Exam 7

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

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Exam 7

Estimation of Policy Liabilities, Insurance Company Valuation, and ERM

May 2, 2019

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 54.75 point examination consists of 21 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 7, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.

   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.

   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.

   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 16, 2019.

END OF INSTRUCTIONS

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1. (7 points)

An insurance company historically never attempted to recover salvage and subrogation ("S&S") on claims. On January 1, 2018, the insurer enters into a one-year agreement with an S&S recovery vendor, requiring the vendor to pursue all S&S opportunities for accident year 2018.

The insurance company's IT department generated the following loss development triangles as of December 31, 2018 (assume no development after 36 months):

Incremental Paid Loss Gross of S&S ($000s) as of (months)			
Accident Year	12	24	36
2016	16,500	6,000	2,500
2017	17,000	5,000	
2018	14,000		

Incremental Paid Loss Net of S&S ($000s) as of (months)			
Accident Year	12	24	36
2016	16,500	6,000	2,500
2017	17,000	5,000	
2018	11,000		

Prior to 2018, the Actuarial department estimated unpaid losses using the following Bayesian model:

- C_{ij} represents the incremental losses for accident year i as of development year j which follow an over-dispersed Poisson ("ODP") distribution with mean $x_i y_j$ and variance $\varphi x_i y_j$.
- x_i represents the expected ultimate losses for accident year i.
- y_j represents the proportion of ultimate losses that emerge in development year j.
- The prior distribution for x_i is gamma with mean $\frac{\alpha_i}{\beta_i}$ and variance $\frac{\alpha_i}{\beta_i^2}$.
- φ represents the dispersion parameter for the ODP distribution, which is 93.
- λ_j represents the incremental chain ladder loss development factor for development year j.
- D_{ij} represents the cumulative losses for accident year i as of development year j.
- M_i represents the value for ultimate losses for accident year i that is obtained using expert knowledge about the losses.
- The mean of C_{ij} for this Bayesian model is:

$$Z_{ij}\,(\lambda_j - 1)D_{i,j-1} + (1 - Z_{ij})(\lambda_j - 1)M_i \frac{1}{\lambda_j\lambda_{j+1}\cdots\lambda_n}, \text{ where } Z_{ij} = \frac{\sum_{k=1}^{j-1} y_k}{\beta_i\varphi + \sum_{k=1}^{j-1} y_k}.$$

The Actuarial department believes that the S&S recovery vendor agreement has resulted in a slowdown of the gross loss payment pattern for calendar year 2018 and will continue in calendar years 2019 and 2020.

To estimate the total unpaid losses gross of S&S for accident year 2018, the Actuarial department is considering the following prior gamma distributions for the Bayesian model above:

Parameter	Option 1	Option 2	Option 3
α	50	200	1,250
β	0.002	0.008	0.100

To estimate the total S&S recoveries for accident year 2018, the Actuarial department analyzed S&S data from competitors of comparable size to find:

- The expected S&S recoveries per accident year were $5M.
- The standard deviation of the S&S recoveries was $1M.
- The expected percent of the S&S recoveries received through 12 months was 75%.
- The standard deviation of the percent of S&S recoveries received through 12 months was 10%.

<QUESTION 1 CONTINUED ON NEXT PAGE>

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1. (continued)

   a. (1 point)

      Select the prior gamma distribution most appropriate to address the concern that the vendor agreement has slowed the gross loss payment pattern. Justify the selection.

   b. (2 points)

      Calculate the incremental losses gross of S&S for accident year 2018 expected to emerge between 12 and 36 months using the Bayesian model with the most appropriate prior distribution from part a. above.

   c. (0.5 point)

      Describe model risk with respect to the Bayesian model for incremental losses gross of S&S.

   d. (1.5 points)

      Calculate the total S&S recoveries for accident year 2018 using Bayesian credibility.

   e. (0.5 point)

      Describe estimation risk with respect to the Bayesian model for S&S recoveries.

   f. (0.5 point)

      Calculate the total unpaid losses net of S&S for accident year 2018.

   g. (1 point)

      Describe two types of operational risk introduced by the vendor agreement and recommend a unique key risk indicator ("KRI") to monitor each risk.

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2. (3 points)

Given the following information as of December 31, 2018:

Accident Year	Earned Premium ($000)	Incremental Paid Loss ($000) as of (months)		
		12	24	36
2016	5,000	1,800	700	500
2017	6,000	2,000	800	
2018	8,000	2,200		

- Assume there is no further development after 36 months.
- $Var[U_i] = Var[U_i^{BC}]$

a. (1.5 points)

Calculate the accident year 2018 Benktander reserve estimate (R^{GB}).

b. (0.5 point)

Calculate the accident year 2018 optimal credible reserve estimate (R_c).

c. (0.5 point)

Identify which of R_c or R^{GB} is the preferable reserve from a statistical point of view and briefly describe a supporting reason.

d. (0.5 point)

Describe the effect on the Benktander credibility for accident year 2018 if the incremental paid loss from 12 to 24 months for accident year 2017 was greater than the value in the table above.

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3. (2.5 points)

Given the following information as of December 31, 2018:

Accident Year	Earned Premium ($000)	Incremental Paid Loss ($000) as of (months)		
		12	24	36
2016	800	320	220	80
2017	600	300	200	
2018	400	280		

- Assume there is no loss development beyond 36 months.

a. (2 points)

Calculate the total Neuhaus loss ratio claims reserve estimate.

b. (0.5 point)

Describe why the Neuhaus method may not be appropriate for the data in the table above.

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4. (2.75 points)

An insurer writes high deductible workers compensation policies with a per-occurrence deductible of $2,000,000.

Given the following information as of December 31, 2018:

Accident Year	Cumulative Reported Losses ($000) Limited to $1,000,000 as of (months)			
	12	24	36	48
2015	450	1,250	2,500	3,300
2016	350	1,500	2,900	
2017	375	1,400		
2018	325			

Accident Year	Limited Expected Values ($000) at $500,000 Basic Limit as of (months)			
	12	24	36	48
2015	99	266	356	385
2016	114	285	370	397
2017	109	282	367	394
2018	118	294	376	402

Limited Expected Values ($000) at $1,000,000 Limit as of (months)			
12	24	36	48
100	330	528	607
115	367	565	641
110	360	558	635
120	385	583	657

Accident Year	Limited Expected Values ($000) at $2,000,000 Limit as of (months)			
	12	24	36	48
2015	100	349	652	809
2016	115	397	724	885
2017	110	388	709	871
2018	120	421	759	921

Unlimited Expected Values ($000) as of (months)			
12	24	36	48
100	350	690	910
115	400	785	1,035
110	390	765	1,010
120	425	835	1,100

- Assume there is no development after 48 months.
- Accident year 2017 reported losses excess of the $2,000,000 deductible at 24 months are $50,000.
- Accident year 2017 reported losses limited to the $2,000,000 deductible at 24 months are $2,400,000.

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4. (continued)

a. (1.75 points)

Calculate the expected ultimate loss to the insurer for accident year 2017 as of December 31, 2018 using the procedure outlined by Sahasrabuddhe.

b. (1 point)

An exponential claim size model was used to produce the limited expected values in the tables above. Explain how the estimate of the ultimate loss to the insurer would change if a smaller model parameter was used.

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5. (2 points)

A Cape Cod loss reserving calculation has the following inputs and estimates:

- Total premium is $10,000,000.
- Estimated ELR is 65%.
- Process variance/mean ratio is 50,000.
- The parameter covariance matrix is:

	ELR	ω	θ
ELR	0.0029	-0.0042	0.19
ω	-0.0042	0.0055	-0.41
θ	0.19	-0.41	25.52

a. (1 point)

Calculate the coefficient of variation of prospective losses.

b. (0.5 point)

Briefly describe what process variance and parameter variance of the prospective losses measure.

c. (0.5 point)

Briefly describe whether the Cape Cod method typically has a higher or lower parameter variance than the chain ladder method.

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6. (2.5 points)

Given the following information as of December 31, 2018:

Accident Year	On-Level Earned Premium ($000,000)	Cumulative Paid Loss ($000,000) as of (months)		
		12	24	36
2016	13,000	360	1,425	2,850
2017	13,250	375	1,375	
2018	13,500	350		

- The expected loss payment pattern follows a loglogistic curve of the form $\frac{x^{\omega}}{x^{\omega}+\theta^{\omega}}$, where
  - $\omega = 1.448$
  - $\theta = 48.021$
- There are no payments after 120 months.
- Accidents occur uniformly throughout the year.
- The scale parameter, σ^2, is 423.

a. (1.75 points)

Calculate the incremental fitted payment and corresponding normalized residual for accident year 2018 at 12 months using the Cape Cod method.

b. (0.75 point)

Calculate ultimate losses for accident year 2016 using the Cape Cod method.

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7. (3.25 points)

Given the following information for a company as of December 31, 2018:

Cumulative Case Incurred Losses ($000,000) as of (months)						
Accident Year	12	24	36	48	60	72
2013	3,384	4,282	4,813	5,408	5,394	5,362
2014	3,499	4,438	5,646	5,532	5,512	
2015	3,483	5,566	5,539	5,398		
2016	6,502	6,197	6,222			
2017	6,656	6,658				
2018	6,595					

Case Incurred Loss Development Factors as of (months)					
Accident Year	12-24	24-36	36-48	48-60	60-72
2013	1.265	1.124	1.124	0.997	0.994
2014	1.268	1.272	0.980	0.996	
2015	1.598	0.995	0.975		
2016	0.953	1.004			
2017	1.000				

<QUESTION 7 CONTINUED ON NEXT PAGE>

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7. (continued)

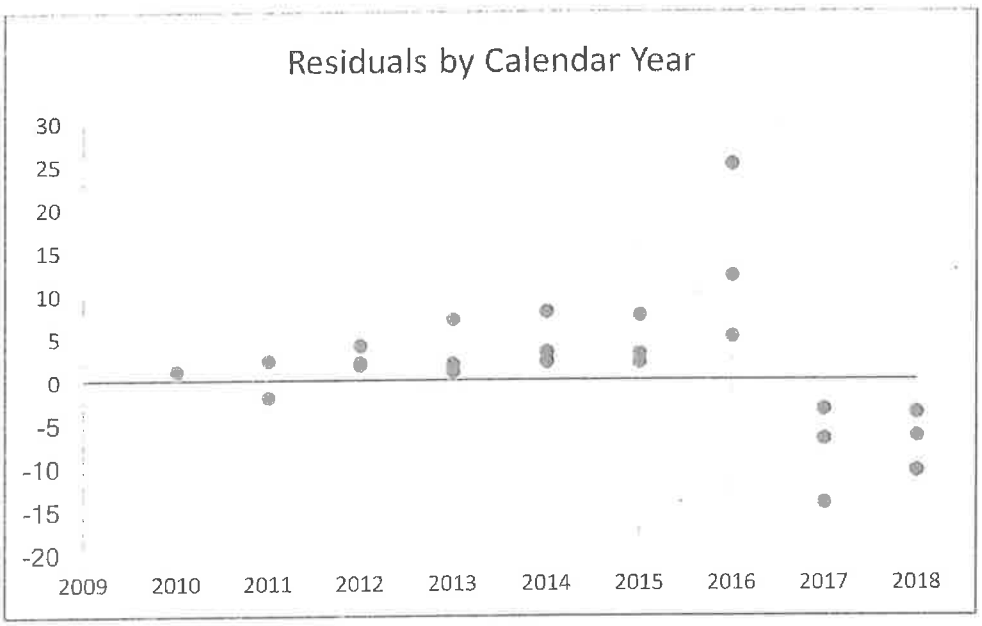
a. (1.75 points)

Test the null-hypothesis that there are no calendar year influences in the data using Mack's methodology assuming an error probability of 5% where:

- $E[Z] = 3.25$
- $Var(Z) = 1.06$
- The 97.5th percentile of the Standard Normal distribution is 1.96.

b. (1.5 points)

The graph below shows the weighted residuals of case incurred losses for maturities of 24, 36, and 48 months.



Discuss whether the results above could be explained by each of the following:

i. Sustained increase in calendar year trend rate in 2016.
ii. Increase in case reserve adequacy in 2016.
iii. Acceleration of loss payment in 2016.

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8. (1.25 points)

   a. (0.25 point)

      Briefly explain when a curve-fitting method for selecting loss emergence patterns will produce a higher mean estimate of ultimate losses than a weighted average method.

   b. (0.5 point)

      Identify one reason why each of the methods in part a. above might be better than the other for estimating the payment pattern.

   c. (0.5 point)

      Briefly explain why the standard deviations of the ultimate losses for each of the scenarios below are narrower than the standard deviation of the ultimate loss for the loss development method using a curve fit to derive the emerged percentages:

      i. Clark Cape Cod method using a curve fit to derive the emerged percentages.
      ii. Loss development method using weighted averages of the development factors.

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9. (2.5 points)

An insurance company specializes in coverage for Workers Compensation, Commercial Auto and General Liability. A new actuary is hired to build a reserving model.

- The model uses both internal and external variables.
- The chosen predictors are different than those selected in a previous model, but have a high correlation with claims experience.
- The model was checked for reasonableness by performing diagnostic tests on valuation outcomes.
- Data has been reconciled against the general ledger and the prior analysis.
- The actuary is not yet familiar with past processes and has not yet met with claims staff.
- The actuary models claims from all lines together.

a. (0.75 point)

Briefly describe three main sources of internal systemic risk according to Marshall.

b. (0.75 point)

Identify a reason why this company would score high in a balanced scorecard approach for each of the three sources of risks identified in part a. above.

c. (0.75 point)

Identify a reason why this company would score low in a balanced scorecard approach for each of the three sources of risks identified in part a. above.

d. (0.25 point)

Briefly describe why a portfolio with the risk indicators identified in part b. above would have a lower Coefficient of Variation (CoV) than those identified in part c. above.

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10. (3.75 points)

Given the following output from a generalized linear model fitted to a triangle of loss development data as of December 31, 2018:

Accident Year	Standardized Pearson Residuals as of (months)		
	12	24	36
2016	-0.54	1.05	3.56
2017	1.43	2.75	
2018	0.00		

Accident Year	Standard Deviation
2016	2.067
2017	0.933
Total	1.570

Accident Year	Fitted Cumulative Losses ($000) as of (months)		
	12	24	36
2016	1,250	2,000	2,150
2017	1,500	2,400	
2018	1,400		

Accident Year	Sample Residual Index (row, column) as of (months)		
	12	24	36
2016	(2, 1)	(1, 1)	(2, 1)
2017	(1, 3)	(2, 2)	
2018	(1, 2)		

- Assume the residuals in accident years 2017 and 2018 are homoscedastic.

a. (2 points)

Construct the triangle of sampled cumulative losses based on the above sampling algorithm and adjusting the residuals as necessary.

Given the following additional information:

Cumulative Distribution Function	Expected Losses
20%	2,500
60%	3,000
100%	3,500

- The tail factor as of 36 months is 1.15.
- 0% loss trend.
- The first stochastically generated random seed for the expected loss is 0.458 and should be applied to the fitted loss triangle.
- The second stochastically generated random seed for the expected loss is 0.812 and should be applied to the sampled loss triangle.

b. (1.75 points)

Calculate the range of stochastic Bornhuetter-Ferguson reserve indications.

