[Z < 2] + P[Z < 1] - 1 \approx 0.9773 + 0.8413 - 1 = 0.8186$.

---

83. Solution: B

Let $X_1, \ldots, X_n$ denote the life spans of the n light bulbs purchased. Since these random variables are independent and normally distributed with mean 3 and variance 1, the random variable $S = X_1 + \ldots + X_n$ is also normally distributed with mean

$$\mu = 3n$$

and standard deviation

$$\sigma = \sqrt{n}$$

Now we want to choose the smallest value for n such that

$$0.9772 \le \Pr[S > 40] = \Pr\left[\frac{S - 3n}{\sqrt{n}} > \frac{40 - 3n}{\sqrt{n}}\right]$$

This implies that n should satisfy the following inequality:

$$-2 \geq \frac{40-3n}{\sqrt{n}}$$

To find such an *n*, let's solve the corresponding equation for *n*:

$$-2 = \frac{40-3n}{\sqrt{n}}$$

$$-2\sqrt{n} = 40 - 3n$$

$$3n - 2\sqrt{n} - 40 = 0$$

$$\left(3\sqrt{n}+10\right)\left(\sqrt{n}-4\right) = 0$$

$$\sqrt{n} = 4$$

$$n = 16$$

----

84. Solution: B

Observe that

$$E\left[X+Y\right] = E\left[X\right] + E\left[Y\right] = 50 + 20 = 70$$

$$Var\left[X+Y\right] = Var\left[X\right] + Var\left[Y\right] + 2\ Cov\left[X,Y\right] = 50 + 30 + 20 = 100$$

for a randomly selected person. It then follows from the Central Limit Theorem that *T* is approximately normal with mean

$$E\left[T\right] = 100\left(70\right) = 7000$$

and variance

$$Var\left[T\right] = 100\left(100\right) = 100^2$$

Therefore,

$$\Pr\left[T < 7100\right] = \Pr\left[\frac{T-7000}{100} < \frac{7100-7000}{100}\right]$$

$$= \Pr\left[Z < 1\right] = 0.8413$$

where *Z* is a standard normal random variable.

---

85. Solution: B

Denote the policy premium by P . Since x is exponential with parameter 1000, it follows from what we are given that E[X] = 1000, Var[X] = 1,000,000, $\sqrt{Var[X]}$ = 1000 and P = 100 + E[X] = 1,100 . Now if 100 policies are sold, then Total Premium Collected = 100(1,100) = 110,000

Moreover, if we denote total claims by S, and assume the claims of each policy are independent of the others then E[S] = 100 E[X] = (100)(1000) and Var[S] = 100 Var[X] = (100)(1,000,000) . It follows from the Central Limit Theorem that S is approximately normally distributed with mean 100,000 and standard deviation = 10,000 . Therefore,

P[S ≥ 110,000] = 1 – P[S ≤ 110,000] = 1 – $P\left[Z \le \frac{110,000-100,000}{10,000}\right]$ = 1 – P[Z ≤ 1] = 1 – 0.841 ≈ 0.159 .

---

86. Solution: E

Let $X_1,\ldots,X_{100}$ denote the number of pensions that will be provided to each new recruit. Now under the assumptions given,

$$X_i = \begin{cases} 0 & \text{with probability} \quad 1-0.4=0.6 \\ 1 & \text{with probability} \quad (0.4)(0.25)=0.1 \\ 2 & \text{with probability} \quad (0.4)(0.75)=0.3 \end{cases}$$

for $i=1,\ldots,100$. Therefore,

$$E[X_i] = (0)(0.6)+(1)(0.1)+(2)(0.3) = 0.7 ,$$

$$E\left[X_i^2\right] = (0)^2(0.6)+(1)^2(0.1)+(2)^2(0.3) = 1.3 , \text{ and}$$

$$\text{Var}[X_i] = E\left[X_i^2\right] - \{E[X_i]\}^2 = 1.3-(0.7)^2 = 0.81$$

Since $X_1,\ldots,X_{100}$ are assumed by the consulting actuary to be independent, the Central Limit Theorem then implies that $S = X_1+\ldots+X_{100}$ is approximately normally distributed with mean

$$E[S] = E[X_1]+\ldots+E[X_{100}] = 100(0.7) = 70$$

and variance

$$\text{Var}[S] = \text{Var}[X_1]+\ldots+\text{Var}[X_{100}] = 100(0.81) = 81$$

Consequently,

$$\begin{aligned}\Pr[S \le 90.5] &= \Pr\left[\frac{S-70}{9} \le \frac{90.5-70}{9}\right] \\ &= \Pr[Z \le 2.28] \\ &= 0.99\end{aligned}$$

---

87. Solution: D

Let X denote the difference between true and reported age. We are given X is uniformly distributed on (–2.5,2.5) . That is, X has pdf f(x) = 1/5, –2.5 < x < 2.5 . It follows that

μ_x = E[X] = 0

σ_x^2 = Var[X] = E[X^2] = $\int_{-2.5}^{2.5} \frac{x^2}{5} dx = \frac{x^3}{15}\Big|_{-2.5}^{2.5} = \frac{2(2.5)^3}{15}$ =2.083

σ_x =1.443

Now $\bar{X}_{48}$, the difference between the means of the true and rounded ages, has a distribution that is approximately normal with mean 0 and standard deviation $\frac{1.443}{\sqrt{48}}$ = 0.2083 . Therefore,

$P\left[-\frac{1}{4} \le \bar{X}_{48} \le \frac{1}{4}\right] = P\left[\frac{-0.25}{0.2083} \le Z \le \frac{0.25}{0.2083}\right]$ = P[–1.2 ≤ Z ≤ 1.2] = P[Z ≤ 1.2] – P[Z ≤ –1.2]

= P[Z ≤ 1.2] – 1 + P[Z ≤ 1.2] = 2P[Z ≤ 1.2] – 1 = 2(0.8849) – 1 = 0.77 .