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11. (2 points)

Given the following information as of December 31, 2018:

Accident Year	Incurred Losses ($000)	Chain Ladder Ultimate ($000)	Bornhuetter-Ferguson Ultimate ($000)	Bayesian Ultimate ($000)
2013	4,400	4,400	4,400	4,400
2014	5,000	6,000	5,833	5,840
2015	2,800	4,000	4,300	4,278
2016	3,000	5,400	5,222	5,301
2017	4,000	10,000	7,000	7,750
2018	750	3,000	4,500	

- $\alpha_{2018} = 12{,}500$
- $\beta_{2018} = 2.5$
- $\varphi_{2018} = 1.1$

a. (0.5 point)

Compare the variance of the prior distributions for accident years 2016 and 2017.

b. (0.5 point)

Calculate the Bayesian ultimate loss estimate for accident year 2018.

c. (0.5 point)

Calculate the accident year 2014 correlated chain ladder log mean ultimate loss given the following parameters:

- $\alpha_{2013} = 8.5172$
- $\alpha_{2014} = 8.5172$
- $\beta_6 = 0$
- $\rho = 0.2$

d. (0.5 point)

Briefly describe one advantage of each of the following methods:

- Correlated chain ladder method
- Bayesian method

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12. (2 points)

The Exponential Dispersion Family ("EDF") is a family of distributions with probability density function $\pi(y;\theta,\phi)$ of the form:

$$\ln \pi\,(y;\theta,\phi) = \frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)$$

and variance function of the form:

$$V(\mu) = b''((b')^{-1}(\mu))$$

a. (0.5 point)

Briefly describe the parameter θ and the function $b(\theta)$ in the equation above.

b. (0.5 point)

Explain how the Tweedie sub-family can be obtained from the EDF above.

c. (0.25 point)

Briefly describe how to select a Tweedie distribution for a heavy-tailed exposure.

Given the following information as of December 31, 2018:

Accident Year	Cumulative Paid Loss ($000) as of (months)		
	12	24	36
2016	180	1,000	1,200
2017	200	600	
2018	150		

A GLM is used to model the incremental paid losses. The GLM uses a log link function and a Tweedie error distribution specified by:

- $p = 1$
- $b(\theta) = exp\,(\theta)$
- $\mu = exp\,(\theta)$
- $ln\,\pi\,(y;\mu,\phi) = \;[y\,ln\,\mu - \mu]/\phi$

d. (0.75 point)

Calculate the accident year 2018 ultimate loss.

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13. (2.25 points)

Given the following information for a reinsurer as of December 31, 2018:

Accident Year	Earned Risk Pure Premium ($000)	Adjusted Premium ($000)	Aggregate Reported Loss ($000)	Aggregate Loss Report Lag
2015	6,800	7,400	4,500	90%
2016	7,200	7,700	3,400	70%
2017	7,600	7,900	2,300	50%
2018	8,000	8,000	3,000	30%

- Report lag reaches 100% in the 5th year.

a. (1 point)

Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using the Stanard-Bühlmann (Cape Cod) method.

b. (0.25 point)

Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using the chain ladder method.

c. (0.5 point)

Calculate the IBNR estimate for accident year 2018 as of December 31, 2018 using a credibility-weighted combination of the Stanard-Bühlmann and chain ladder methods given a selected credibility factor of 0.6.

d. (0.5 point)

Describe a situation in which an actuary would choose to use a credibility-weighted combination of the Stanard-Bühlmann and chain ladder methods.

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14. (2 points)

Identify and briefly describe four major problems that make reinsurance loss reserving more difficult than primary insurer loss reserving.

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15. (2.25 points)

Given the following information for a retrospectively rated book of business:

- Basic Premium is $25 million.
- Standard Premium is $100 million.
- Tax Multiplier is 1.03.
- Loss Conversion Factor is 1.18.
- Expected Loss Ratio is 0.8.

Retro Adjustment Period	Incremental Loss Capping Ratio	Age-to-Ultimate Loss Development Factor
1st	0.95	2.00
2nd	0.75	1.08
3rd	0.55	1.01

a. (1.5 points)

Calculate the cumulative premium development to loss development (PDLD) ratio at the 1st retro adjustment.

b. (0.25 point)

Briefly describe a different approach for estimating PDLD ratios for retrospectively rated policies.

c. (0.5 point)

Briefly describe an advantage and a disadvantage of using the method in part a. above over the method described in part b. above.

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16. (1.5 points)

Given the following financial information for an insurance company as of December 31, 2018:

- Beginning US GAAP equity as of January 1, 2019 = $195 million.
- US GAAP net income forecast for 2019 = $43 million.
- Change in reserves included in net income forecast for 2019 = $11 million.
- Capital needed to maintain target rating at December 31, 2019 = $220 million.
- Risk free interest rate = 3.0%.
- Expected equity market risk premium = 6.5%.
- Insurance company equity beta = 0.8.

a. (0.5 point)

Calculate the Free Cash Flow to Equity (FCFE) for 2019.

b. (0.5 point)

Calculate the reinvestment rate for 2019.

c. (0.5 point)

Briefly describe two reasons why the FCFE approach is preferred to the Free Cash Flow to the Firm (FCFF) approach for valuing insurance companies.

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17. (3 points)

Given the following financial information for a property and casualty insurance company:

- Projected ROE = 9.0%.
- Projected dividend payout rate = 40.0%.
- Yield on long-term T-bonds = 4.7%.
- Term premium for long-term T-bonds = 1.3%.
- Expected equity risk premium = 7.0%.
- Insurer's equity beta = 0.8.

a. (1 point)

Calculate the company's Price to Earnings (P-E) Ratio assuming projections continue in perpetuity.

b. (0.5 point)

Explain the effect on the company's P-E ratio assuming the dividend payout rate changes to 60%.

c. (0.5 point)

Explain the effect on the company's P-E ratio assuming the yield on long-term T-bonds is 4% and the equity risk premium is unchanged.

d. (1 point)

Describe two alternative uses for the P-E ratio.

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18. (1 point)

    a. (0.5 point)

       Briefly describe two difficulties when using the Equity as a Call Option method to value a P&C insurance company.

    b. (0.5 point)

       Discuss a common difficulty when using the Dividend Discount Model, Discounted Cash Flow, and Abnormal Earnings methods to value a P&C insurance company.

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19. (3.25 points)

An actuary is reviewing results of a hurricane model on a potential new reinsurance program. The actuary simulates five years of losses with the following results:

Combined Ratios			
Simulation	Direct	Net of Current Program	Net of Proposed Program
#1	130%	105%	115%
#2	180%	140%	130%
#3	80%	90%	100%
#4	90%	95%	105%
#5	200%	160%	150%

a. (1.25 points)

Construct a coverage option cumulative distribution function chart using the simulations above.

b. (0.5 point)

Select a reinsurance program and briefly explain your selection based on the simulations above if an actuary is primarily concerned about losses in the tail.

c. (0.75 point)

Calculate the 80th percentile results using direct data for the following:

i. V@R
ii. TV@R
iii. XTV@R

d. (0.25 point)

Define WTV@R.

e. (0.5 point)

Briefly describe a weakness of TV@R and briefly explain how WTV@R corrects the weakness.

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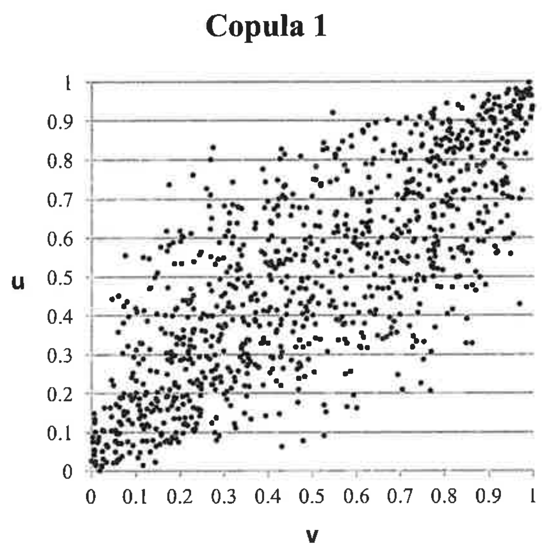
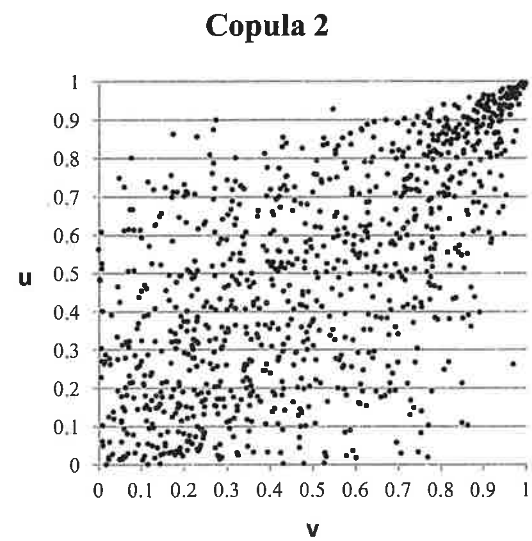
20. (2.25 points)

The following scatter plots were obtained from simulations of two different copulas with the same Kendall τ.





a. (0.75 point)

Identify which copula above is the Normal copula and which is the HRT copula. Fully justify your answer.

b. (0.5 point)

Briefly describe one advantage for each of the two copulas above when used to aggregate the insured losses from multiple lines of business subject to catastrophe events.

c. (1 point)

Explain which of the two copulas above will produce the higher result on the joint distribution for the following two metrics:

i. Expected value
ii. $V@R_{0.9}$

CONTINUED ON NEXT PAGE

21. (2.75 points)

a. (2 points)

Describe a possible explanation why markets may soften for each of the following theories for the underwriting cycle:

i. Institutional factors
ii. Competition
iii. Capacity and capital levels
iv. Economic linkages

b. (0.75 point)

Identify two components of an econometric model of the underwriting cycle and briefly explain how the two components are linked.

END OF EXAMINATION

Exam 7
Estimation of Policy Liabilities, Insurance Company Valuation, and Enterprise Risk Management

POINT VALUE OF QUESTIONS

QUESTION	TOTAL POINT VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	7.00	1.00	2.00	0.50	1.50	0.50	0.50	1.00
2	3.00	1.50	0.50	0.50	0.50			
3	2.50	2.00	0.50					
4	2.75	1.75	1.00					
5	2.00	1.00	0.50	0.50				
6	2.50	1.75	0.75					
7	3.25	1.75	1.50					
8	1.25	0.25	0.50	0.50				
9	2.50	0.75	0.75	0.75	0.25			
10	3.75	2.00	1.75					
11	2.00	0.50	0.50	0.50	0.50			
12	2.00	0.50	0.50	0.25	0.75			
13	2.25	1.00	0.25	0.50	0.50			
14	2.00	2.00						
15	2.25	1.50	0.25	0.50				
16	1.50	0.50	0.50	0.50				
17	3.00	1.00	0.50	0.50	1.00			
18	1.00	0.50	0.50					
19	3.25	1.25	0.50	0.75	0.25	0.50		
20	2.25	0.75	0.50	1.00				
21	2.75	2.00	0.75					
TOTAL	54.75							

SAMPLE ANSWERS AND EXAMINER'S REPORT – SPRING 2019 EXAM 7

SPRING 2019 EXAM 7 EXAMINER'S REPORT

The Syllabus and Examination Committee has prepared this Examiner's Report as a tool for candidates preparing to sit for a future offering of this exam. The Examiner's Report provides:

- A summary of exam statistics.
- General observations by the Syllabus and Examination Committee on candidate performance.
- A question-by-question narrative, describing where points were commonly achieved and missed by the candidates.

The report is intended to provide insight into what the graders for each question were looking for in responses that received full or nearly-full credit. This includes an explanation of common mistakes and oversights among candidates. We hope that the report aids candidates in mastering the material covered on the exam by providing valuable insights into the differences between responses that are comprehensive and those that are lacking in some way.

Candidates are encouraged to review the Future Fellows article from June 2013 entitled "Getting the Most out of the Examiner's Report" for additional insights.

EXAM STATISTICS:

- Number of Candidates: 880
- Available Points: 54.75
- Passing Score: 40.5
- Number of Passing Candidates: 380
- Raw Pass Ratio: 43.2%
- Effective Pass Ratio: 45.0%

GENERAL COMMENTS:

- Candidates should note that the instructions to the exam explicitly say to show all work; graders expect to see enough support on the candidate's answer sheet to follow the calculations performed. While the graders made every attempt to follow calculations that were not well-documented, lack of documentation may result in the deduction of points where the calculations cannot be followed or are not sufficiently supported.
- Integrative Questions (IQs) were first introduced to Exam 7 in 2018 and are being used to test candidates' ability to apply and synthesize multiple advanced reserving, valuation, and/or ERM ideas in addressing complex business problems. The IQ this sitting was based on a real-world scenario and was designed to test multiple syllabus learning objectives at higher cognitive (Bloom's) levels. Candidates should expect to encounter similar sorts of questions in future sittings.

- Candidates should justify all selections when prompted to do so. For example, if the candidate selects an all year average and the question prompts a justification of all selections, a brief explanation should be provided for the reasoning behind this selection. Candidates should note that a restatement of a numerical selection in words is not a justification.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- Candidates should try to be cognizant of the way an exam question is worded. They must look for key words such as "briefly" or "fully" within the problem. We refer candidates to the Future Fellows article from December 2009 entitled "The Importance of Adverbs" for additional information on this topic.
- Some candidates provided lengthy responses to a "briefly describe" question, which does not provide extra credit and only takes up additional time during the exam.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- In cases where a given number of items were requested (e.g., "three reasons" or "two scenarios"), the examiner's report often provides more sample answers than the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses. Candidates are reminded that, per the instructions to the exam, when a specific number of items is requested, only the items adding up to that number will be graded (i.e., if two items are requested and three are provided, only the first two are graded).
- It should be noted that all exam questions have been written and graded based on information included in materials that have been directly referenced in the official syllabus, which is located on the CAS website. The CAS takes no responsibility for the content of supplementary study materials and/or manuals produced by outside corporations and/or individuals which are not directly referenced in the official syllabus.

QUESTION 1	
TOTAL POINT VALUE: 7.0	**LEARNING OBJECTIVE(S): A1, A8, A10, C6, C7**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

Mean (α/β):
Option 1: 25,000
Option 2: 25,000
Option 3: 12,500

Variance ($\frac{\alpha}{\beta^2}$)
Option 1: 12,500,000
Option 2: 3,125,000
Option 3: 125,000

Select Option 2. Mean value is more reasonable. Variance is smaller which gives less credibility to LDF, and more credibility to prior estimate.

Sample 2

We want to select Option 2.
Option 3 results in an expected loss that is half of the other options at 12,500. Based on historical data, an ultimate loss estimate of 25,000 is more reasonable.
Option 2 has higher beta vs Option 1. Thus results in a smaller variance, stronger prior. We want to rely more on our expert opinion and less on historical data. So Option 2 provides the most weight on our expert opinion.

Sample 3

Expected values:
Option 1: 50/0.002 = 25,000
Option 2: 25,000
Option 3: 12,500

Since 2018 is already at 14M as gross loss, I selected Option 1 or 2.
Assuming I am confident in my prior estimate, I choose the one with higher β.
Picking Option 2.