---

88. Solution: C

Let *X* denote the waiting time for a first claim from a good driver, and let *Y* denote the waiting time for a first claim from a bad driver. The problem statement implies that the respective distribution functions for *X* and *Y* are

$F(x) = 1 - e^{-x/6}$, $x > 0$ and

$G(y) = 1 - e^{-y/3}$, $y > 0$

Therefore,

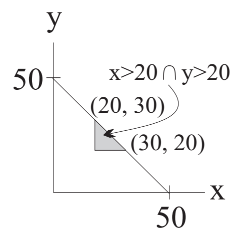
$$\Pr\left[(X \le 3) \cap (Y \le 2)\right] = \Pr\left[X \le 3\right]\Pr\left[Y \le 2\right]$$

$$= F(3)G(2) = \left(1 - e^{-1/2}\right)\left(1 - e^{-2/3}\right) = 1 - e^{-2/3} - e^{-1/2} + e^{-7/6}$$

89. Solution: B

We are given that $f(x,y) = \begin{cases} \frac{6}{125,000}(50-x-y) & \text{for } 0 < x < 50 - y < 50 \\ 0 & \text{otherwise} \end{cases}$

and we want to determine P[X > 20 ∩ Y > 20] . In order to determine integration limits, consider the following diagram:



We conclude that P[X > 20 ∩ Y > 20] = $\frac{6}{125,000}\int_{20}^{30}\int_{20}^{50-x}(50-x-y)$ dy dx .

----

90. Solution: C

Let T_1 be the time until the next Basic Policy claim, and let T_2 be the time until the next Deluxe policy claim. Then the joint pdf of T_1 and T_2 is

$f(t_1,t_2) = \left(\frac{1}{2}e^{-t_1/2}\right)\left(\frac{1}{3}e^{-t_2/3}\right) = \frac{1}{6}e^{-t_1/2}e^{-t_2/3}$, $0 < t_1 < \infty$, $0 < t_2 < \infty$ and we need to find

P[$T_2 < T_1$] = $\int_0^\infty\int_0^{t_1}\frac{1}{6}e^{-t_1/2}e^{-t_2/3}dt_2dt_1 = \int_0^\infty\left[-\frac{1}{2}e^{-t_1/2}e^{-t_2/3}\right]_0^{t_1} dt_1$

$= \int_0^\infty\left[\frac{1}{2}e^{-t_1/2} - \frac{1}{2}e^{-t_1/2}e^{-t_1/3}\right]dt_1 = \int_0^\infty\left[\frac{1}{2}e^{-t_1/2} - \frac{1}{2}e^{-5t_1/6}\right]dt_1 = \left[-e^{-t_1/2} + \frac{3}{5}e^{-5t_1/6}\right]_0^\infty = 1 - \frac{3}{5} = \frac{2}{5}$

= 0.4 .

----

91. Solution: D

We want to find P[X + Y > 1] . To this end, note that P[X + Y > 1]

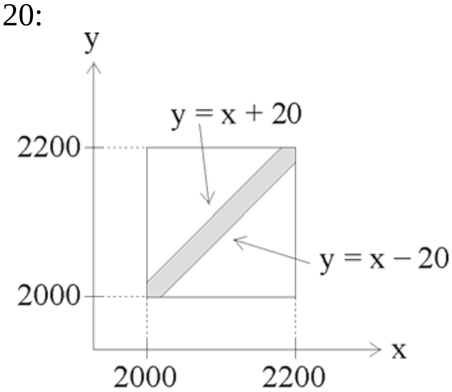
$= \int_0^1\int_{1-x}^2\left[\frac{2x+2-y}{4}\right]dydx = \int_0^1\left[\frac{1}{2}xy + \frac{1}{2}y - \frac{1}{8}y^2\right]_{1-x}^2 dx$

$= \int_0^1\left[x + 1 - \frac{1}{2} - \frac{1}{2}x(1-x) - \frac{1}{2}(1-x) + \frac{1}{8}(1-x)^2\right]dx = \int_0^1\left[x + \frac{1}{2}x^2 + \frac{1}{8} - \frac{1}{4}x + \frac{1}{8}x^2\right]dx$

$= \int_0^1\left[\frac{5}{8}x^2 + \frac{3}{4}x + \frac{1}{8}\right]dx = \left[\frac{5}{24}x^3 + \frac{3}{8}x^2 + \frac{1}{8}x\right]_0^1 = \frac{5}{24} + \frac{3}{8} + \frac{1}{8} = \frac{17}{24}$

92. Solution: B

Let X and Y denote the two bids. Then the graph below illustrates the region over which X and Y differ by less than 20:



Based on the graph and the uniform distribution:

$$\Pr\left[\left|X-Y\right|<20\right]=\frac{\text{Shaded Region Area}}{\left(2200-2000\right)^2}=\frac{200^2-2\cdot\frac{1}{2}\left(180\right)^2}{200^2}$$

$$=1-\frac{180^2}{200^2}=1-\left(0.9\right)^2=0.19$$

More formally (still using symmetry)

$$\Pr\left[\left|X-Y\right|<20\right]=1-\Pr\left[\left|X-Y\right|\geq 20\right]=1-2\Pr\left[X-Y\geq 20\right]$$

$$=1-2\int_{2020}^{2200}\int_{2000}^{x-20}\frac{1}{200^2}\,dydx=1-2\int_{2020}^{2200}\frac{1}{200^2}\,y\Big|_{2000}^{x-20}dx$$

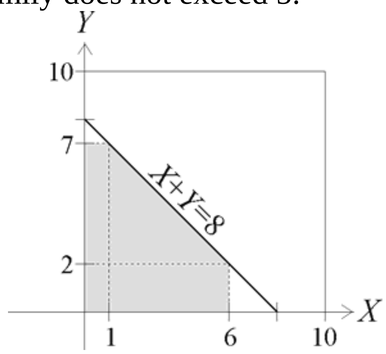
$$=1-\frac{2}{200^2}\int_{2020}^{2200}\left(x-20-2000\right)dx=1-\frac{1}{200^2}\left(x-2020\right)^2\Big|_{2020}^{2200}$$

$$=1-\left(\frac{180}{200}\right)^2=0.19$$

----

93. Solution: C

Define X and Y to be loss amounts covered by the policies having deductibles of 1 and 2, respectively. The shaded portion of the graph below shows the region over which the total benefit paid to the family does not exceed 5:



We can also infer from the graph that the uniform random variables X and Y have joint density function $f(x, y) = \frac{1}{100}$, $0 < x < 10$, $0 < y < 10$