Part b: 2 points

Sample 1

Calculated LDFs: 12-24 LDF = (22500 + 22000)/(16500 + 17000) = 1.3284
24-36 LDF = 25000/22500 = 1.111
LDF_{12} = 1.111*1.328 = 1.475, $\%paid_{12}$ = 1/1.475 = 0.678
LDF_{24} = 1.111, $\%paid_{12}$ = 1/1.111 = 0.900

Calculate Z_{12} = 0.678/(0.008*93 + 0.678) = 0.477
Z_{24} = 0.900/(0.008*93 + 0.900) = 0.547

E[incremental loss at 24 mo] = 0.477*14000*(1.328-1) + (1-0.477)*25000*(0.9-0.68) = 5093
E[incremental loss at 36 mo] = 0.547*(1.111-1)*(14000 + 5093) + (1-0.547)*25000*(1-0.9) = 2291.77

Total expected emergence for AY 2018 = 5093 + 2291.77 = 7384.77 (in 000s)

Sample 2
E(Ult) = 25000
LDF 12-24 = (16500 + 6000 + 17000 + 5000)/(16500 + 17000) = 1.328
24-36 = 1.111
12-ult = 1.475
% rept at 12 = 1/(1.328*1.111) = 68%
z = (%rept)/(beta*phi + %rept) = 0.68/(0.008*93 + 0.68) = 0.478
0.478*(1.475 – 1)*(14000) + (1-0.478)*(1.475 – 1)*(25000)*(0.68) = 7394

Sample 3
Gamma Mean = 25000
LDFs: Age 12 = 1.328. Age 24 = 1.111
CLDFs: Age 12 = 1.475, Age 24 = 1.111
% Paid: Age 12 = 0.678, Age 24 = 0.9
% Emerged: Age 12-24 = 0.222, Age 24-36 = 0.1
Z_{12} = 0.678/(0.008*93 + 0.678) = 0.477
CL Ultimate = 20649
$CL_{12\text{-}24}$ = 4584, $CL_{24\text{-}36}$ = 20649*0.1 = 2065
$BF_{12\text{-}24}$ = 5500, $BF_{24\text{-}36}$ = 25000*0.1 = 2500
Wtd12-24 = 5063, Wtd24-36 = 2065*(0.477) + 2500*(1 – 0.477) = 2292.5

Part c: 0.5 point

Sample 1
1. The prior distribution of Gamma might not be true
2. The incremental loss might not follow an ODP distribution
3. Bayesian model won't perfectly reflect the process. Model risk is the uncertainty that the model won't perfectly reflect the insurance process

Sample 2
Model risk is risk of not specifying the correct model. In Bayesian model, we weigh our faith in the specified model via the β parameter, which controls how much weight is given to the model (chain ladder) versus our a priori estimate. Higher β reduces the credibility measure, thus giving more weight to our a priori estimate.

Sample 3
The competitor experience may not apply to the insurer. If it is the case, the prior distribution selected may not be accurate. And hence the model can't incorporate the expert opinion very well.

Sample 4

Model risk is the risk of selecting the incorrect loss distribution for the a priori expectation or that we are not using the correct credibility procedure.

Sample 5

The Bayesian model is putting low credibility on actual experience and high credibility on the gamma distribution estimate. This is risky to put a lot of credibility on the assumption of a slow down in a green year with no historical data to back it up.

Part d: 1.5 point

Sample 1

VHM = Var [E(X/Y)] = $d^2 \cdot \sigma^2 = (0.75)^2 \times 1^2 = 0.563$

EVPV = E [Var (X/Y)] = $\sigma_d^2 (\sigma_y^2 + E(Y)^2 = (0.1)^2 \times (1^2 + 5^2) = 0.26$

Z = 0.563 / (0.653 + 0.26) = 68.4%

Ult_{SS} = 68.4% x [(14 – 11) / 0.75] + (1 – 68.4%) x 5 = 4.31 million

Sample 2

E(Y) = 5,000,000 σ(Y) = 1,000,000 E(X/Y) = 0.75 σ(X/Y) = 0.1

VHM = [.75 (1,000000) $]^2 = 5.625 \times 10^{11}$

EVPV = .12 $(5M^2 + 1M^2) = 2.6 \times 10^4$

Z = VHM / (VHM + EVPV) = .684

S&S to date ($000) = 14,000 – 11,000 = 3,000

S&S Recov for AY 2018 ($000) = 3,000 / .75 x (.684) + (1 - .684)(5,000) = $4,316

Part e: 0.5 point

Sample 1

- Estimation risk is the risk that the forms and parameters chosen don't reflect the "true" form and parameters, due to estimating from a sample of the data.
- For S&S, the department took data from competitors of comparable size. However, there could be a difference in their mix of business, their credibility of data, and assumptions/definitions that can cause our estimates from it to misrepresent the data.

Sample 2

As data obtained from competitors, the E(S&S) and σ might be wrong for our company (different make-ups, lines, practice, etc.)

Sample 3

Estimation risk is risk we have not correctly estimated a parameter for the loss emergence, and specifically for this, there is a risk that our estimate of E[X/Y] (% S&S emerged @12mo.) is inaccurate, especially since we don't have our own past data to go on, only competitor data which might not reflect our business.

Sample 4

Estimation risk – the risk that the selected parameters of the model are not the "true" parameters of the distribution, would result from the appropriateness of the selected competitors for comparison regarding the similarity in lines of business, states written and vendor selection to the company.

Part f: 0.5 point

Sample 1

Future Expected Recoveries = 4.316 – 3M = 1.316M

Total Unpaid Net of S&S = 7.396M – 1.316M
= 6.08M

Sample 2

AY 2018 Ultimate Loss Gross S&S = 14M + 7.395M = 21.395M
AY 2018 Total S&S = 4.316M
AY 2018 Ultimate Loss Net S&S = 21.395M – 4.316M = 17.079M
Paid Loss Net S&S = 11M
Unpaid Net S&S = 6.079M

Part g: 1 point

Any two of the following samples:

Clients & Business Practices – not meeting professional obligations to clients; could be trying to over recover and receiving complaints to the DOI. (KRI – monitor number of complaints)

External Fraud – S&S company may put falsified information into the claims and return less money to the company (KRI – external auditor's reports)

Execution & Process Change Management – employees have never done S&S before; this could lead to people getting upset over the added work or over glitches that may occur during implementation (KRI – monitor employee turnover ratio)

Internal Fraud – since there will be an increased number of transactions with another company, there may be more opportunities for accounting people to slip in fraudulent transactions to steal money from the insurer (KRI – keep track of the number of ledger entries for transactions with the vendor and look for any unexpected increases)

Recovery Risk – comes from uncertainty of recovery which could be a significant amount of total gross claim (KRI – recovery amount / total gross claim amount)

System Failure – integrating our systems within the new S&S processes could cause problems, perhaps from tying our reporting systems & the vendors payments, and tracking systems; this could possibly cause processing center down time (KRI – processing center down time)

EXAMINER'S REPORT

Candidates were expected to evaluate models across multiple areas of risk, and select the most appropriate model based on the relationship between model parameters, desired credibility, and changes in the expected claims environment. Candidates were also expected to apply prior distributions and mean/variance assumptions to estimate reserves after appropriately blending this information with the indications implied by historical claims development.

Part a

Candidates were expected to demonstrate knowledge with identifying data issues and related model adjustments for reserving models, as well as testing assumptions underlying reserving models.

Common mistakes included:

- Selected Option 3 with the lowest variance (the highest β) without considering the reasonability of mean under Option 3
- Correctly selected Option 2 with lower variance, however, incorrectly linked lower variance (higher β) to more weight to Chain Ladder (weaker prior), or incorrectly linked lower variance to reflect slow down payment pattern
- Incorrectly selected Option 1 with larger variance (lower β) to put less weight on prior estimate
- Selected Option 2 as the variance is in the middle of three options
- Calculated standard deviation from three options, and selected Option 2 as it is the closest to competitor's standard deviation of 1M
- Calculated variance using Accident Year 2016 and 2017 ultimate losses, and selected Option 2 as it has the closest variance
- Mean or variance were calculated incorrectly

Part b

Candidates were expected to combine information from the loss triangle and prior gamma distribution to calculate future incremental payments based on the provided credibility formula.

Common mistakes include

- Using the Net of S&S instead of the Gross of S&S triangle
- Estimating %pd as the reciprocal of age-to-age LDFs instead of age-to-ultimate LDFs
- Only estimating incremental paid losses for 12-24 months and not 12-36 months
- When using different credibility weights for the 12-24 and 24-36 periods, using the wrong estimate for $D_{2018,24}$ in the credibility formula, most commonly by using the CL method indicated incremental losses instead of applying the CL development factors to 14k + the credibility wtd mean of $C_{2018,24}$
- Using incremental %pd amounts instead of cumulative %pd amounts in the calculation of z factors
- Using one Gamma prior distribution to calculate M_i and a different Gamma prior distribution to estimate Z
- Applying LDFs to M_i without adjusting M_i for the %pd implied by the LDFs
- When applying the credibility formula to ultimate loss estimates instead of reserve estimates, candidates often applied the complement of credibility to the prior Gamma mean instead of the BF method (i.e. 14k + Gamma Apriori*%unpaid)

Part c

Candidates were expected to define model risk and relate it to the context of this problem's Bayesian model for losses gross of S&S. However, full credit was also given to candidates who explained how model risk related to this problem without defining it, as the question did not explicitly ask for a definition.

Common mistakes include

- Failing to link model risk back to the question, and only defining it
- Only discussing S&S model risk
- Discussing estimation risk. Although estimation risk is a subset of model risk, estimation risk is asked directly later in the problem.

Part d

Candidates were expected to understand and apply a Bayesian credibility method as presented in the Brosius paper to the specific situation described in this question and provide an ultimate S&S estimate using that approach.

Common mistakes included:

- Errors in calculating the VHM and EVPV values, either from applying the wrong formulas, using the incorrect inputs or having calculation errors.
- Errors in credibility weighting a developed loss amount and an expected loss amount. These included not properly developing the $3M of S&S paid to date to an ultimate value or not incorporating the appropriate expected ultimate amount into the formula presented in the syllabus reading.

Part e

Candidates were expected to understand the estimation risk, and why the insurer would have estimation risk from using only competitor's data.

Common mistakes included:

- Only provided definition of estimation risk
- Failed to comment on insurer using competitor's data to estimate the S&S parameters

Part f

Candidates were expected to recognize that the total unpaid losses gross of S&S was calculated in part b. and the ultimate S&S recoveries was calculated in part d., and then to combine those responses to come up with the total unpaid losses net of S&S.

Common mistakes included:

- Not subtracting the paid-to-date losses
- Calculating the ultimate losses net of S&S instead of the unpaid losses net of S&S
- Using the Bayesian model for losses gross of S&S to estimate the losses net of S&S

Part g

Candidates were expected to define two operational risks and discuss how the risk is related to the agreement with the S&S vendor. Also, candidates were expected to identify quantifiable KRIs that related to the operational risk described.

Common mistakes include:

- Identifying an operational risk, such as 'External Fraud', but failing to describe the risk
- Providing insufficient detail for a KRI, such as stating 'monitor S&S'
- Identifying the following as operational risks.

<u>Expense risk</u> – risk that expenses increase as a result of vendor agreement

- Expenses are expected to increase with the agreement. An operational risk would be if the expenses increased more than expected

Reserving process risk – slowing down the claim payment patterns and affecting reserve projections

- It was stated in the scenario that claim payment patterns are slowing. An operational risk would be if this slowdown upsets insureds and impacts customer retention

QUESTION 2	
TOTAL POINT VALUE: 3 points	LEARNING OBJECTIVE(S): A1
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

$$m_1 = \frac{1800+2000+2200}{5000+6000+8000} = 0.316$$
$$m_2 = \frac{700+800}{5000+6000} = 0.136$$
$$m_3 = \frac{500}{5000} = 0.1$$

$$ELR = 0.316 + 0.136 + 0.1 = 0.552$$
$$p_1 = 0.316/0.552 = 0.572 \quad p_2 = (0.316 + 0.136)/(0.552) = 0.819 \quad p_3 = 1$$
$$R^{GB} = q_k * U^{BF}$$
$$U^{BF} = 2200 + (1 - 0.572) * 0.552 * 8000 = 4090$$
$$R^{GB} = (1 - 0.572) * 4090 = 1750.52$$

Sample 2

$$m_1 = \frac{1800+2000+2200}{5000+6000+8000} = 0.3158$$
$$m_2 = \frac{700+800}{5000+6000} = 0.1364$$
$$m_3 = \frac{500}{5000} = 0.1$$

$$ELR = \sum m_i = 0.5522$$
$p_1 = m_1/ELR = 0.5719 \quad q_1 = 0.4281$ → 2018

2018: $R^{ind} = \frac{Loss}{p_i} * q_i = \frac{2200}{0.5719} * 0.4281 = 1646.67$

$$R^{Coll} = EP * ELR * q_i = 8000 * 0.5522 * (0.4281) = 1890.91$$
$$Z^{GB} = p_i = 0.5719$$
$$R^{GB} = Z^{GB} * R^{ind} + (1 - Z^{GB}) * R^{Coll} = (0.5719) * 1646.67 + (1 - 0.5719) * 1890.91 = 1751.22\ (in\ 000s)$$

Sample 3

$$m_1 = (1800 + 2000 + 2200)/(5000 + 6000 + 8000) = 0.316$$
$$m_2 = (700 + 800)/(5000 + 6000) = 0.136$$
$$m_3 = 500/5000 = 0.1$$
$$ELR = \sum m_i = 0.552$$
$$p_1 = 57.2\% \quad p_2 = 81.9\%$$
$$R^{CL} = \frac{(2200)}{57.2\%} * (1 - 57.2\%) = 1646$$
$$R^{BF} = 8000 * (0.136 + 0.1) = 1888$$
$$Z = p_1 = 0.572$$
$$R^{GB} = (0.572) * 1646 + (1 - 0.572) * 1888 = 1750$$

Sample 4

$ULT^{GB} = CL\ Ult\ (1-q^2) + U_0 * (q^2)$

$m_1 = \frac{1800+2000+2200}{5000+6000+8000} = 0.316$

$m_2 = \frac{700+800}{5000+6000} = 0.136$

$m_3 = \frac{500}{5000} = 0.1$

$ELR = \sum m = 0.552$

$p_1 = 0.316/0.552 = 0.572 \quad q_1 = 1 - 0.572 = 0.428$

$AY\ 2018\ R^{GB} = (1-0.428^2) * (2200) * \left(\frac{1}{0.572} - 1\right) + 0.428^2 * (0.552 * 8000 - 2200) = 1750$

Sample 5

Since exposure has been increasing, I'm going to model loss ratio.

Cumulative Loss	12	24	36
2016	1,800	2500	3000
2017	2,000	2800	
2018	2,200		

$12 - 24 factor = \frac{(2500+2800)}{(1800+2000)} = 1.395 \quad 24 - 36 factor = \frac{3000}{2500} = 1.395$

$12 - Ult = 1.395 * 1.2 = 1.674$

$CL\ Ult\ for\ AY\ 2018 = 2200 * 1.674 = 3682.8$

Since we are not giving an expectation at losses, I'm going to use Hurlimann's approach to estimate loss ratio.

Loss Ratio

$m_k = \frac{1800+2000+2200}{5000+6000+8000} + \frac{700+800}{5000+6000} + \frac{500}{5000} = 0.552$

$U_0\ for\ AY\ 2018 = 8000 * 0.552 = 4416$

$Paid\ \%\ for\ AY\ 2018 = \frac{1}{1.674} = 0.597$

$U^{BF} = 2200 + (1-0.597) * 4416 = 3979.65$

$U^{GB} = 2200 + (1-0.597) * 3979.65 = 3803.8$

$R^{GB} = 3803.8 - 2200 = 1603.80$

Part b: 0.5 point

Sample 1

Since Var (U_i)=Var (U_i^{BC}) $Z^{opt} = \frac{p_i}{p_i+\sqrt{p_i}}$ because $t_i = \sqrt{p_i}$

$R_c = Z^{opt} * R^{Ind} + (1 - Z^{opt}) * R^{Coll}$

$R_c = 1646.67 * 0.4307 + 1890.91 * (1 - 0.4307) = 1785.74$ (in 000s)

Sample 2

$Z^c = \frac{p_k}{p_k+\sqrt{p_k}} = 0.572/(0.572 + \sqrt{0.572}) = 0.431$

$R^{ind} = \frac{2200}{0.572} - 2200 = 1646.15$

$R_c = Z^c * R^{Ind} + (1 - Z^c) * R^{Coll} = 0.431 * 1646.15 + (1 - 0.431) * (4090 - 2200) = 1784.9$

Sample 3

$U^{CL} = \dfrac{2200}{57.2\%} = 3846$

$U^{BF} = 4090$

Z = $\frac{P}{P+\sqrt{P}}$ =43.1%

$U_c = Z * U^{CL} + (1 - Z) * U^{BF} = 3988$

$R_c = 3985 - 2200 = 1785$

Sample 4

CDF = 1.2*1.3947 = 1.67364 $p_i = \frac{1}{1.67364} = 59.75\%$

Z = $\frac{p_i}{p_i+\sqrt{p_i}}$ = $0.5975/(0.5975 + \sqrt{0.5975}$) =0.436

Res CL = 2200 * (1.6736 -1) = 1482.00

Res BF = (1-0.5975)*(0.552*8000) = 1777.44

AY 2018 opt cred res = 1482*0.436 + (1-0.436)* 1777.44 = 1648.63K

Part c: 0.5 point

Sample Responses

- R_c (optimal credibility) is a preferable reserve from a statistical stand point because it minimizes the MSE of the estimate, $MSE_c < MSE_{GB}$
- R_c is preferred because optimal credibility has the lowest of variance.
- R_c is the preferable reserve because it minimizes the MSE of the reserve.