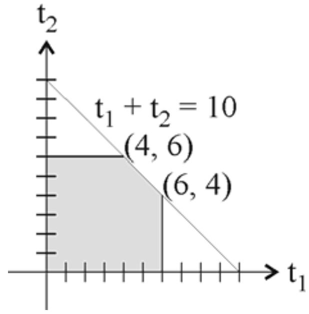
We could integrate f over the shaded region in order to determine the desired probability. However, since X and Y are uniform random variables, it is simpler to determine the portion of the 10 x 10 square that is shaded in the graph above. That is,

$$\Pr(\text{Total Benefit Paid Does not Exceed 5})$$
$$= \Pr(0 < X < 6,\ 0 < Y < 2) + \Pr(0 < X < 1,\ 2 < Y < 7) + \Pr(1 < X < 6,\ 2 < Y < 8 - X)$$
$$= \frac{(6)(2)}{100} + \frac{(1)(5)}{100} + \frac{(1/2)(5)(5)}{100} = \frac{12}{100} + \frac{5}{100} + \frac{12.5}{100} = 0.295$$

----

94. Solution: C

Let $f(t_1, t_2)$ denote the joint density function of T_1 and T_2 . The domain of f is pictured below:



Now the area of this domain is given by

$$A = 6^2 - \frac{1}{2}(6-4)^2 = 36 - 2 = 34$$

Consequently, $f(t_1,t_2)=\begin{cases}\dfrac{1}{34} & ,\ \ 0<t_1<6\ ,\ 0<t_2<6\ ,\ t_1+t_2<10\\ 0 & \text{elsewhere}\end{cases}$

and

$$E[T_1+T_2]=E[T_1]+E[T_2]=2E[T_1] \qquad \text{(due to symmetry)}$$

$$=2\left\{\int_0^4 t_1\int_0^6 \frac{1}{34}dt_2dt_1+\int_4^6 t_1\int_0^{10-t_1}\frac{1}{34}dt_2dt_1\right\}=2\left\{\int_0^4 t_1\left[\frac{t_2}{34}\Big|_0^6\right]dt_1+\int_4^6 t_1\left[\frac{t_2}{34}\Big|_0^{10-t_1}\right]dt_1\right\}$$

$$=2\left\{\int_0^4\frac{3t_1}{17}dt_1+\int_4^6\frac{1}{34}\left(10t_1-t_1^2\right)dt_1\right\}=2\left\{\frac{3t_1^2}{34}\Big|_0^4+\frac{1}{34}\left(5t_1^2-\frac{1}{3}t_1^3\right)\Big|_4^6\right\}$$

$$=2\left\{\frac{24}{17}+\frac{1}{34}\left[180-72-80+\frac{64}{3}\right]\right\}=5.7$$

----

95. Solution: E

$$M(t_1,t_2)=E\left[e^{t_1W+t_2Z}\right]=E\left[e^{t_1(X+Y)+t_2(Y-X)}\right]=E\left[e^{(t_1-t_2)X}e^{(t_1+t_2)Y}\right]$$

$$=E\left[e^{(t_1-t_2)X}\right]E\left[e^{(t_1+t_2)Y}\right]=e^{\frac{1}{2}(t_1-t_2)^2}e^{\frac{1}{2}(t_1+t_2)^2}=e^{\frac{1}{2}\left(t_1^2-2t_1t_2+t_2^2\right)}e^{\frac{1}{2}\left(t_1^2+2t_1t_2+t_2^2\right)}=e^{t_1^2+t_2^2}$$

----

96. Solution: E

Observe that the bus driver collect 21x50 = 1050 for the 21 tickets he sells. However, he may be required to refund 100 to one passenger if all 21 ticket holders show up. Since passengers show up or do not show up independently of one another, the probability that all 21 passengers will show up is $(1-0.02)^{21}=(0.98)^{21}=0.65$. Therefore, the tour operator's expected revenue is $1050-(100)(0.65)=985$.

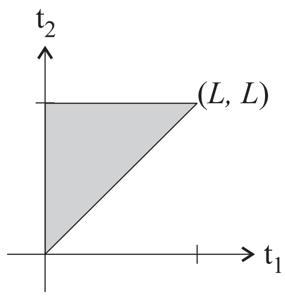
97. Solution: C

We are given $f(t_1, t_2) = 2/L^2$, $0 \le t_1 \le t_2 \le L$.

Therefore, $E[T_1^2 + T_2^2] = \int_0^L \int_0^{t_2} (t_1^2 + t_2^2) \frac{2}{L^2} dt_1 dt_2 =$

$$\frac{2}{L^2}\left\{\int_0^L \left[\frac{t_1^3}{3} + t_2^2 t_1\right]_0^{t_2} dt_1\right\} = \frac{2}{L^2}\left\{\int_0^L \left(\frac{t_2^3}{3} + t_2^3\right) dt_2\right\}$$

$$= \frac{2}{L^2}\int_0^L \frac{4}{3} t_2^3 dt_2 = \frac{2}{L^2}\left[\frac{t_2^4}{3}\right]_0^L = \frac{2}{3}L^2$$



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98. Solution: A

Let $g(y)$ be the probability function for $Y = X_1X_2X_3$. Note that $Y = 1$ if and only if $X_1 = X_2 = X_3 = 1$. Otherwise, $Y = 0$. Since $P[Y = 1] = P[X_1 = 1 \cap X_2 = 1 \cap X_3 = 1]$ $= P[X_1 = 1]\, P[X_2 = 1]\, P[X_3 = 1] = (2/3)^3 = 8/27$.

We conclude that $g(y) = \begin{cases} \frac{19}{27} & \text{for } y = 0 \\ \frac{8}{27} & \text{for } y = 1 \\ 0 & \text{otherwise} \end{cases}$

and $M(t) = E\left[e^{y_t}\right] = \frac{19}{27} + \frac{8}{27}e^t$

99. Solution: C

We use the relationships $\text{Var}(aX+b)=a^2\ \text{Var}(X)$, $\text{Cov}(aX,bY)=ab\ \text{Cov}(X,Y)$, and $\text{Var}(X+Y)=\text{Var}(X)+\text{Var}(Y)+2\ \text{Cov}(X,Y)$. First we observe

$17,000=\text{Var}(X+Y)=5000+10,000+2\ \text{Cov}(X,Y)$, and so $\text{Cov}(X,Y)=1000$.