Part d: 0.5 point

Sample Responses

- If that loss was greater, then the overall loss ratio would be larger. Since the incremental LR at 12 months stays the same, p would decrease, which means the credibility also decreases. If the incremental loss was greater than above, this would increase m_2, which would also increase the ELR. m_1 would stay the same so p_{2018} = m_1/ELR would decrease. Since the credibility for Benktender is Z=p_{2018} the credibility would decrease as well.

- If 12-24 incremental paid loss increased, this increase your 12-24 LDF. Since the % paid is based off of the calculated CDFs (1/CDF), the % paid at 12 months would decrease. Since your credibility factor is % paid, the credibility going to the current paid losses would decrease.

- So if it wasn't 800, if it was bigger:
  - m_1 would remain the same
  - m_2 would increase
  - m_3 would remain the same
  - ELR = $\sum m_k$'s would increase because m_2 is larger

Cred Z^{GB} = p_1 = $\frac{m1}{ELR}$ = $\frac{m1\ remains\ the\ same}{ELR\ would\ increase}$ → p_1 would decrease so Z decreases
Answer: The credibility Z^{GB} would decrease

- ELR would increase and m_1 would be unaffected, leading to a decrease in p and a following decrease in Z (Z^{GB} = p). So credibility would decrease (more weight R^{Coll})

EXAMINER'S REPORT

Candidates were expected to know the key assumptions underlying the Benktander and Optimal reserve estimates, how to apply credibility, including mechanics of the methods (e.g., loss ratio based payout factors), be able to determine whether a particular reserve estimate is or is not preferable for the given condition, and understand the impact on Benktander credibility if loss condition has changed.

Part a

Candidates were expected to know how to calculate Benktander reserve estimate correctly, including all of the required components such as Hurlimann loss ratio payout factors, ELR and p (which is the Benktander credibility). They were expected to know how to calculate individual reserve estimate, collective reserve estimate, and application of credibility to these methods to get the Benktander reserve estimate.

Common mistakes included:

- Not using the right Benktander reserve estimate formulae
- Calculation error in the components in Benktander reserve estimate formulae
- Mistakenly considered BF reserve estimate as Benktander reserve estimate
- U^0 was incorrectly used instead of U^{Coll}
- Application of the wrong credibility (e.g., use (1-p) as credibility on Individual reserves).

Part b

Candidates were expected to know how to calculate optimal credible reserve estimates correctly. They were also expected to be able to calculate optimal credibility given assumption in question and apply credibility correctly.

Common mistakes included:

- Not using the correct optimal credible reserve estimate formulae (e.g., R^{GB} was mistakenly used instead of R^{BF})
- Calculation error in the components in optimal credible reserve estimate formulae
- Application of the wrong credibility (e.g., use (1-p)) as credibility on Individual reserves

Part c
Candidates were expected to know which reserve estimate (Benktander or optimal credible) is preferable and the reason of such conclusion, from a statistical point of view, given the assumption in the question. Common mistakes included: • Misstating that Benktander reserve estimate is preferable • Not able to provide the rationale from statistical point of view • Calculating MSE for Benktander and optimal credible reserve estimate with the wrong formula or calculation error
Part d
Candidates were expected to assess how the changes in incremental losses can impact Benktander credibility and provide the rationale for it. Common mistakes included: • Incorrect conclusion regarding the credibility impact • Using the wrong credibility to base conclusion on • Not providing any supporting reason for conclusion

QUESTION 3	
TOTAL POINT VALUE: 2.5	LEARNING OBJECTIVE(S): A1, A2
SAMPLE ANSWERS	
Part a: 2.0 points	

Sample 1

Neuhaus credibility weights CL and BF estimates using z = ELR * pi

$m_1 = (320 + 300 + 280) / (800 + 600 + 400) = 0.5$
$m_2 = (220 + 200) / (800 + 600) = 0.3$
$m_3 = (80) / (800) = 0.1$

ELR = $\sum mi = 0.90$

$p_1 = m_1 / ELR = 0.556$
$p_2 = (m_1 + m_2) / ELR = 0.889$
$p_3 = 1$

AY	CL Reserve	BF Reserve	z	Neuhaus Reserve
2016	0	0	0.90	0
2017	62.43	59.94	0.80	61.932
2018	223.60	159.84	0.50	191.720
				253.652 ($000)

Example calcs for AY 2017:

- CL Reserve = (300 + 200) / 0.889 – (300 + 200)
- BF Reserve = 600 * (0.90) (1 - 0.889)
- Z = 0.90 (0.889)
- Neuhaus Reserve = 0.80 (62.43) + (1 - 0.8) (59.94)

Sample 2

	12	24	36	L/R
Mk	.5	.3	.1	.9
Pk	.556	.889	1	
Incremental payout	.556	.333	.111	

AY	ULT ind	ULT coll	Pk * L/R = zNH	ULT NH	Reserve NH
2016	620	620	0	620	0
17	562	560	.8	561.6	61.6
18	504	440	.5	472	192
					253.6

Sample 3

Neuhaus: $z_k = p_k * ELR$

$m_{12} = (320 + 300 + 280) / (800 + 600 + 400) = 0.5$
$m_{24} = (220 + 200) / (800 + 600) = 0.3$
$m_{36} = (80) / (800) = 0.1$

ELR = 0.5 + 0.3 + 0.1 = 0.9 = 90%

$p_{12} = 0.5 / 0.9 = 55.569\%$
$p_{24} = 0.3 / 0.9 = 33.33\%$
$p_{36} = 0.1 / 0.9 = 11.11\%$

$R^{coll}{}_{2017} = 11.11\% * 90\% * 600 = 60$
$R^{coll}{}_{2018} = (11.11\% + 33.33\%) * 90\% * 400 = 160$

$R^{ind}{}_{2017} = (500 * 11.11\%) / 88.89\% = 62.5$
$R^{ind}{}_{2018} = (280 * 44.44\%) / 55.56\% = 224$

$z_{2017} = 0.8 = 0.9 * 8/9$
$z_{2018} = 0.5 = 0.9 * 5/9$

$R^{Neuhaus}{}_{2017} = 0.8 * 62.5 + 0.2 * 60 = 62$
$R^{Neuhaus}{}_{2018} = 0.5 * 224 + 0.5 * 160 = 192$
Total Reserve = 62 + 192 = 254

Part b: 0.5 point

Sample Responses

- It might not be appropriate because it assumes a constant ELR for all AY's, and as the book shrinks the incremental LR's seem to be increasing
  Incremental LR @ 12 months
  2016 = 0.4
  2017 = 0.5
  2018 = 0.7
- Neuhaus uses a common LR for all years to compute the credibility factor z but here loss ratio seems to be changing largely year over year as EP declines so using one LR in the credibility weight (as well as to estimate R_{coll}) is not appropriate.
- WN assume constant ELR across AY's.
  The data exhibits increasing trend for ELR's
  WN is not appropriate

AY	OLEP	p_i	Used Up P	RL	ELRi = RL / Used Up P
2016	800	1.0	800	620	=620/800 = 77.5%
2017	600	0.889	533	500	93.8%
2018	400	0.556	222	280	126.1%

- Using an expected loss ratio should mean you expect a consistent loss ratio and/or exposure base. This doesn't seem to be the case here. Premiums have dropped 50% in 2 years, meanwhile the losses have not dropped substantially meaning loss ratios appear to be increasing. This actually has the effect of giving high credibility to a shrinking line.
- Neuhaus works for stable business which is not the case here as the premium is decreasing from 2016-2018
- Premium has been declining every year, may indicate change in mix of business. Neuhaus gives weight to the collective loss ratio reserve which assumes a constant loss ratio every year. Declining premium may indicate that constant loss ratio cannot be assumed.
- The Neuhaus may not be appropriate because there is a decline in earned prem over the 3 accident years, and the credibility calculation may distort due to the weight placed on the collected method.
- The premium volume is shrinking over time. This may indicate historical premium/losses are not representative of current/future premium and loss. The Neuhaus method assumes the overall ELR, calculated from the data, is appropriate for all years. This may not be the case here if the mix of business / overall loss ratio is changing over time.

EXAMINER'S REPORT

Candidates were expected to be able to calculate Neuhaus reserve estimates, understand the mechanics of the method, and test the suitability of this model to the given data set.

Part a

The candidate was expected to know how to estimate reserves according to the Neuhaus method.

Common mistakes included:

- Not using cumulative paid losses in the 2017 year when estimating individual reserves.
- Only calculating reserves for a single year (2017 or 2018)
- Calculating a non-zero reserve for the 2016 accident year
- When estimating reserves for either the collective or individual method, neglecting to multiply by q (the loss ratio reserve factor)
- Deriving payout factors and/or individual reserve estimates based on chain ladder LDF method
- Calculating reserves based on the expected loss ratio method rather than the BF reserve as the collective reserve estimate
- Misinterpreting the claims triangle as cumulative instead of incremental losses

Part b

Candidates were expected to identify patterns in the premium or loss and their fit to the Neuhaus model assumptions. A variety of responses were accepted in regard to tying the observation back to the Neuhaus method, including responses where candidates commented on either how the Neuhaus method works or underlying assumptions of the method.

Common mistakes included:

- Not specifically commenting on the trends evident in the data.
- Not tying the trends in the data back to the Neuhaus method.

QUESTION 4	
TOTAL POINT VALUE: 2.75	**LEARNING OBJECTIVE(S): A4**
SAMPLE ANSWERS	
Part a: 1.75 points	

Sample 1

Adjusted Reported Loss @ Basic Limit @ 2018 Level

	@24	@36	@48	
2015:	1114	1780	2186	1780 = 2500 x 376 / 528
2016:	1202	1930		1930 = 2900 x 376 / 565

	24-36	36-48	48-Ult	
LDF	1.602	1.228	1.0	LDF = (1780 + 1930) / (1114 + 1202) = 1.602
CDF	1.967	1.228	1.0	CDF = 1.602 x 1.228 x 1.0 = 1.967

Adjusted CDF

1.967 x [(1010 – 871) / 402] / [(390 – 388) / 294] = 99.98

2017 Ultimate XS Loss of $2M

50,000 x 99.98 = 4,998,969

Sample 2

15: 1250(294/330) = 1114 2500(376/528) = 1780 3300(402/607) = 2185

16: 1500(294/367) = 1202 2900(376/565) = 1930

	24-36	36-48
Development Factors:	1.602	1.228

LDF to Ultimate = 1.602 x 1.228 = 1.967

Ult XS Loss = 50k x 1.967 x [(1010/402)/(390/294)] x [(1- 871/1010) / (1 – 388/390)] = 4.999M

Sample 3

Basic limit triangle at the latest limit (AY 2018 level)

	12	24	36	48
2015:	Not used	1114	1780	2186
2016:	Not used	1202	1930	

2015:24 = 1250 x 294/330 = 1114 2015:36 = 2500 x 376 / 528 = 1780

2015:48 = 3300 x 402/607 = 2186

2016:24 = 1500 x 294/367 = 1202 2016:36 = 2900 x 376 / 565 = 1930

CDF = 1.602 x 1.228 = 1.967

1.602 x 1.228 x [(1010 – 871) / 402] / [(390 – 388) / 294] = 1.967 x 0.346 / 0.007 = 97.226

50,000 x 97.226 = $4,861,300 → Ultimate excess loss for a 2M deductible.

Part b: 1 point

Sample Responses

- A smaller model parameter would lead to a smaller difference between the LEVs. If the LEVs are closer, the CDF adjustment will be larger and the ultimate will therefore be larger.
- If a smaller model parameter were used, that would mean the expected unlimited value was smaller, thus less claims would be expected to be capped. The XS LDF would be larger (the denominator would shrink more than the numerator in the adjustment) so the expected ultimate loss would increase.
- If a smaller model parameter was used, then there would be less impact of limits. This would make the limited LEVs closer to the unlimited expected value. The XS losses would be a smaller portion. This would make XS losses more leveraged, increasing the XS CDF, increasing the XS ultimate.

EXAMINER'S REPORT

Candidates were expected to know how to calculate the expected ultimate for a given layer of losses and understand the relationship between model parameters, development patterns and estimated ultimate losses.

Part a

Candidates were expected to calculate the expected ultimate loss using the Sahasrabuddhe approach for an insurer that writes $2,000,000 high deductible workers compensation policies.

Common mistakes included:

- Not converting the reported losses limited to $1M to 2018 basic limits when calculating loss development factors.
- Not subtracting the AY 2017 limited expected values at $2M from the AY 2017 unlimited expected values to calculate the 24-ultimate LDF excess of $2M.
- Calculating the total ultimate loss and subtracting the limited loss at $2M to get the insurer's expected ultimate losses.
- Not using the method prescribed by Sahasrabuddhe.

Part b

Candidates were expected to understand how a change to the claim size model parameter would affect the limited expected values and impact the estimated ultimate loss to an insurer writing high deductible workers compensation policies.

Common mistakes included:

- Not knowing that a smaller parameter of an exponential will cause a lower percentage of losses eliminated and the relationship between the limited expected values and the unlimited expected values changes.
- Not considering the impact that the changing limited expected values has to the LDF adjustment factor. Specifically, the change at earlier maturities of the limited expected value at $2M to the unlimited expected value is smaller than at later maturities.

QUESTION 5	
TOTAL POINT VALUE: 2.0	**LEARNING OBJECTIVE(S): A2, A3**
SAMPLE ANSWERS	
Part a: 1.0 point	

Sample 1

$$E[Loss] = ELR \times Premium = 10{,}000{,}000 \times 0.65 = 6{,}500{,}000$$

$$Process\ Variance = \sigma^2 \times E[Loss] = 50{,}000 \times 6{,}500{,}000 = 3.25 * 10^{11}$$

$$Parameter\ Variance = Var[ELR] \times Premium^2 = 0.0029 \times 10{,}000{,}000^2 = 2.9 * 10^{11}$$

$$Std\ Dev = \sqrt{Process\ Var. + Parameter\ Var.} = \sqrt{3.25x10^{11} + 2.9x10^{11}} = 784{,}219$$

$$CV = \frac{Std\ Dev}{E[Loss]} = \frac{784{,}219}{6{,}500{,}000} = 12.06\%$$

Sample 2

$$Mean = 10M \times 0.65 = 6.5M$$

$$\frac{Process\ Var}{Mean} = 50{,}000; Process\ Var = 3.25 * 10^{11} = Process\ Std\ Dev = 570{,}088$$

$$CoVprocess = \frac{Process\ Std\ Dev}{Mean} = \frac{570{,}088}{6{,}500{,}000} = 0.0877$$

$$CoVparameter = \frac{\sqrt{0.0029}}{0.65} = 0.0828$$

$$CoV = \sqrt{CoVprocess^2 + CoVparameter^2} = \sqrt{0.0877^2 + 0.0828^2} = 0.121$$

Part b: 0.5 point

Sample Responses

- Process variance measures the uncertainty that arises due to random fluctuations in the data, while parameter variance measures uncertainty due to parameter estimates differing from the true parameters of the distribution.
- Process variance: risk from inherent randomness of the insurance process. Parameter variance: risk from the incorrect parameterization of all predictors.
- Process variance: randomness inherent in the insurance modeling process. Parameter variance: uncertainty arising from parameter selection/estimation.
- Process variance is natural uncertainty due to inability to predict the insurance process. Parameter variance is uncertainty in the parameters used to estimate prospective losses.