We want to find $\text{Var}\left[(X+100)+1.1Y\right]=\ \text{Var}\left[(X+1.1Y)+100\right]$

$=\text{Var}\left[X+1.1Y\right]=\text{Var}\,X+\text{Var}\left[(1.1)Y\right]+2\ \text{Cov}(X,1.1Y)$

$=\text{Var}\,X+(1.1)^2\ \text{Var}\,Y+2(1.1)\text{Cov}(X,Y)=5000+12,100+2200=19,300.$

---

100. Solution: B

Note

P(X = 0) = 1/6

P(X = 1) = 1/12 + 1/6 = 3/12

P(X = 2) = 1/12 + 1/3 + 1/6 = 7/12 .

E[X] = (0)(1/6) + (1)(3/12) + (2)(7/12) = 17/12

$E[X^2] = (0)^2(1/6) + (1)^2(3/12) + (2)^2(7/12) = 31/12$

$\text{Var}[X] = 31/12 - (17/12)^2 = 0.58$.

---

101. Solution: D

Note that due to the independence of X and Y

$\text{Var}(Z) = \text{Var}(3X - Y - 5) = \text{Var}(3X) + \text{Var}(Y) = 3^2\ \text{Var}(X) + \text{Var}(Y) = 9(1) + 2 = 11$.

---

102. Solution: E

Let *X* and *Y* denote the times that the two backup generators can operate. Now the variance of an exponential random variable with mean β is β^2. Therefore,

$$\text{Var}\left[X\right]=\text{Var}\left[Y\right]=10^2=100$$

Then assuming that *X* and *Y* are independent, we see

$$\text{Var}\left[X+Y\right]=\text{Var}\left[X\right]+\text{Var}\left[Y\right]=100+100=200$$

103. Solution: E

Let $X_1, X_2,$ and X_3 denote annual loss due to storm, fire, and theft, respectively. In addition, let $Y = Max\left(X_1, X_2, X_3\right)$.

Then

$$\Pr\left[Y > 3\right] = 1 - \Pr\left[Y \le 3\right] = 1 - \Pr\left[X_1 \le 3\right]\Pr\left[X_2 \le 3\right]\Pr\left[X_3 \le 3\right]$$

$$= 1 - \left(1 - e^{-3}\right)\left(1 - e^{-3/1.5}\right)\left(1 - e^{-3/2.4}\right) \quad *$$

$$= 1 - \left(1 - e^{-3}\right)\left(1 - e^{-2}\right)\left(1 - e^{-5/4}\right)$$

$$= 0.414$$

* Uses that if X has an exponential distribution with mean μ

$$\Pr\left(X \le x\right) = 1 - \Pr\left(X \ge x\right) = 1 - \int_x^\infty \frac{1}{\mu} e^{-t/\mu} dt = 1 - \left(-e^{-t/\mu}\right)\Big|_x^\infty = 1 - e^{-x/\mu}$$

----

104. Solution: B

Let us first determine k:

$$1 = \int_0^1 \int_0^1 kxdxdy = \int_0^1 \frac{1}{2} kx^2 \Big|_0^1 \, dy = \int_0^1 \frac{k}{2} dy = \frac{k}{2}$$

$$k = 2$$

Then

$$E\left[X\right] = \int_0^1 \int_0^1 2x^2 dydx = \int_0^1 2x^2 dx = \frac{2}{3} x^3 \Big|_0^1 = \frac{2}{3}$$

$$E\left[Y\right] = \int_0^1 \int_0^1 y \; 2x \; dxdy = \int_0^1 ydy = \frac{1}{2} y^2 \Big|_0^1 = \frac{1}{2}$$

$$E\left[XY\right] = \int_0^1 \int_0^1 2x^2 ydxdy = \int_0^1 \frac{2}{3} x^3 y \Big|_0^1 \, dy = \int_0^1 \frac{2}{3} ydy$$

$$= \frac{2}{6} y^2 \Big|_0^1 = \frac{2}{6} = \frac{1}{3}$$

$$\text{Cov}\left[X, Y\right] = E\left[XY\right] - E\left[X\right]E\left[Y\right] = \frac{1}{3} - \left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{3} - \frac{1}{3} = 0$$

(Alternative Solution)

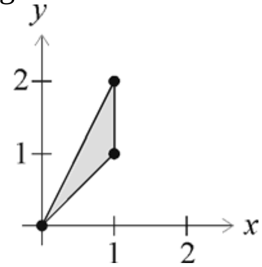
Define $g(x) = kx$ and $h(y) = 1$. Then

$f(x,y) = g(x)h(x)$

In other words, $f(x,y)$ can be written as the product of a function of x alone and a function of y alone. It follows that X and Y are independent. Therefore, Cov[X, Y] = 0 .

105. Solution: A

The calculation requires integrating over the indicated region.



$$E(X)=\int_0^1\int_x^{2x}\frac{8}{3}x^2y\,dy\,dx=\int_0^1\frac{4}{3}x^2y^2\Big|_x^{2x}dx=\int_0^1\frac{4}{3}x^2\left(4x^2-x^2\right)dx=\int_0^1 4x^4dx=\frac{4}{5}x^5\Big|_0^1=\frac{4}{5}$$

$$E(Y)=\int_0^1\int_x^{2x}\frac{8}{3}xy^2dy\,dx=\int_0^1\frac{8}{9}xy^3\Big|_x^{2x}dy\,dx=\int_0^1\frac{8}{9}x\left(8x^3-x^3\right)dx=\int_0^1\frac{56}{9}x^4dx=\frac{56}{45}x^5\Big|_0^1=\frac{56}{45}$$

$$E(XY)=\int_0^1\int_x^{2x}\frac{8}{3}x^2y^2dy\,dx=\int_0^1\frac{8}{9}x^2y^3\Big|_x^{2x}dx=\int_0^1\frac{8}{9}x^2\left(8x^3-x^3\right)dx=\int_0^1\frac{56}{9}x^5dx=\frac{56}{54}=\frac{28}{27}$$

$$\mathrm{Cov}(X,Y)=E(XY)-E(X)E(Y)=\frac{28}{27}-\left(\frac{56}{45}\right)\left(\frac{4}{5}\right)=0.04$$

----

106. Solution: C

The joint pdf of X and Y is $f(x,y) = f_2(y|x)\, f_1(x)$

$= (1/x)(1/12)$, $0 < y < x$, $0 < x < 12$.