Part c: 0.5 point

Sample Responses

- Cape Cod has a lower parameter variance since it is using more information (premium exposure).
- Cape Cod method has lower parameter variance as less parameters are being estimated.

- Cape Cod usually has a lower parameter variance because LDF method overparameterizes the data. Cape Cod uses less parameters. Cape Cod also incorporates more information (premium/exposure) which also leads to a lower parameter variance.

EXAMINER'S REPORT

Part a

Candidates were expected to understand the relationship that the process and parameter variances had to the coefficient of variation of prospective losses. Additionally, they were expected to be able to calculate both variances and the expected value of prospective losses based on the information provided, and then correctly combine them for the total CV.

Common mistakes included:

- Using an incorrect equation for parameter variance (e.g. trying to calculate the parameter variance of reserves instead of prospective losses).
- Using the correct parameter variance equation but replacing the premium with expected losses.

Part b

Candidates were expected to be able to properly define both process and parameter variance in sufficient detail.

A common mistake included defining parameter variance as "the variance of the parameters", instead of mentioning that it is a result of the estimated/selected parameters.

Part c

Candidates were expected to understand the differences in parameters between the Cape Cod and the chain ladder methods, and how the differing number of parameters influences the size of the parameter variance.

A common mistake included correctly identifying that the Cape Cod method has fewer parameters than the chain ladder method, but then stating that this equated to a higher parameter variance.

QUESTION 6

TOTAL POINT VALUE: 2.5 | **LEARNING OBJECTIVE(S): A2**

SAMPLE ANSWERS

Part a: 1.75 points

Sample 1

G(X) = $\frac{x^{\omega}}{x^{\omega}+\theta^{\omega}}$ Used Prem = G(X) * Prem

Ult = ELR * EP

AY	X	G(X)	"Used Prem"	Ult
trunc	114	0.776		
2016	30	0.336	4,367.92	7,847.81
2017	18	0.195	2,577.45	7,998.73
2018	6	0.047	633.17	8,149.65
			7,578.54	

ELR = (2,850 + 1,375 + 350) / 7,578.54 = .604

Expected Inc. $_{2018,12}$ = .047(8149.65) = 382.23

Norm Resid = $\frac{actual - expected}{\sqrt{\sigma^2 \times expected}}$ = $\frac{350 - 382.23}{\sqrt{423 \times 382.23}}$ = -.08

Sample 2

AY	Avg Age	G(X)	G(X)/ 0.776	"Used Prem"
trunc	114	0.776	1.0000	
2016	30	0.336	0.4321	5,617
2017	18	0.195	0.2501	3,314
2018	6	0.047	0.0603	814
				9,745

ELR = (2,850 + 1,375 + 350) / 9,745 = .4695

Expected Inc. $_{2018,12}$ = = 13,500 x 0.4695 x .0603 = 382.2

Norm Resid = $\frac{actual - expected}{\sqrt{\sigma^2 \times expected}}$ = $\frac{350 - 382.2}{\sqrt{423 \times 382.2}}$ = -.08

Part b: 0.75 point

Sample 1

Reserve_{2016} = Ult * [$G(X_{trunc})$ – G(X)]
= 7,847.81(.776 - .336) = 3,465.84
Ult Loss_{2016} = 2,850 + 3,465.84 = 6,315.84

Sample 2

Ult Loss_{2016} = 13,500 x 0.4965 x (1-0.4321) + 2,850 = 6,316

EXAMINER'S REPORT

Candidates were expected to be able to test how well the Cape Cod Method performed using a truncated development pattern by producing a normalized residual for the 2018 accident year. Candidates were also expected to produce an estimate of ultimate losses using the Cape Cod Method for the 2016 year.

Part a

Candidates were expected to produce a normalized residual using a truncated payment pattern.

Candidates were also expected to derive the payment pattern based on the given curve form and loss data and utilize that pattern to produce an overall estimated loss ratio (ELR). Using the overall ELR, candidates were expected to apply it to the 2018 earned premium along with the 2018 paid development pattern to produce the fitted incremental payment which is the underlying input to the normalized residual formula.

Common mistakes included:

- Applying a normalized truncated pattern to an ELR that was derived using a non-normalized pattern.
- Using the Cape Cod ultimate losses as the seed for the incremental payment calculation

Part b

Candidates were expected to produce ultimate losses for the 2016 accident year using the Cape Cod Method.

Common mistakes included:

- Producing ultimate losses for a year other than 2016
- Applying an incorrect incremental payment pattern
- Calculating the estimated ultimate losses as the ELR x Premium.

QUESTION 7	
TOTAL POINT VALUE: 3.25	LEARNING OBJECTIVE(S): A2
SAMPLE ANSWERS	

Part a: 1.75 points

Sample 1

AY	12-24	24-36	36-48	48-60	60-72
13	*	L	L	L	*
14	L	L	*	S	
15	L	S	S		
16	S	S			
17	S				

L => GT median – column
S => LT median – column
* => median

CY	Z = min(#L, #S) along diagonal
13	N/A (one observation)
14	0
15	0
16	1
17	0
ΣZ_i = Z =	1

95% Confidence Interval for T
$3.25 \pm 1.96\sqrt{1.06} = (1.232, 5.268)$

Because our test statistic, T=1, lies outside the bounds of our confidence interval, we reject H_0 that there are no significant CY effects in the data.

Sample 2

AY	12-24	24-36	36-48	48-60	60-72
13	*	H	H	H	*
14	H	H	*	L	
15	H	L	L		
16	L	L			
17	L				

CY	H	L	min(H, L)
2	2	0	0
3	3	0	0
4	1	2	1
5	0	4	0
			1

C.I. : $3.25 \pm 1.96\sqrt{1.06} = (1.232, 5.268)$
1 is not in the Confidence interval, so we reject the null hypothesis that there are no calendar year effects.

Sample 3

	12-24	24-36	36-48	48-60	60-72
2013	*	L	L	L	*
2014	L	L	*	S	
2015	L	S	S		
2016	S	S			
2017	S				

$Z^* = 0 + 0 + 0 + 1 + 0 = 1$
$C.I: (3.25 - 1.96\sqrt{1.06}, 3.25 + 1.96\sqrt{1.06}$
$= (1.232, 5.268)$
$Z^* = 1$ is not in the 95% C.I., we can therefore reject the null hypothesis that there are no calendar year influences.

Sample 4

AY	12-24	24-36	36-48	48-60	60-72
2013	*	H	H	H	*
2014	H	H	*	L	
2015	H	L	L		
2016	L	L			
2017	L				

* = median in column
H = above col median
L = below col median

Z_i = min{#H, #L} in ith diagonal

$Z_1 =$ 0
$Z_2 =$ 0 $\quad Z = \Sigma Z_i = 1$
$Z_3 =$ 0
$Z_4 =$ 1
$Z_5 =$ 0

$Z \pm 1.96sd(Z)$
$= 1 \pm 1.96\sqrt{1.06}$
$= -1.018, 3.0179$

95% CI excludes E(Z), so reject null! Conclude there are significant CY effects.

Sample 5

Triangle of rank, with median circled

AY	12-24	24-36	36-48	48-60	60-72
2013	3	3	3	2	1
2014	4	4	2	1	
2015	5	1	1		
2016	1	2			
2017	2				

Triangle of smaller vs larger

CY	AY	12-24	24-36	36-48	48-60	60-72
1	2013	*	L	L	L	*
2	2014	L	L	*	S	
3	2015	L	S	S		
4	2016	S	S			
5	2017	S				

CY	nb S	nb L	Z
1	0	0	0
2	0	2	min(0,2) = 0
3	0	3	min(0,3) = 0
4	2	1	min(2,1) = 1
5	4	0	min(4,0) = 0
			0 + 0 + 0 + 1 + 0 = 1

We have $Z\ score = \frac{Z-E[Z]}{\sqrt{Var[Z]}} = \frac{1-3.25}{\sqrt{1.06}} = -2.1854$

Since |Z-score| = 2.1854 > 1.96, we reject H_0.

There are no calendar year effects.

Part b: 1.50 points

Sample Responses for [i]

- No. This would cause losses in CY2017 & 2018 to be higher than expected, resulting in positive/higher residuals, not negative like in graph.
- No, a sustained calendar year trend would show consistent under prediction and not the reversal we see in 2017 and 2018.
- A sustained trend increase would imply 2017 & 2018 actuals are also greater than expected, so this does not explain the data.
- If we experienced a CY trend increase in 2016 and beyond then we would expect our actuals to be above expected so we should expect positive residuals in 2017 & 2018. Therefore this does not explain the results we see.

Sample Responses for [ii]

- Yes. If case reserves are larger (more adequate), Mack model will overestimate development, resulting in negative residuals in CY2017 and 2018.
- Yes. A sudden increase in case reserves would initially show underprediction. If 2017 and 2018 CR return to normal, the large 2016 losses would increase LDF which would overpredict 17 and 18.
- Strengthening of case reserves in 2016 would cause positive residuals as shown in the graph. It would also lead to less future incurred, so 2017 and 2018 would have negative residuals. Thus this occurrence would explain the results in the graph.
- An increase in case reserve adequacy in 2016 would result in higher actual incurreds in CY 2016 and therefore higher positive residuals in 2016, which we see in the graph. If this increased case reserve adequacy did not persist in 2017 and 2018 when we have negative residuals, then this could explain the result in we see.

- Yes, residuals are high in 2016 showing the reserve increase, then low in later years since reserves will develop less going forward.

Sample Responses for [iii]

- No. This impacts paid, rather than reported claims, and this data is for reported losses.
- No. The triangle is based on incurred data. Assuming case reserves are adequate, an acceleration at payments will not affect incurred development.
- If claims are adequately reserved, acceleration of loss payments would not have an effect on incurred losses and would thus not explain the results.
- An acceleration in loss payment wouldn't have any effect on these residuals because these are for incurred losses. As long as the increase in paid is offset by reduction to case reserves then we wouldn't necessarily see any CY impact to the residuals. Therefore does not explain.
- No. If losses are paid faster, but the case incurred amount does not change, residuals here shouldn't be impacted.

EXAMINER'S REPORT

Candidates were expected to understand and demonstrate how to perform Mack's methodology of testing for calendar year influences in data, and to analyze a graph of case incurred residuals for the type of change that could create a certain residual pattern.

Part a

Candidates were expected to convert the triangle of case incurred loss development factors into a triangle of large/small, hi/low, or similar indicators consistent with Mack's methodology, then calculate the test statistic Z as the sum of the minimum of the count of large and small indicators on each diagonal. Candidates were then expected to calculate the confidence interval for the test statistic, compare the test statistic to the confidence interval, and reject the null hypothesis since the test statistic falls outside the confidence interval. Since the null hypothesis was explicitly provided in the question, candidates were not required to restate the null hypothesis in their conclusion for full credit.

Some candidates failed to notice that E[Z] and Var[Z] were provided in the question and calculated them directly. While candidates did not lose credit for this so long as the values were calculated correctly, they may have lost valuable time.

Common mistakes included:

- Incorrectly assigning large and small indicators.
- Applying large and small indicators to the case incurred triangle instead of the case incurred development factor triangle.
- Using a calculation of Z other than the minimum of the count of S and L indicators by diagonal, such as the maximum.
- Not taking the square root of the variance when calculating the range.
- Providing a conclusion that was made without comparison of the test statistic to the confidence interval.
- Misstating the null hypothesis in the conclusion. Examples include stating that the null hypothesis is that there are calendar year influences, that calendar years are independent, or that columns of development factors are uncorrelated.

- Performing a different test, such as Mack's test for correlation of columns of development factors.

Part b

Candidates were expected to understand the effects of the following and to make relevant observations from the graph to support their conclusions:

- A sustained increase in calendar year trend.
- An increase in case reserve adequacy.
- An acceleration of loss payments on case incurred loss development and residuals.

When discussing a sustained increase in calendar year trend or an increase in case reserve adequacy, candidates needed to discuss calendar years 2017 and 2018 in order to receive full credit.

When discussing the impact of an acceleration in loss payments, some candidates stated that residuals are, or should be, positive or negative. This was not considered a sufficient response because an increase in payments should be offset by a reduction in case reserves, resulting in no impact on case incurred all other factors being constant.

Common mistakes included:

- Not supporting their conclusion by relating it to the graph
- Not addressing calendar years 2017 and 2018
- Not understanding that calendar years are incremental
- Not understanding that an increase in case reserve adequacy would cause residuals for 2017 and 2018 to decrease, following the increase in 2016
- Trying to draw conclusions about payment patterns from case incurred residuals
- Misinterpreting case incurred as case reserves
- Misinterpreting the x-axis as Accident Year instead of Calendar Year

QUESTION 8	
TOTAL POINT VALUE: 1.25	**LEARNING OBJECTIVE(S): A2**
SAMPLE ANSWERS	
Part a: 0.25 point	

Sample Responses

- When weighted average LDF won't generate a tail factor (not enough data to model tail factor). But, curve-fitting will naturally generate a tail factor > 1, causing the mean to be higher than weighted average.
- If there is a negative development due to salvage and subrogation. Curve fitting method assumes the % paid will always go up, but weighted average will recognize it (negative development).
- If your weighted average method is weighting more recent years at a higher weight, and loss developments have dropped off drastically in recent years.
- If there is an outlier age to age factor that is really low and has a lot of weight it will sway the weighted average method downward while the curve fit doesn't react to deviations as much.

Part b: 0.5 point

Sample Responses

Weighted average method

- Weighted average estimate relies upon actual data of company to produce estimates
- Weighted average method is easy to understand and explain to people.
- Weighted average makes more sense when there's negative development due to S&S since curve-fitting assumes increasing development.
- Weighted average is easier to calculate, as you don't have to fit a curve at all.
- If the payment patterns are relatively stable and sufficient data, then the weighted average method is likely better.
- Weighted average is good when there is a change in the development pattern, since the weighted average allows us to put more weight on recent years.

Curve fit method

- Curve fit provides an estimate of development after the end of the available data.
- Curve fit smooths out any spurious points in the data.
- Curve fitting is better because it can work with any development age, while weighted average will only work with the same development age as the data.
- Curve fitting: less parameters reduce the variance of the estimation (parameter variance).
- Can be used on data in other than triangle format

Part c: 0.5 point

Sample responses for part (i): vs. Clark Cape Cod method using a curve fit

- The cape cod method uses less parameters and that will reduce variability (and also standard deviation).
- Cape Cod uses additional info, like exposure and on-level premium.

Sample responses for part (ii) vs. Loss development method using weighted averages of the development factors

- Using weighted averages of development factors produces a narrower range than using a curve fit because it ignores volatility in the tail.
- In this case, we don't have the additional uncertainty of estimating parameters for the development pattern curve, so parameter variance and therefore standard deviation of total losses, is reduced.

EXAMINER'S REPORT

Candidates were expected to know the pros and cons of using the two curve fitting methods described in the Clark paper and the impact of using such curves vs the simpler weighted average LDFs method.

Part a

Candidates were expected to give one example of when the curve fit method would produce a higher mean estimate of ultimate losses than the weighted average LDFs method.

Common mistakes included:

- Explaining that the weighted average LDFs method would respond to an accident year or a calendar year trend better than the curve fitting method. This is true only in certain circumstances, when we are putting more weight on relevant experience.

Part b

Candidates were expected to provide an example of situation were <u>each</u> of the curve fit method and the weighted average LDFs method is superior.

Common mistakes included:

- Only providing an example for one of the two methods.
- Explaining that the weighted average LDFs method would respond to an accident year or a calendar year trend better than the curve fitting method (similar to part a). This is true only in certain circumstances, when we are putting more weight on relevant experience.

Part c

Candidates were expected to explain why the standard deviation of the two methods provided was lower than the standard deviation of the LDF curve fit method.

Candidates had more difficulty responding for the LDF method with weighted average development patterns than the Cape Cod method with curve fit.