Therefore,

E[X] = $\int_0^{12}\int_0^x x\cdot\frac{1}{12x}dydx=\int_0^{12}\frac{y}{12}\Big|_0^x\;dx=\int_0^{12}\frac{x}{12}dx=\frac{x^2}{24}\Big|_0^{12}\;=6$

E[Y] = $\int_0^{12}\int_0^x\frac{y}{12x}dydx=\int_0^{12}\left[\frac{y^2}{24x}\right]_0^x\;dx=\int_0^{12}\frac{x}{24}dx=\frac{x^2}{48}\Big|_0^{12}\;=\frac{144}{48}\;=3$

E[XY] = $\int_0^{12}\int_0^x\frac{y}{12}dydx=\int_0^{12}\left[\frac{y^2}{24}\right]_0^x\;dx=\int_0^{12}\frac{x^2}{24}dx=\frac{x^3}{72}\Big|_0^{12}\;=\frac{(12)^3}{72}\;=24$

Cov(X,Y) = E[XY] – E[X]E[Y] = 24 – (3)(6) = 24 – 18 = 6 .

107. Solution: A

$$\operatorname{Cov}(C_1, C_2) = \operatorname{Cov}(X + Y,\ \ X + 1.2Y)$$

$$= \operatorname{Cov}(X, X) + \operatorname{Cov}(Y, X) + \operatorname{Cov}(X, 1.2Y) + \operatorname{Cov}(\mathrm{Y}, 1.2\mathrm{Y})$$

$$= \operatorname{Var} X + \operatorname{Cov}(X, Y) + 1.2\operatorname{Cov}(X, Y) + 1.2\operatorname{Var} Y$$

$$= \operatorname{Var} X + 2.2\operatorname{Cov}(X, Y) + 1.2\operatorname{Var} Y$$

$$\operatorname{Var} X = E(X^2) - (E(X))^2 = 27.4 - 5^2 = 2.4$$

$$\operatorname{Var} Y = E(Y^2) - (E(Y))^2 = 51.4 - 7^2 = 2.4$$

$$\operatorname{Var}(X + Y) = \operatorname{Var} X + \operatorname{Var} Y + 2\operatorname{Cov}(X, Y)$$

$$\operatorname{Cov}(X, Y) = \frac{1}{2}\left(\operatorname{Var}(X + Y) - \operatorname{Var} X - \operatorname{Var} Y\right) = \frac{1}{2}(8 - 2.4 - 2.4) = 1.6$$

$$\operatorname{Cov}(C_1, C_2) = 2.4 + 2.2(1.6) + 1.2(2.4) = 8.8$$

----

107. Alternate solution:

We are given the following information:

$$C_1 = X + Y$$

$$C_2 = X + 1.2Y$$

$$E[X] = 5$$

$$E[X^2] = 27.4$$

$$E[Y] = 7$$

$$E[Y^2] = 51.4$$

$$\operatorname{Var}[X + Y] = 8$$

Now we want to calculate

$$\operatorname{Cov}(C_1, C_2) = \operatorname{Cov}(X + Y, X + 1.2Y)$$

$$= E[(X + Y)(X + 1.2Y)] - E[X + Y] \cdot E[X + 1.2Y]$$

$$= E[X^2 + 2.2XY + 1.2Y^2] - (E[X] + E[Y])(E[X] + 1.2E[Y])$$

$$= E[X^2] + 2.2E[XY] + 1.2E[Y^2] - (5 + 7)(5 + (1.2)7)$$

$$= 27.4 + 2.2E[XY] + 1.2(51.4) - (12)(13.4)$$

$$= 2.2E[XY] - 71.72$$

Therefore, we need to calculate $E[XY]$ first. To this end, observe

$$8 = \mathrm{Var}[X+Y] = E\left[(X+Y)^2\right] - (E[X+Y])^2$$
$$= E\left[X^2 + 2XY + Y^2\right] - (E[X] + E[Y])^2$$
$$= E\left[X^2\right] + 2E[XY] + E\left[Y^2\right] - (5+7)^2$$
$$= 27.4 + 2E[XY] + 51.4 - 144$$
$$= 2E[XY] - 65.2$$
$$E[XY] = (8 + 65.2)/2 = 36.6$$

Finally, $\mathrm{Cov}(C_{1,}C_2) = 2.2(36.6) - 71.72 = 8.8$

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108. Solution: A

The joint density of T_1 and T_2 is given by

$$f(t_1, t_2) = e^{-t_1} e^{-t_2} \quad , \quad t_1 > 0 \quad , \quad t_2 > 0$$

Therefore,

$$\Pr[X \le x] = \Pr[2T_1 + T_2 \le x]$$
$$= \int_0^x \int_0^{\frac{1}{2}(x-t_2)} e^{-t_1} e^{-t_2}\, dt_1 dt_2 = \int_0^x e^{-t_2} \left[-e^{-t_1} \Big|_0^{\frac{1}{2}(x-t_2)} \right] dt_2$$
$$= \int_0^x e^{-t_2} \left[1 - e^{-\frac{1}{2}x + \frac{1}{2}t_2} \right] dt_2 = \int_0^x \left(e^{-t_2} - e^{-\frac{1}{2}x} e^{-\frac{1}{2}t_2} \right) dt_2$$
$$= \left[-e^{-t_2} + 2e^{-\frac{1}{2}x} e^{-\frac{1}{2}t_2} \right] \Big|_0^x = -e^{-x} + 2e^{-\frac{1}{2}x} e^{-\frac{1}{2}x} + 1 - 2e^{-\frac{1}{2}x}$$
$$= 1 - e^{-x} + 2e^{-x} - 2e^{-\frac{1}{2}x} = 1 - 2e^{-\frac{1}{2}x} + e^{-x} \quad , \quad x > 0$$

It follows that the density of X is given by

$$g(x) = \frac{d}{dx}\left[1 - 2e^{-\frac{1}{2}x} + e^{-x} \right] = e^{-\frac{1}{2}x} - e^{-x} \quad , \quad x > 0$$

109. Solution: B

Let

u be annual claims,
v be annual premiums,
g(*u*, *v*) be the joint density function of *U* and *V*,
f(*x*) be the density function of *X*, and
F(*x*) be the distribution function of *X*.

Then since U and V are independent,

$$g(u,v) = \left(e^{-u}\right)\left(\frac{1}{2}e^{-v/2}\right) = \frac{1}{2}e^{-u}e^{-v/2} \quad , \quad 0 < u < \infty \quad , \quad 0 < v < \infty$$

and

$$F(x) = \Pr[X \le x] = \Pr\left[\frac{u}{v} \le x\right] = \Pr[U \le Vx]$$

$$= \int_0^\infty \int_0^{vx} g(u,v)\,du\,dv = \int_0^\infty \int_0^{vx} \frac{1}{2}e^{-u}e^{-v/2}\,du\,dv$$

$$= \int_0^\infty -\frac{1}{2}e^{-u}e^{-v/2}\Big|_0^{vx}\,dv = \int_0^\infty \left(-\frac{1}{2}e^{-vx}e^{-v/2} + \frac{1}{2}e^{-v/2}\right)dv$$

$$= \int_0^\infty \left(-\frac{1}{2}e^{-v(x+1/2)} + \frac{1}{2}e^{-v/2}\right)dv$$

$$= \left[\frac{1}{2x+1}e^{-v(x+1/2)} - e^{-v/2}\right]_0^\infty = -\frac{1}{2x+1} + 1$$

Finally, $f(x) = F'(x) = \dfrac{2}{(2x+1)^2}$

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110. Solution: C

Note that the conditional density function

$$f\left(y \,\middle|\, x = \frac{1}{3}\right) = \frac{f(1/3, y)}{f_x(1/3)}, \quad 0 < y < \frac{2}{3},$$

$$f_x\left(\frac{1}{3}\right) = \int_0^{2/3} 24(1/3)\,y\,dy = \int_0^{2/3} 8y\,dy = 4y^2\Big|_0^{2/3} = \frac{16}{9}$$

It follows that $f\left(y \,\middle|\, x = \frac{1}{3}\right) = \frac{9}{16} f(1/3, y) = \frac{9}{2}y \quad , \quad 0 < y < \frac{2}{3}$

Consequently, $\Pr\left[Y < X \,\middle|\, X = 1/3\right] = \int_0^{1/3} \frac{9}{2}y\,dy = \frac{9}{4}y^2\Big|_0^{1/3} = \frac{1}{4}$

111. Solution: E

$$\Pr\left[1<Y<3 \middle| X=2\right]=\int_1^3 \frac{f(2,y)}{f_X(2)}dy$$

$$f(2,y)=\frac{2}{4(2-1)}y^{-(4-1)/2-1}=\frac{1}{2}y^{-3}$$

$$f_X(2)=\int_1^\infty \frac{1}{2}y^{-3}dy=-\frac{1}{4}y^{-2}\Big|_1^\infty=\frac{1}{4}$$

Finally, $\Pr\left[1<Y<3 \middle| X=2\right]=\dfrac{\int_1^3 \frac{1}{2}y^{-3}dy}{\frac{1}{4}}=-y^{-2}\Big|_1^3=1-\dfrac{1}{9}=\dfrac{8}{9}$

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112. Solution: D

We are given that the joint pdf of X and Y is f(x,y) = 2(x+y), 0 < y < x < 1 .

Now $f_x(x) = \int_0^x (2x+2y)dy=\left[2xy+y^2\right]_0^x$ $= 2x^2 + x^2 = 3x^2$, 0 < x < 1

so f(y|x) = $\dfrac{f(x,y)}{f_x(x)}=\dfrac{2(x+y)}{3x^2}=\dfrac{2}{3}\left(\dfrac{1}{x}+\dfrac{y}{x^2}\right)$, 0 < y < x

f(y|x = 0.10) = $\dfrac{2}{3}\left[\dfrac{1}{0.1}+\dfrac{y}{0.01}\right]=\dfrac{2}{3}\left[10+100y\right]$, 0 < y < 0.10

P[Y < 0.05|X = 0.10] = $\int_0^{0.05}\dfrac{2}{3}\left[10+100y\right]dy=\left[\dfrac{20}{3}y+\dfrac{100}{3}y^2\right]_0^{0.05}=\dfrac{1}{3}+\dfrac{1}{12}=\dfrac{5}{12}$ = 0.4167 .

---

113. Solution: E

Let

W = event that wife survives at least 10 years
H = event that husband survives at least 10 years
B = benefit paid
P = profit from selling policies

Then $\Pr\left[H\right]=P\left[H\cap W\right]+\Pr\left[H\cap W^c\right]=0.96+0.01=0.97$

and

$$\Pr\left[W \mid H\right]=\frac{\Pr\left[W\cap H\right]}{\Pr\left[H\right]}=\frac{0.96}{0.97}=0.9897$$

$$\Pr\left[W^c \mid H\right]=\frac{\Pr\left[H\cap W^c\right]}{\Pr\left[H\right]}=\frac{0.01}{0.97}=0.0103$$

It follows that

$$E[P]=E[1000-B]=1000-E[B]=1000-\{(0)\Pr[W|H]+(10{,}000)\Pr[W^c|H]\}$$
$$=1000-10{,}000(0.0103)=1000-103=897$$

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114. Solution: C

Note that

$P(Y = 0 | X = 1) = \dfrac{P(X=1,Y=0)}{P(X=1)}=\dfrac{P(X=1,Y=0)}{P(X=1,Y=0)+P(X=1,Y=1)}=\dfrac{0.05}{0.05+0.125}$

$= 0.286$

$P(Y = 1 | X=1) = 1 – P(Y = 0 | X = 1) = 1 – 0.286 = 0.714$

Therefore, $E(Y | X = 1) = (0)\, P(Y = 0 | X = 1) + (1)\, P(Y = 1 | X = 1) = (1)(0.714) = 0.714$

$E(Y^2 | X = 1) = (0)^2\, P(Y = 0 | X = 1) + (1)^2\, P(Y = 1 | X = 1) = 0.714$

$Var(Y | X = 1) = E(Y^2 | X = 1) – [E(Y | X = 1)]^2 = 0.714 – (0.714)^2 = 0.20$

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115. Solution: A

Let $f_1(x)$ denote the marginal density function of X. Then

$$f_1(x)=\int_x^{x+1}2x\,dy=2xy\,\Big|_x^{x+1}=2x(x+1-x)=2x \quad , \quad 0<x<1$$