Common mistakes included:

- Stating that the weighted average method was the Minimum Variance Unbiased Linear Estimator as per the Mack paper. This is not a relevant reason because the question is not comparing two linear estimators (LDF with curve fit method is not a linear method).
- Explaining that the regular weighted average method relied more on actual data, thus reducing the SSE, but the LDF curve fit also relies on actual data to the same extent. Also, having a lower SSE does not necessarily lead to a lower variance: the Clark paper specifically states that overfitting the data usually leads to higher parameter variance, thus increasing overall variance.

QUESTION 9	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): A5**
SAMPLE ANSWERS	

Part a: 0.75 point

Sample 1
- Specification Error: Error that arises because model is unable to perfectly model the insurance process.
- Parameter Selection Error: Error that arises because model is unable to adequately measure all predictors of future claim costs or trends in these predictors.
- Data Error: Error that arises due to lack of credible data or lack of understanding or knowledge about the book of business.

Sample 2
- Specification error: unable to perfectly model underlying insurance process.
- Parameter selection error: unable to select correct parameters & trends.
- Data error: lack of data or lack knowledge.

Sample 3
- Specification error: the risk that the selected model cannot accurately estimate losses generated by the loss process.
- Parameter error: the risk that the best parameters are not used or have not been identified.
- Data error: the risk that an effective model can't be fitted because the data is of poor quality.

Part b: 0.75 point

Sample 1
- Specification Error: The model was checked for reasonableness by performing diagnostic tests.
- Parameter Selection Error: Since the model captures both internal & external variables, it will better capture all predictors of future claim cost.
- Data Error: Since the data is reconciled to general ledger & prior data analysis, it is reliable & consistent with prior analysis.

Sample 2
- Specification: model was checked for reasonableness by performing diagnostic tests on outcomes.
- Parameter selection: chosen predictors have high correlation with claims experience. Suggests best predictors selected.
- Data error: data reconciled against ledger & prior analysis.

Sample 3
- Specification error: diagnostic tests on valuation results.
- Parameter error: use internal & external variables.
- Data error: data reconciled.

Part c: 0.75 point

Sample 1

- Specification Error: Since a single model is used for all LOBs the specific patterns unique to each LOB will not be captured.
- Parameter Selection Error: Since predictors are different from prior valuations, they are not consistent over time.
- Data Error: Since the actuary is not familiar with past processes & hasn't spoken to claims he/she may not be aware of any significant episodes or systematic changes & thus may not use the data correctly.

Sample 2

- Specification → Actuary models all lines together.
- Parameter Selection → Predictors are different than those used in past analysis.
- Data → Actuary is not familiar w/ past processes & has not met with claims staff.

Sample 3

- Specification: Modeling all LOBs together.
- Parameter: Predictors have changed.
- Data: Actuary is not particularly knowledgeable with the process.

Part d: 0.25 point

Sample Responses

- Since the risk indictors in (b) follow best practices the magnitude of risk is reduced. Thus variability in predicted losses is reduced which leads to lower CoV, as compared to risk indicators in part (c).
- The portfolio with good scores will be mapped to a lower CV. The portfolio with bad scores will be mapped to a higher CV.
- Higher score (in part b) means lower volatility → it results in lower CoV.

EXAMINER'S REPORT

Candidates were expected to briefly describe the three sources of internal systemic risk according to Marshall, identify a high scoring and low scoring characteristic from the listed process, as well as describe how scores in the balanced scorecard approach impact CoV.

Part a

Candidates were expected to briefly describe all three sources of internal systemic risk.

Common mistakes included:

- Describing specification error as the risk due to selecting the wrong model or distribution without mentioning the insurance process.
- Describing parameter selection error as the risk of selecting the incorrect parameter values or not outputting the true values of the parameters.
- Listing risk other than internal systemic risks from Marshall.
- Listing the risk without a brief description.

Part b

Candidates were expected to identify a high scoring characteristic of the described modeling process in the question.

<table>
<tr><td>
Common mistakes included:

Listing high scoring criteria that were not supported by the question.
Listing low scoring criteria.
Listing both a high and low scoring criteria such as parameters being different, but highly correlated with claims.

</td></tr>
<tr><td>Part c</td></tr>
<tr><td>
Candidates were expected to identify a low scoring characteristic of the described modeling process in the question.

Common mistakes included:

Listing low scoring criteria that were not supported by the question.
Listing high scoring criteria.
Listing both a high and low scoring criteria such as parameters being different, but highly correlated with claims.
Listing the low score for specification error as only using one model.

</td></tr>
<tr><td>Part d</td></tr>
<tr><td>
Candidates were expected to describe why a high scoring portfolio would have a lower CoV than a high scoring portfolio.

Common mistakes included:

Not describing why variance, or CoV, would be lower for a high scoring portfolio.
Giving decreased parameter and/or process risk as the reason for lower CoV.

</td></tr>
</table>

QUESTION 10	
TOTAL POINT VALUE: 3.75	LEARNING OBJECTIVE(S): A7, A8, A10
SAMPLE ANSWERS	

Part a: 2 points

Sample 1:

AY	h_i
16	1.57 / 2.067 = 0.7596
17 - 18	1.57 / 0.933 = 1.6827

Adj Residuals = $r \times h_i$			
AY	12	24	36
16	-0.410	0.798	2.704
17	2.406	4.627	
18	0.000		

Sampled Incr. $m_{w,d} + r/h_i\,(m_{w,d})^{0.5}$			
AY	12	24	36
16	1362	735	189
17	1562	982	
18	1418		

Example Calc.	
AY16,12M	$1250 + (2.406 / 0.7596) \times (1250)^{0.5} =$ 1362
AY17,12M	$1500 + (2.704 / 1.6827) \times (1500)^{0.5} =$ 1562

Sampled Cumulative			
AY	12	24	36
16	1362	2097	2286
17	1562	2545	
18	1418		

Sample 2:

Fitted Incre.			
AY	12	24	36
16	1250	750	150
17	1500	900	
18	1400		

Residuals in 2016 & 2017/2018 are heteroscedastic

$h_{16} = 1.57 / 2.067 = 0.76$

$h_{17/18} = 1.57 / 0.933 = 1.683$

Sampled Residual		
12	24	36
2.41 / 0.76 = 3.17	-0.54	3.17
2.71 / 1.683 = 1.61	2.75	
0.798 / 1.683 = 0.474		

Incremental		
12	24	36
$1250+3.17\times(1250)^{0.5} = 1362$	$750+ -0.54\times(750)^{0.5} = 735$	$150+3.17\times(150)^{0.5} = 189$
$1500+1.61\times(1500)^{0.5} = 1562$	$900+2.75\times(900)^{0.5} = 983$	
$1400+0.474\times(1400)^{0.5} = 1418$		

Cumulative		
12	24	36
1362	2097	2286
1562	2545	
1418		

Sample 3:

This calculation is the same as the calculation from Solution 1, but is done in $000's instead of taking the values as shown.

Fitted Incre.			
AY	12	24	36
16	1250000	750000	150000
17	1500000	900000	
18	1400000		

Residuals in 2016 & 2017 are heteroscedastic

$h_{16} = 1.57 / 2.067 = 0.76$

$h_{17} = 1.57 / 0.933 = 1.683$

Sampled Residual		
12	24	36
2.41 / 0.76 = 3.17	-0.54	3.17
2.71 / 1.683 = 1.61	2.75	
0.798 / 1.683 = 0.474		

Incremental		
12	24	36
$1250000+3.17\times(1250000)^{0.5}$ = 1,253,542	$750000+ -0.54\times(750000)^{0.5}$ = 749,532	$150000+3.17\times(150000)^{0.5}$ = 151,227
$1500000+1.61\times(1500000)^{0.5}$ = 1,501,968	$900000+2.75\times(900000)^{0.5}$ = 902,609	
$1400000+0.474\times(1400000)^{0.5}$ = 1,400,561		

Cumulative		
12	24	36
1,253,542	2,003,074	2,154,301
1,501,968	2,404,577	
1,400,561		

Part b: 1.75 points

Sample 1:

First Sim

E{L} = 3000

$LDF_{12\text{-}24}$ = 1.600

$LDF_{24\text{-}36}$ = 1.075

CDF = 1.6 × 1.075 × 1.15 = 1.978

AY	E(L)	(1 – p) = 1 – 1 / CDF	CDF	R
16	3k	0.130	1.15	390
17	3k	0.191	1.236	573
18	3k	0.494	1.978	1482
			Total	2445

Second Sim

AY	E(L)	(1 – p) = 1 – 1 / CDF	CDF	R
16	3500	0.130	1.15	455
17	3500	0.203	1.254	710.5
18	3500	0.497	1.990	1739.5
			Total	2905

The range of stochastic BF reserve indications is 2,445,000 to 2,905,000

Sample 2:

First Sim

E(L) = 2500 + 500 × [(0.458 – 0.200) / (0.6 – 0.2)] = 2822.5

$LDF_{12\text{-}24}$ = 1.600

$LDF_{24\text{-}36}$ = 1.075

CDF = 1.6 × 1.075 × 1.15 = 1.978

AY	E(L)	(1 – p) = 1 – 1 / CDF	CDF	R
16	2822.5	0.130	1.15	368
17	2822.5	0.191	1.236	539
18	2822.5	0.494	1.978	1396
			Total	2303

Second Sim
E(L) = 3000 + 500 × [(0.812 – 0.600) / (0.6 – 0.2)] = 3265

AY	E(L)	(1 – p) = 1 – 1 / CDF	CDF	R
16	3265	0.130	1.15	426
17	3265	0.203	1.254	661
18	3265	0.497	1.990	1624
			Total	2711

The range of stochastic BF reserve indications is 2,303 to 2,711

EXAMINER'S REPORT

Candidates were expected to adjust the development triangle with heteroscedastic data and use the fitted triangle and the result from part a) to calculate the range of BF method reserve indications.

Candidates had more difficulty with the calculation of the BF method reserve indications in part b) than the adjustment of the heteroscedastic data in part a).

Part a

Candidates were expected to note that the accident year standard deviations were significantly different for 2016 and 2017, and calculate the heteroscedastic adjustment factors to put the residuals on the same variance level so random sampling is appropriate. These factors needed to be correctly applied to the residual triangle to calculate the incremental sampled triangle that was then used to calculate the cumulative sampled triangle.

Common mistakes included:

- Incorrectly adjusting the residual triangle
- Skipping the calculation of the incremental fitted triangle and going straight for the cumulative sampled triangle.
- Not readjusting the variance when calculating the sample incremental triangle (dividing by the appropriate heteroscedasticity adjustment factor for the sampled accident year)
- Adjusting 2016 to the total variance level and stating that 2017 and/or 2018 did not require a heteroscedasticity adjustment.
- Not calculating the cumulative triangle and leaving the calculation at the incremental sample triangle.

Part b

Candidates were expected to use the provided seed information to look up the expected losses, calculate the LDFs / CDFs for both the fitted and sampled triangle from part a), then calculate the BF reserve for each accident year and sum to find the total range of reserves.

Common mistakes included:

- Using the incorrect expected loss on the fitted / sampled triangles
- Using LDFs from only one of the fitted / sampled triangles to calculate the range for both simulations
- Incorrectly applying the BF methodology.

- Missing the tail factor when calculating the cumulative loss development factor
- Calculating the range for each individual accident year, but not calculating the total overall range of reserves.
- Misusing the information provided to estimate the range of reserves.

QUESTION 11	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): A5, A6, A8, A10**
SAMPLE ANSWERS	
Part a: 0.5 point	

Sample 1
AY 2016: Bayesian estimate is slightly closer to BF method than C-L. Variance is moderately small, but not too small
AY 2017: Bayesian estimate is very close to BF method. Thus, strong opinion, small variance
The variance for AY 2016 is larger than AY 2017

Sample 2
2016: 5301 = 5400 * Z + 5222 * (1 – Z) => Z = 44.4%
2017: 7750 = 10,000 * Z + 7000 * (1 – Z) => Z = 25%

Since Z is smaller for 2017, β is bigger, thus the variance (α/β^2) is smaller in 2017

Sample 3
AY 2016: Z*(5400) = (1-Z)*5222 = 5301
Z = .4438
p_{2016} = 3000/5400 = .5556
.5556/[βϕ + .5556] = .4438
βϕ = .696

AY 2017: Z*(10,000) + (1-Z)*(7000) = 7,750
Z = .25
p_{2017} = 4000/10000 = .4
.4/[βϕ + .4] = .25
βϕ = 1.2

Assuming the dispersion parameter used in 2016 is not significantly smaller than that used in AY 2017, we can assume a higher β value for AY 17 & thus a lower variance of the prior distribution in AY 17

Part b: 0.5 point

p_k = 750/3000 = 0.25
Z = .25/[.25 + 1.1*2.5] = 0.083
0.083*3000 + (1 – 0.083)*4500 = 4376

Part c: 0.5 point

Sample 1
μ_{2013} = 8.5172 + 0 = 8.5172
μ_{2014} = 8.5172 + 0 + 0.2 (ln(4400) - 8.5172)
= 8.4916 (ult = $e^{8.4916}$ = 4874)

Sample 2 [assumed different scale was used for α and β]

$\mu_{2013} = 8.5172 + 0 = 8.5172$

$\mu_{2014} = 8.5172 + 0 + 0.2\ (\ \ln(4{,}400{,}000) - 8.5172\ )$

$= 9.873$

Part d: 0.5 point

Sample Responses for Correlated Chain Ladder advantages

- Incorporates correlation between AYs
- Shown by Meyers to accurately describe the distribution of reserves

Sample Responses for Bayesian advantages

- Can incorporate expert opinion to the simulation
- Can produce full predictive distribution in addition to point estimate

EXAMINER'S REPORT

Candidates were expected to know the definition, assumptions, calculation procedures and advantages for the Bayesian and Correlated Chain Ladder reserving methods.

Part a

Candidates were expected to know that when the distribution of initial expected ultimate has a high variance, the Bayesian ultimate will be closer to the chain ladder ultimate and then apply that to compare the variances of the distributions of initial expected ultimate losses between accident years. Alternatively, candidates could calculate the variances of the prior distributions and compare.

Common mistakes included:

- Stating that when the distribution of initial expected ultimate losses has a high variance, then the Bayesian ultimate will be closer to the Bornhuetter-Ferguson (B-F) ultimate.
- Defining closeness based on subtracting Bayesian from B-F and not as a ratio to the range of Chain ladder to B-F

Part b

Candidates were expected to be able to calculate the Bayesian ultimate loss estimate.

Common mistakes included:

- Using an incorrect formula for calculating credibility
- Using expected ultimate claims as the complement of credibility instead of the B-F estimate

Part c

Candidates were expected to be able to calculate the logged mean ultimate of the Correlated Chain Ladder model using given parameters.

Common mistakes included:

- Using an incorrect equation

Part d
Candidates were expected to know advantages of the Correlated Chain Ladder and Bayesian methods. Common mistakes for CCL included: • Correlated Calendar years • Incorporate calendar year effects (trend, claim settlement rate – see CIT and CSR models) • Passes the K-S test Common mistakes for Bayesian included: • It credibility weights chain ladder and Bornhuetter-Ferguson • Minimizes MSE • You can assume an a priori distribution • Adds stability

QUESTION 12

TOTAL POINT VALUE: 2.0 | **LEARNING OBJECTIVE(S): A8,A9**

SAMPLE ANSWERS

Part a: 0.5 point

Sample Responses for parameter θ

- Location parameter
- Canonical parameter
- $\theta = (b')^{-1}(\mu)$

Sample Responses for $b(\theta)$

- Cumulant function
- $b(\theta)$ describes the mean of the EDF: $\mu = b'(\theta)$
- Shape function

Part b: 0.5 point

Sample Responses

- Tweedie subfamily can be obtained by allowing the variance function to be $V(\mu) = \mu^p$, where p is not between 0 and 1.
- Tweedie can be obtained from the EDF by restriction of the variance function: $V(\mu) = \mu^p,\ p \leq 0\ or\ p \geq 1$

Part c: 0.25 point

Sample Responses

- Choose a larger value for p
- Select a Tweedie with p = 2, since this is the Gamma which is heavy-tailed
- For heavy-tailed, use a Tweedie with p = 3 (inverse Gaussian)

Part d: 0.75 point

Sample 1

This is an over dispersed Poisson model, if the Mack assumptions hold, which we assume they do, we can solve this using CL development factors.