Consequently,

$$f(y|x)=\frac{f(x,y)}{f_1(x)}=\begin{cases}1 & \text{if:} \quad x<y<x+1\\ 0 & \text{otherwise}\end{cases}$$

$$E[Y|X]=\int_x^{x+1}y\,dy=\frac{1}{2}y^2\,\Big|_x^{x+1}=\frac{1}{2}(x+1)^2-\frac{1}{2}x^2=\frac{1}{2}x^2+x+\frac{1}{2}-\frac{1}{2}x^2=x+\frac{1}{2}$$

$$E[Y^2|X]=\int_x^{x+1}y^2\,dy=\frac{1}{3}y^3\,\Big|_x^{x+1}=\frac{1}{3}(x+1)^3-\frac{1}{3}x^3$$
$$=\frac{1}{3}x^3+x^2+x+\frac{1}{3}-\frac{1}{3}x^3=x^2+x+\frac{1}{3}$$

$$\mathrm{Var}[Y|X]=E[Y^2|X]-\{E[Y|X]\}^2=x^2+x+\frac{1}{3}-\left(x+\frac{1}{2}\right)^2$$
$$=x^2+x+\frac{1}{3}-x^2-x-\frac{1}{4}=\frac{1}{12}$$

116. Solution: D

Denote the number of tornadoes in counties P and Q by N_P and N_Q, respectively. Then
$E[N_Q|N_P = 0]$
$= [(0)(0.12) + (1)(0.06) + (2)(0.05) + 3(0.02)] / [0.12 + 0.06 + 0.05 + 0.02] = 0.88$
$E[N_Q^2|N_P = 0]$
$= [(0)^2(0.12) + (1)^2(0.06) + (2)^2(0.05) + (3)^2(0.02)] / [0.12 + 0.06 + 0.05 + 0.02]$
$= 1.76$ and $Var[N_Q|N_P = 0] = E[N_Q^2|N_P = 0] – \{E[N_Q|N_P = 0]\}^2 = 1.76 – (0.88)^2$
$= 0.9856$.

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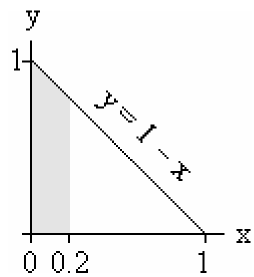
117. Solution: C

The domain of X and Y is pictured below. The shaded region is the portion of the domain over which $X<0.2$.



Now observe

$$\Pr\left[X < 0.2\right] = \int_0^{0.2}\int_0^{1-x} 6\left[1-\left(x+y\right)\right]dydx = 6\int_0^{0.2}\left[y - xy - \frac{1}{2}y^2\right]_0^{1-x} dx$$

$$= 6\int_0^{0.2}\left[1 - x - x\left(1-x\right) - \frac{1}{2}\left(1-x\right)^2\right]dx = 6\int_0^{0.2}\left[\left(1-x\right)^2 - \frac{1}{2}\left(1-x\right)^2\right]dx$$

$$= 6\int_0^{0.2}\frac{1}{2}\left(1-x\right)^2 dx = -\left(1-x\right)^3 |_0^{0.2} = -\left(0.8\right)^3 + 1 = 0.488$$

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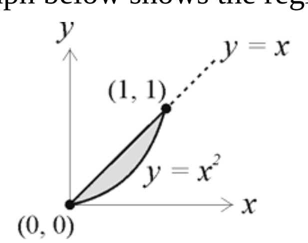
118. Solution: E

The shaded portion of the graph below shows the region over which $f\left(x, y\right)$ is nonzero:



We can infer from the graph that the marginal density function of Y is given by

$$g\left(y\right) = \int_y^{\sqrt{y}} 15y\,dx = 15xy\Big|_y^{\sqrt{y}} = 15y\left(\sqrt{y} - y\right) = 15y^{3/2}\left(1 - y^{1/2}\right),\ 0 < y < 1$$

or more precisely, $g(y) = \begin{cases} 15y^{3/2}(1-y)^{1/2}, & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

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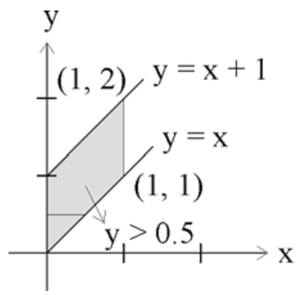
119. Solution: D

The diagram below illustrates the domain of the joint density $f(x,y)$ of X and Y.



We are told that the marginal density function of X is $f_X(x) = 1$, $0 < x < 1$

while $f_{y|x}(y|x) = 1$, $x < y < x+1$

It follows that $f(x,y) = f_X(x)\, f_{y|x}(y|x) = \begin{cases} 1 & \text{if} \quad 0 < x < 1,\ x < y < x+1 \\ 0 & \text{otherwise} \end{cases}$

Therefore,

$$\Pr[Y > 0.5] = 1 - \Pr[Y \le 0.5] = 1 - \int_0^{1/2}\int_x^{1/2} dy\,dx$$

$$= 1 - \int_0^{1/2} y\Big|_x^{1/2} dx = 1 - \int_0^{1/2}\left(\frac{1}{2} - x\right)dx = 1 - \left[\frac{1}{2}x - \frac{1}{2}x^2\right]\Bigg|_0^{1/2} = 1 - \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$

[Note since the density is constant over the shaded parallelogram in the figure the solution is also obtained as the ratio of the area of the portion of the parallelogram above $y = 0.5$ to the entire shaded area.]

120. Solution: A

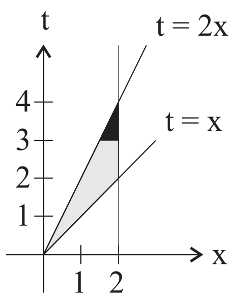
We are given that X denotes loss. In addition, denote the time required to process a claim by T.

Then the joint pdf of X and T is $f(x,t)=\begin{cases}\frac{3}{8}x^2\cdot\frac{1}{x}=\frac{3}{8}x, & x<t<2x, 0\le x\le 2\\ 0, & \text{otherwise.}\end{cases}$

Now we can find P[T ≥ 3] =

$$\int_3^4\int_{t/2}^2 \frac{3}{8}x\,dx\,dt=\int_3^4\left[\frac{3}{16}x^2\right]_{t/2}^2 dt=\int_3^4\left(\frac{12}{16}-\frac{3}{64}t^2\right)dt=\left[\frac{12}{16}-\frac{1}{64}t^3\right]_3^4=\frac{12}{4}-1-\left(\frac{36}{16}-\frac{27}{64}\right)$$

= 11/64 = 0.17 .



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121. Solution: C

The marginal density of *X* is given by

$$f_x(x)=\int_0^1\frac{1}{64}\left(10-xy^2\right)dy=\frac{1}{64}\left(10y-\frac{xy^3}{3}\right)\Bigg|_0^1=\frac{1}{64}\left(10-\frac{x}{3}\right)$$

Then $E(X)=\int_2^{10}x\,f_x(x)dx=\int_2^{10}\frac{1}{64}\left(10x-\frac{x^2}{3}\right)dx=\frac{1}{64}\left(5x^2-\frac{x^3}{9}\right)\Bigg|_2^{10}$

$$=\frac{1}{64}\left[\left(500-\frac{1000}{9}\right)-\left(20-\frac{8}{9}\right)\right]=5.778$$

122. Solution: D

The marginal distribution of Y is given by $f_2(y) = \int_0^y 6\, e^{-x}\, e^{-2y}\, dx = 6\, e^{-2y} \int_0^y e^{-x}\, dx$

$= -6\, e^{-2y}\, e^{-y} + 6e^{-2y} = 6\, e^{-2y} - 6\, e^{-3y}$, $0 < y < \infty$

Therefore, $E(Y) = \int_0^\infty y\ f_2(y)\, dy = \int_0^\infty (6ye^{-2y} - 6ye^{-3y})\, dy = 6 \int_0^\infty ye^{-2y}\ dy - 6 \int_0^\infty y\ e^{-3y}\, dy =$

$\frac{6}{2}\int_0^\infty 2\ ye^{-2y}\, dy - \frac{6}{3}\int_0^\infty 3y\, e^{-3y}\, dy$

But $\int_0^\infty 2\, y\, e^{-2y}\, dy$ and $\int_0^\infty 3y\, e^{-3y}\, dy$ are equivalent to the means of exponential random variables with parameters 1/2 and 1/3, respectively. In other words, $\int_0^\infty 2\, y\, e^{-2y}\, dy = 1/2$ and $\int_0^\infty 3y\, e^{-3y}\, dy = 1/3$. We conclude that E(Y) = (6/2) (1/2) – (6/3) (1/3) = 3/2 – 2/3 = 9/6 – 4/6 = 5/6 = 0.83 .

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123. Solution: C

Observe

$$\begin{aligned}\Pr\left[4<S<8\right] &= \Pr\left[4<S<8 \middle| N=1\right]\Pr\left[N=1\right]+\Pr\left[4<S<8 \middle| N>1\right]\Pr\left[N>1\right]\\ &= \frac{1}{3}\left(e^{-4/5}-e^{-8/5}\right)+\frac{1}{6}\left(e^{-1/2}-e^{-1}\right)\ *\\ &= 0.122\end{aligned}$$

*Uses that if X has an exponential distribution with mean μ

$$\Pr\left(a \le X \le b\right) = \Pr\left(X \ge a\right) - \Pr\left(X \ge b\right) = \int_a^\infty \frac{1}{\mu}e^{-t/\mu}dt - \int_b^\infty \frac{1}{\mu}e^{-t/\mu}dt = e^{-\frac{a}{\mu}} - e^{-\frac{b}{\mu}}$$

124. Solution: A

Because $f(x,y)$ can be written as $f(x)\,f(y) = e^{-x}\;\;2e^{-2y}$ and the support of $f(x,y)$ is a cross product, X and Y are independent. Thus, the condition on X can be ignored and it suffices to just consider $f(y) = 2e^{-2y}$.

Because of the memoryless property of the exponential distribution, the conditional density of Y is the same as the unconditional density of Y+3.

Because a location shift does not affect the variance, the conditional variance of Y is equal to the unconditional variance of Y.

Because the mean of Y is 0.5 and the variance of an exponential distribution is always equal to the square of its mean, the requested variance is 0.25.

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125. Solution: E

The support of (X,Y) is $0 < y < x < 1$.

$f_{X,Y}(x,y) = f(y|x)\,f_X(x) = 2$ on that support. It is clear geometrically

(a flat joint density over the triangular region $0 < y < x < 1$) that when Y = y

we have X ~ U(y, 1) so that $f(x|y) = \dfrac{1}{1-y}$ *for* $y < x < 1$.

By computation:

$$f_Y(y) = \int_y^1 2dx = 2-2y \Rightarrow f(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{2}{2-2y} = \frac{1}{2-y} \;\; for \;\; y < x < 1$$

126. Solution: C

Using the notation of the problem, we know that $p_0 + p_1 = \frac{2}{5}$ and

$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 = 1$.

Let $p_n - p_{n+1} = c$ for all $n \le 4$. Then $p_n = p_0 - nc$ for $1 \le n \le 5$.

Thus $p_0 + (p_0 - c) + (p_0 - 2c) + \mathbf{...} + (p_0 - 5c) = 6p_0 - 15c = 1$.

Also $p_0 + p_1 = p_0 + (p_0 - c) = 2p_0 - c = \frac{2}{5}$. Solving simultaneously $\begin{cases} 6p_0 - 15c = 1 \\ 2p_0 - c = \frac{2}{5} \end{cases}$

$$\begin{array}{r} 6p_0 - 3c = \frac{6}{5} \\ \Rightarrow \underline{-6p_0 + 15c = -1} \\ 12c = \frac{1}{5} \end{array}$$

So $c = \frac{1}{60}$ and $2p_0 = \frac{2}{5} + \frac{1}{60} = \frac{25}{60}$. Thus $p_0 = \frac{25}{120}$.

We want $p_4 + p_5 = (p_0 - 4c) + (p_0 - 5c) = \frac{17}{120} + \frac{15}{120} = \frac{32}{120} = 0.267$.

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127. Solution: D

Because the number of payouts (including payouts of zero when the loss is below the deductible) is large, we can apply the central limit theorem and assume the total payout S is normal. For one loss there is no payout with probability 0.25 and otherwise the payout is U(0, 15000). So,

$E[X] = 0.25*0 + 0.75*7500 = 5625$,

$E[X^2] = 0.25*0 + 0.75*(7500^2 + \frac{15000^2}{12}) = 56{,}250{,}000$, so the variance of one claim is

$Var(X) = E[X^2] - E[X]^2 = 24{,}609{,}375$.

Applying the CLT,

$$P[1{,}000{,}000 < S < 1{,}200{,}000] = P\left[-1.781741613 < \frac{S - (200)(5625)}{\sqrt{(200)(24{,}609{,}375)}} < 1.069044968\right]$$

which interpolates to 0.8575-(1-0.9626)=0.8201 from the provided table.