12-24 = $\frac{1000+600}{180+200} = 4.21$; 24-34 = $\frac{12001000+600}{1000} = 1.2$

2018 ultimate = 4.21 x 1.2 x 150 = 757.89

Sample 2

ODP dist.

$\propto_{2016} = 1200, \beta_3 = 200/1200 = 0.167, \propto_{2017} = 600/1 - 0.167 = 720,$

$\beta_2 = (820 + 400)/(1200 + 720) = 0.635, \propto_{2018} = 150/(1 - 0.167 - 0.635) = 758,$

$\beta_1 = (180 + 200 + 150)/(1200 + 720 + 758) = 0.198$

2018 $Ult = 150 + 758 \times (0.635 + 0.167) = 758$

EXAMINER'S REPORT

Candidates were expected to recall facts associated with the Tweedie distribution, and to recognize the relationship between a reserve derived using a GLM versus the standard chain ladder approach.

Part a
Candidates were expected to recall the name of θ and $b(\theta)$ in the context of an EDF probability density function, or state how they affect an EDF distribution. Common mistakes included: • stating that they are the dispersion parameter • stating that they are the scale parameter
Part b
Candidates were expected to recall the restriction on the variance function of an EDF distribution that would give rise to a Tweedie distribution. Both the form of the variance function and the restriction on p must be identified. Specific results of the Tweedie distribution, such as $\mu = [(1-p)\theta]^{\frac{1}{1-p}}$ were given credit. Responses that did not identify the correct restriction on p did not receive full credit. Common mistakes included: • not providing the restriction on p • confusing the concept of variance versus the variance function
Part c
Candidates were expected to recognize that a higher value of p will increase the variance of a Tweedie distribution and thus generate a distribution with a heavier tail. Alternatively, candidates can identify which member of a Tweedie distribution has a heavy tail, and how such a distribution would be generated by changing p. Responses that stated selecting a higher ϕ will lead to higher variance and thus a heavier tail did not receive credit. ϕ is a parameter of a distribution of the EDF family, so it does not affect the selection of a distribution. Common mistakes included: • stating decreasing p will increase tail heaviness • stating a specific value of p without clear justification why it would lead to a heavy tail
Part d
Candidates were expected to recognize when p = 1, the Tweedie distribution is an over dispersed Poisson (ODP) and it replicates the standard chain ladder method. Common mistakes include • using incorrect MLEs for the ODP cross-classified model • not stating the reason why the CL method applies

QUESTION 13	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE(S): A13**

SAMPLE ANSWERS

Part a: 1 point

Sample 1

ELR = (4500 + 3400 + 2300 + 3000) / (7400 x 90% + 7700 x 70% + 7900 x 50% + 8000 x 30%) = 0.717

8000 x 0.717 x (1 – 0.3) = 4015.2

Sample 2

Accident Year	Used Up Premium	Loss
2015	7400 x 0.9 = 6660	4500
2016	5390	3400
2017	3950	2300
2018	2400	3000
Total	18400	13200

ELR = 13200 / 18400 = 71.7%

AY 2018 IBNR = (8000 – 2400) x 71.7% = 4015

Part b: 0.25 point

Sample 1

R^{CL} = 3000 x (1/0.3 – 1) = 7000

Sample 2

LDF = 1/30% = 3.33

3000 x 3.33 – 3000 = 7000 (000)

Part c: 0.5 point

Sample 1

Z = 0.6 x 30% = 18%

R = 18% x 7000 + (1 – 18%) x 4015 = 4552

Sample 2

Ult^{2018} = Lag x Credibility x (CL Ult) + (1 – Lag x Credibility) x (SB Ult)

= 0.3 x 0.6 x 10000 + (1 – 0.18) x 7017.3973 = 7554.96

IBNR = 7554.96 – 3000 = 4554.96

Part d: 0.5 point

Sample Responses

- When the actuary wants to give more weight to the CL in older periods because they have more confidence in the loss data and less confidence in the on-leveled premiums used. Also want to give more weight to the SB method in the most recent period where they have more confidence in the on-level premium estimate and less confidence in the loss data.
- When the actuary doesn't have full faith in the chain ladder method b/c of highly leveraged LDFs but also doesn't have full faith in the on-level adjustments needed for the Cape Cod calculation.

- If losses for the year are coming in higher than expected → CL method will more quickly reflect this so give some weight to CL and rest of weight to SB to not overreact and maintain stability.
- For a long tail line where the chain ladder factors are highly leveraged, like workers comp. This would help moderate the early accident years where the LDFs are high but also respond to the emerging losses by gradually moving toward the CL as the ages mature.

EXAMINER'S REPORT

Candidates were expected to estimate IBNR estimates using the Cape Cod method, the chain ladder method, and the credibility-weighted combination of the Cape Cod and chain ladder methods. Candidates were also expected to describe a situation where the actuary would use the credibility-weighted method instead of only the Cape Cod method or the chain ladder method.

Part a

Candidates were expected to estimate the IBNR for accident year 2018 using the Cape Cod method.

Common mistakes included:

- Using the earned risk pure premium instead of the adjusted risk pure premium in the ELR calculation.
- Subtracting the reported losses from the Cape Cod ultimate estimate instead of multiplying by 1 minus the report lag.
- Calculating the ultimate loss estimate for accident year 2018 instead of the IBNR.
- Calculating the IBNR estimate for all accident years instead of accident year 2018.

Part b

Candidates were expected to estimate the IBNR for accident year 2018 using the chain ladder method.

Common mistakes included:

- Calculating the ultimate loss estimate for accident year 2018 instead of the IBNR.
- Calculating the IBNR estimate for all accident years instead of accident year 2018.

Part c

Candidates were expected to estimate the IBNR for accident year 2018 using the credibility-weighted combination of the Cape Cod and chain ladder methods.

Common mistakes included:

- Forgetting to multiply the credibility factor by the report lag.
- Applying the credibility factor to the Cape Cod estimate and the remaining weight to the chain ladder method instead of the opposite.
- Multiplying the credibility factor by the ELR instead of the report lag.
- Calculating the ultimate loss estimate for accident year 2018 instead of the IBNR.
- Calculating the IBNR estimate for all accident years instead of accident year 2018.

Part d

Candidates were expected to describe a situation where the actuary would use the credibility-weighted combination of the Cape Cod and chain ladder methods.

Common mistakes included:

- Identifying a situation without providing any description as to why the credibility-weighted method is appropriate.
- Discussing credibility-weighting in general without describing the relative advantages and disadvantages of the Cape Cod and chain ladder methods.
- Briefly describing a situation in which the credibility-weighted method should be used without providing enough detail.

QUESTION 14	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): A11**

SAMPLE ANSWERS

Sample 1

- Report lags are much longer for reinsurers. Claim needs to be reported to primary insurer. Then after that, it has to reach a certain threshold before it's reported to reinsurer & there could be intermediaries between the reinsurer & primary insurer too.
- There is persistent upward development. This is caused by primary insurers under-reserving for ALAE & setting case reserves to mode.
- Industry data is not useful due to heterogeneity of reinsurers exposures so reinsurers have hard time comparing data.
- IT and technological issues. It's hard for IT departments to keep up with data coding since information required to be captured is always changing.

Sample 2

- Non-homogeneous data/reporting patterns – exposures vary significantly by reinsurance products due to retention limits, attachment points, policy terms, etc.
- Reporting delays tend to be longer for reinsurance due to: cedant needs to recognize claim. Then it must flow thru pipeline to reinsurer; cedant may under-value claim for a long time; mass torts (like asbestos) cause major delays.
- Industry statistics not as useful for reinsurance lines due to non-homogenous data issue mentioned above. Aggregated industry values don't have appropriate level of detail.
- Data provided by primary via reporting can lack crucial details, such as AY, forcing reinsurer to impute it. May also be heavily summarized or have missing info that would help reinsurer properly reserve.

Sample 3

- Due to heterogeneity of coverage, data coding and IT system are not updated frequently and may have errors - reserve data might not be accurate.
- Persistent increase in reserves. Claims are usually recorded at modal values; ALAE understated; there is social and economic inflation.
- Due to the uncertainty of reinsurance reserves the reserve-to-surplus ratio is higher than that of a primary. Management may not believe the higher reserves and may be difficult to convince.
- Since there are different forms of reinsurance; different attachment points and coverage, data is not homogenous, so industry data is less useful.

EXAMINER'S REPORT

Candidates were expected to identify and briefly describe the differences between reinsurers and primary insurers relating to reserving methods, impact on assumptions from differences in information available to reinsurers and the underlying business characteristics of reinsurance contracts.

Common errors include:

- Incorrectly stating that RAA publishing their study ever 2 years is a problem
- Stating persistent upward trend instead of upward development
- Using the word "heteroskedasticity" instead of the word "heterogeneity"
- Identifying a reason without briefly describing
- Stating that reinsurers have a higher surplus to reserve ratio
- Stating that reserves are homogeneous instead of heterogeneous

QUESTION 15	
TOTAL POINT VALUE: 2.25	**LEARNING OBJECTIVE(S): A14**

SAMPLE ANSWERS

Part a: 1.5 points

Sample 1

$$PDLD1 = \left(\frac{25}{100}\right) * \frac{1.03}{.8 * .5} + (.95 * 1.18 * 1.03) = 1.798$$
$$PDLD2 = .75 * 1.18 * 1.03 = .912$$
$$PDLD3 = .55 * 1.18 * 1.03 = .668$$
$$CPDLD1 = \frac{(1.798 * .5 + .912 * .426 + .668 * .064)}{.5 + .426 + .064 + .01} = 1.330$$

Adj #	*Emergence*
1	*.5*
2	*.426*
3	*.064*
4	*.01*

Sample 2

$$PDLD1 = \left(\frac{25}{100}\right) * \frac{1.03}{.8 * .5} + (.95 * 1.18 * 1.03) = 1.798$$

$$PDLD2 = .75 * 1.18 * 1.03 = .912$$
$$PDLD3 = .55 * 1.18 * 1.03 = .668$$
$$CPDLD1 = \frac{(1.798 * .5 + .912 * .426 + .668 * .064)}{.5 + .426 + .064} = 1.34$$

Adj #	*Emergence*
1	*.5*
2	*.426*
3	*.064*
4	*.01*

Sample 3

(Same Calculations for PDLD1, PDLD2, PDLD3 as above)

$$CPDLD3 = \frac{(.668 * .064)}{.064 + .01} = .578$$

$$CPDLD2 = \frac{.912 * .426 + .578 * (.064 + .01)}{.426 + .064 + .01} = .863$$
$$CPDLD1 = 1.798 + .863 * .5 = 1.33$$

Part b: 0.25 point

Sample Responses

- Use Booked Premium Triangle and Loss Development Triangle to generate PDLDs
- Select PDLD ratios based on empirical analysis
- Use Fitzgibbon method P=a+bx
- Fit a linear model comparing UltLosses(X) to Ult Premium(Y) -> Y=aX+b

Part c: 0.5 point

Sample Responses

If part b was the empirical method:

Advantages of formula method over empirical

- Responds to changing retro parameters that are being sold
- More stable than empirical approach
- Easier to explain to underwriters
- Hard to estimate retro parameters if there is a mix of different retro polices
- Can calculate PDLD faster, do not have to wait for premium lag

Disadvantages of formula to empirical

- Difficult to estimate the retro parameters (Tax multiplier by state etc.)
- May not reflect trends in the data in a timely manner
- Ignores the actual relationship between the premium emerged and the reported losses
- Using average factors could lead to bias in results
- Portfolio level factors may not be appropriate for individual accounts

If part b was Fitzgibbon/Linear approximation:
Advantage of formula over Fitzgibbon:

- Recognizes that responsiveness of premium development to loss development is not linear
- Formula recognizes declining premium responsiveness in later adjustments

Disadvantage of formula to Fitzgibbon

- Difficult to measure parameters
- More calculation intensive
- More time consuming

EXAMINER'S REPORT

The candidates need to be able to calculate the cumulative premium development to loss development ratio using the formula method laid out in the Teng and Perkins paper. They also need to be aware of a separate method to estimate the PDLD.

Part a

Candidates were expected to know how to calculate the PDLD at the first adjustment. They were expected to know how to use the incremental percent reported to get to the cumulative PDLD (CPDLD) for the first adjustment.

The question required candidates to calculate the cumulative premium development to loss development ratio. Some candidates incorrectly interpreted "(PDLD)" in the question to mean that only the PDLD at the first adjustment was required, and did not calculate the cumulative premium development to loss development ratio as written in the question. Because of the perceived ambiguity in the question wording, a scoring adjustment was made for candidates that ignored the word cumulative and mistakenly stopped calculations at PDLD1. This adjustment ensured this perceived ambiguity would not cause a candidate to fail that would have otherwise passed; note filing grade reports and scores may not reflect this grading adjustment.

Part b

The candidates were expected to know one additional method. Most candidates responded with either the empirical method or the Fitzgibbon approach.

Common mistakes included:

- The Feldblum adjustment (removing the fixed portion) as that is not a separate method
- Developing premium directly without any mention to loss

Part c
Candidates were expected to give one advantage and one disadvantage of the formula method. A common mistake was switching the advantage and disadvantage, or being unclear as to which was the advantage or disadvantage.

QUESTION 16

TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): B1, B2**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

FCFE = 43 – (220-195) = $18M

Sample 2

k = .03 + .8*.065 = .082

18/1.082 = $16.64M

Part b: 0.5 point

Sample 1

(220 – 195)/ 43 = 58.14%

Sample 2

g = ROE * p

g = (220 – 195)/195; ROE = 43/195; p = 25/43 = 58.14%

Part c: 0.5 point

Sample 1

- FCFF values equity and debt combined and then subtracts the market value of the debt to get to the equity. This is a complication for insurance companies because policyholder liabilities are indistinguishable from other forms of debt.
- Policyholder liabilities make it difficult to calculate the WACC or APV for discounting.

Sample 2

Both reasons have to do with the difficulty in distinguishing between debt/policyholder liability.

- WACC needs an estimate of debt
- The FCFF approach subtracts market value of debt

EXAMINER'S REPORT

Candidates were expected to understand how to use the Discounted Cash Flow (DCF) Valuation using free cash flow to equity (FCFE), including reasons why this method is preferred over the free cash flow to the firm (FCFF) method for P&C insurers.

Part a

Candidates were expected to correctly calculate the FCFE for 2019. Credit was given if the candidate had the correct answer but added an additional step in discounting the FCFE.

Common mistakes included:

- Only calculating change in required surplus
- Incurred adjustments for reserve changes as no adjustment should be made

Part b

Candidates were expected to correctly calculate the reinvestment rate for 2019.

Common mistakes included:

- Calculating the growth rate
- Calculating the required return on equity

Part c
Candidates were expected to understand that policyholder liabilities can be viewed as arbitrary from debt and therefore make the FCFF difficult to calculate due to the necessary subtraction of debt in the formula as well as the required discount rate of the debt.

QUESTION 17	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): B2, B3**
SAMPLE ANSWERS	
Part a: 1 point	
Sample 1 P/E = 1-ρ / (k - ρ *ROE) ROE = 0.09 r_f = 0.047 – 0.013 = 0.034 k = 0.034 + 0.8 (0.07) = 0.09 ρ = 1 – 0.4 = 0.6 P/E = (1-0.6) / (0.09 – 0.6(0.09)) = 11.11 *Sample 2* P-E = Div / (k-g) k = 0.047 – 0.013 + 0.8(0.07) = 0.09 g = 0.09(1-0.4) = 0.054 P-E = 0.4 / (0.09 – 0.054) = 11.11	
Part b: 0.5 point	
Sample 1 Div rate ↑ 60%, plowback rate ↓ 40%, g = 0.09 * 0.4 = 3.6% P/E = 0.6/(0.09 – 0.036) = 11.111 Assuming that what's not paid in dividend is reinvested in the firm, the P/E ratio stays constant as increase in dividend rate is offset by a decrease in growth rate. *Sample 2* Since the discount rate equals the ROE (both 9%), the dividend payout rate has no effect on the P/E ratio. It will remain 11.11. *Sample 3* P/E = 0.6 / (0.09 – 0.036) = 11.11 -> No Effect G = 0.09(0.4) = 0.036	
Part c: 0.5 point	
Sample 1 r_f = 0.04 - 0.013 = 0.027 K = 0.027 + 0.8(0.07) = 0.083 P/E = 0.4 / (0.083 – 0.054) = 13.79 When r_f decreases => cost of capital decrease => PV of dividends increase providing higher multiple	

Sample 2
This would decrease r_f, which would decrease k, ultimately decreasing the denominator of the ratio -> Increases P/E ratio.

Sample 3
r = 0.04 – 0.013 = 0.027
k = 0.027 + 0.8(0.07) = 0.083
P/E = 0.4 / (0.083-0.054) = 13.793
Increases it

Part d: 1 point

Sample 1
- Validation of assumptions. You can compare your P/E ratio to that of similar firms. If they are reasonably close, then you will feel more confident in your assumptions. If they are not close, then this may indicate a need to revisit assumptions
- The P/E ratio can assist in the calculation of the terminal value. The terminal value has a large influence in the overall value of the company & is highly leveraged. You could use the P/E ratio to remove some of the uncertainty in this calculation

Sample 2
- Shortcut to valuation -> allows us to quickly compute company value without needing detailed forecast of financial projections (time consuming & difficult to project NI over multiple years
- Diagnostic tool for valuation -> Can compare to peer companies to assess reasonability. Also, can compare to result from DDM AE, FCFE method to compare/assess reasonability of terminal value assumptions

Sample 3
- You can use P-E ratios to check assumptions used in other valuation models. If the results of the valuation models don't produce P-E ratios in line with peer companies, you may want to revisit model assumptions
- You can price/value your company quickly using peer/competitor multiples of P-E ratios. Finding comparable peers by looking at premium volume, type of business and expected return will help you select which peer P-E ratios are appropriate

Sample 4
- The P-E ratio can be used as a diagnostic check on an alternative valuation, such as the dividend discount model.
- The P-E ratio could be used for the terminal value calculation in conjunction with another valuation method. For example, it could be used with the FCFE method, to calculate the terminal value

Sample 5
- Can use the P-E ratio to compare valuation with similar firms and validate assumptions
- Can use the P-E ratio to guide the calculation of the terminal value in other valuation methods

EXAMINER'S REPORT

The candidate was expected to demonstrate knowledge of firm valuation and how the P/E ratio can play an integral role in this exercise

Part a

The candidate was expected to determine the discount rate using the Capital Asset Pricing Model and apply the Dividend Discount Model to calculate the company's Price to Earnings Ratio

Common mistakes included:

- Neglecting to subtract the term premium when calculating the risk-free rate
- Subtracting the term premium from the equity risk premium in the CAPM formula
- Failing to recognize the application of the Dividend Discount Model to calculate the P/E ratio (using other valuation models)
- Using incorrect formula for Dividend Discount Model

Part b

The candidate was expected to recognize that the dividend payout ratio has no impact on the P/E ratio when the discount rate equals the company's return on equity

Common mistakes included:

- Failing to adjust the growth rate for the increased dividends
- Only recognizing the impact on growth, but not reflecting the increased dividends
- Failing to recognize the significance of the discount rate being equal to the ROE

Part c

The candidate was expected to recognize that the P/E ratio increases when the discount rate decreases

Common mistakes included:

- Adjusting the ROE as well as the discount rate
- Failing to recognize the lower discount rate would increase the PV of the future dividends & earnings

Part d

The candidate was expected to list and describe two alternative uses of the P/E ratio

Candidates did not receive credit for use cases that did not relate to valuation, as the P/E ratio is a relative measure of company value to net income

Common mistakes included:

- Failing to recognize the use of P/E as a valuation metric
- Failing to elaborate on the uses provided
- Misinterpreting question and providing other valuation ratios (e.g. P/BV)
- Vagueness in answers (e.g. "benchmarking")

QUESTION 18	
TOTAL POINT VALUE: 1	**LEARNING OBJECTIVE(S): B2**

SAMPLE ANSWERS

Part a: 0.5 point

Sample Responses

Any combination of two of the sample answers below.

- It's hard to distinguish between debt and policyholder liabilities.
- Policyholder liabilities do not have a single expiration date.
- Often priced using Black-Scholes model. Black-Scholes inputs like risk free rate, volatility, time horizon is hard to estimate.

Part b: 0.5 point

Sample Responses

Varying combination of these concepts received credit.

- These methods require forecasts of financial variables which is hard as we do not know the company's plan for growth nor do we have detailed information for forecasting.
- It may be difficult to calculate the terminal value since it's difficult to project the company's future earnings.
- All of these methods require discounting cash flows to a single point in time. Must decide between using firm beta to calculate k which may be volatile or use industry beta which may not perfectly reflect the risk profile of any given P&C company. K equals discount rate based on CAPM.

EXAMINER'S REPORT

Candidates were expected to understand the Dividend Discount Model, Discounted Cash Flow, Abnormal Earnings and Equity as a Call Option methods to value a P&C insurance company and be able to discuss the difficulties of using each method.

Part a

Candidates were expected to provide two valid difficulties when using the Equity as a Call Option.

Common mistakes included:

- Providing answers which were not specific enough.
- Discussing difficulties with Real Options Valuation rather than the Equity as a Call Option method.

Part b

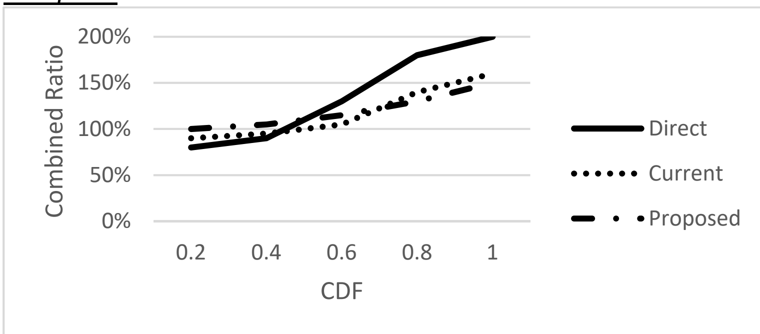
Candidates were expected to discuss a valid difficulty that is common to the three methods listed in the question.

Common mistakes included:

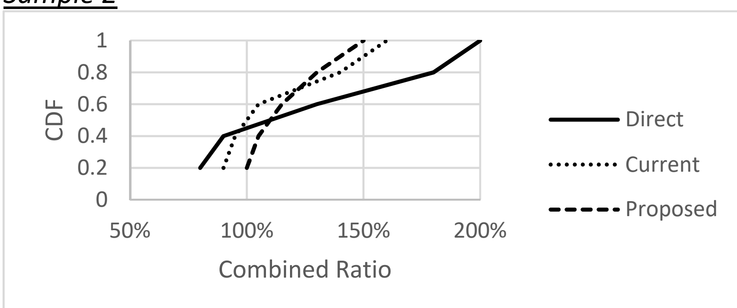
- Providing answers which were not specific enough, such as saying the methods are complex, difficult to parameterize or sensitive to inputs without further explanation.
- Providing difficulties that were not common to all three methods.

SPRING 2019 EXAM 7, QUESTION 19	
TOTAL POINT VALUE: 3.25	**LEARNING OBJECTIVE(S): C2, C3**
SAMPLE ANSWERS	
Part a: 1.25 points	

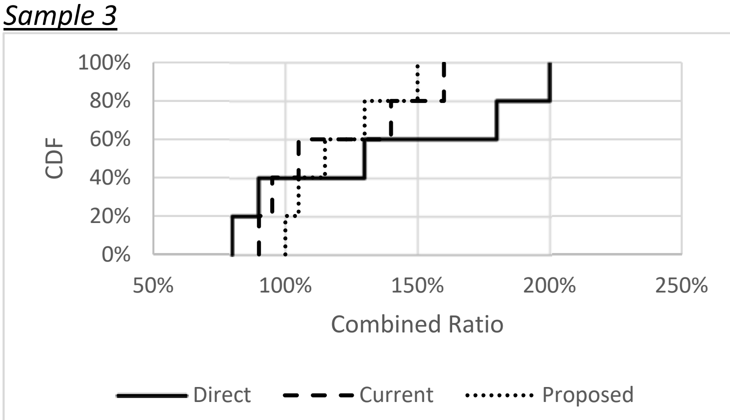
Sample 1



Sample 2



Sample 3



Part b: 0.5 point

Sample 1

Proposed program has lower Combined Ratio in the tail, use proposed.

Sample 2

I choose proposed because it outperforms current & bare at around α = .7 and above, even though it performs worse at lower α's.

Part c: 0.75 point

Sample 1

- V@R = 80th percentile = 180%
- TV@R = (180 + 200)/2 = 190%
- XTV@R = TV@R – mean = 190% - $\frac{(80+90+130+180+200)}{5}$ = 54%

Sample 2

- VaR = E(X| X = 80%) = 180%
- TVaR = E(X| X>80%) = 200%
- XTVaR = TVaR – mean = 200% - 136% = 64%

Sample 3

- V@R = 180%
- TV@R = Area under curve where CDF > 80% = 180%
- XTV@R = TV@R – mean = 180% - 136% = 44%

Part d: 0.25 point

Sample Responses

- WTVaR is TVaR calculated with probability distribution weighted to emphasize large loss.
- WTVaR is similar to TVaR, however the probabilities assigned to the simulations are assigned through a Wang Transformation instead of being uniform like TVaR.

Part e: 0.5 point

Sample Responses

- TVaR treats all losses equally. This doesn't always reflect true risk appetite. WTVaR corrects this by placing more weight on extreme values.
- TVaR treats losses linearly in the tail (i.e., a loss 2x as large is 2x as bad), which does not accurately reflect market attitudes. WTVaR is not linear and places higher weights on more extreme losses, which is more appropriate.

EXAMINER'S REPORT

Candidates were expected to compare and select a reinsurance program and provide statistics based off of a small set of simulations.

Part a

Candidates were expected to provide a plot of Combined Ratios vs CDF with the three sets of simulations plotted.

Common mistakes included:

- Providing alternative charts such as a Program vs Combined Ratio plot
- Plotting simulations on an axis
- Not sorting the observations
- Only providing discrete points for CDF

Part b

Candidates were expected to identify the proposed reinsurance program that would satisfy the actuary's concern about tail risk.

Common mistakes included: • Not selecting the program based on the concern over tail risk
Part c
Candidates were expected to provide a numerical solution for each of the three statistics using the direct data. There were several acceptable answers for TV@R and they were all given full credit. Common mistakes included: • V@R set at the 3rd observation (60th percent) rather than the 4th observation • Incorrect formula used for XTV@R • Calculating statistics for the current or proposed program instead of the direct book
Part d
Candidates were expected to describe that WTVaR is the same calculation as TVaR with transformed probabilities to put additional weight on the tail values. Common mistakes included: • Responding with only the acronym fully spelled out.
Part e
Candidates were expected to describe that TVaR does not properly reflect the risk averse attitude towards risk and that WTVaR corrects for this by applying additional weight on larger losses to reflect their increasing impact to the company. Common mistakes included: • Not stating the risk averseness or market attitude of increasingly larger losses.

QUESTION 20	
TOTAL POINT VALUE: 2.25	LEARNING OBJECTIVE(S): C5, C6

SAMPLE ANSWERS

Part a: 0.75 point

Sample 1

Copula 1 is normal copula as it's more symmetric and has more points bunched up together in the lowest probabilities (0, 0.1), as well as highest probabilities (0.9, 1), indicating tail dependency on both tails, which normal has.
Copula 2 is HRT copula as has points bunched up primarily in right tail (0.9, 1.0), implying dependency only in right-tail, which is heavy.

Sample 2

Copula 1 is the normal copula as it is more uniform across the graph.
Copula 2 has higher density in the top corner which indicates an HRT copula.

Sample 3

Copula 1 is the Normal copula. It is symmetrical, and appears to have lower correlation in the right tail.
Copula 2 is HRT. Much higher correlation in the right tail (top right corner).

Part b: 0.5 point

Sample 1

- HRT copula is advantageous for LOBs that are generally not correlated (workers comp & homeowners) but show high correlation in the cases of extreme events (earthquake).
- Normal copula is easier to generalize when we want to join more than 2 lines of business.

Sample 2

- Normal can handle more than two lines, HRT can't.
- HRT has much heavier density in right tail and it is the case that dependency is highest in tail for cat risk.

Sample 3

- Normal: easy to invert and simulate losses
- HRT: CATs are tail events so higher tail weight on right is appropriate

Part c: 1 point

Sample 1

- Expected value: Same result since copula only influence correlation and thus the variance part. Will not influence expect value.
- $VaR_{0.9}$: HRT copula since the risks correlate more under HRT the $VaR_{0.9}$ could be much larger since it captures the loss at tail.

Sample 2

- Expected value – These should produce the same expected value given that the overall Kendall t value is the same.
- $VaR_{0.9}$ – HRT will provide a higher value than Normal, given there is more correlation in the tail for HRT than Normal.

Sample 3

- Copula should not change the expected value of the LOBs it is correlating. Correlation impacts variability. Thus, this should be the same.
- HRT -> $VaR_{0.9}$ is a right tail event. More substantial correlation can be seen at this point in the scatter plot. Higher likelihood at both LOB's being bad thus, HRT.

EXAMINER'S REPORT

Given two scatter plots, candidates were expected to correctly identify which plot is the Normal copula and which is the HRT, to name an advantage of each copula in the case that two lines of business are subject to catastrophe events, to identify which copula will produce the highest joint expected value and joint $VaR_{0.9}$, and to explain why.

Part a

Candidates were expected to correctly identify which scatter plot corresponds to the Normal copula and which one is the HRT copula.

To obtain full credit, candidates should have correctly identified the two copulas and justified their choice by stating a reason for each copula.
The reasons can be about correlation and/or observations on the scatter plots.

Common mistakes included:

- Misidentification of the copulas
- Same reason given for both copula
- Reason(s) given for HRT copula only

Part b

Candidates were expected to name an advantage of each copula in the case that two lines of business are subject to catastrophe events.

If candidates gave two advantages for one copula, only the first one was graded.

Common mistakes included (Normal copula):

- Talked about the left tail correlation (good for non-cat years)
- Also has a right tail correlation (not an advantage over the HRT for catastrophe events)
- Easy to invert (with no other reason)
- Easy to explain (with no other reason)

Common mistakes included (HRT copula):

- No recurring mistakes were noted by the graders

Part c

Candidates were expected to identify which copula will produce the highest expected value and which one will produce the highest $VaR_{0.9}$.

Candidates generally performed better on the $VaR_{0.9}$ part of the question than the expected value part.

Common mistakes included (expected value):

- Candidates named one copula or the other (the correct response was that choice of copula does not affect expected values)

Common mistakes included ($Var_{0.9}$):

- No recurring mistakes were noted by the graders

QUESTION 21	
TOTAL POINT VALUE: 2.75	LEARNING OBJECTIVE(S): C8
SAMPLE ANSWERS	
Part a: 2 points	

Sample responses are separated into the four theories listed in the question:

Theory i

- There is a data lag as pricing indications are done on historical data which might indicate a rate decrease, however the rate may be inadequate but due to the lag we don't see it in the data yet. A further rate decrease brings rates too low.
- Data lags can make insurers slow to react to changes in trends. Using lagged data with low frequency and severity trends could cause prices to fall even when more recent trends (not contemplated yet) have risen. Therefore, business is being written below adequacy.
- There is a lag in data aggregation by the statistical bureau, creating a lag in rate regulation, which may result in the regulator approving inadequate rates.
- Regulators may refuse to approve or take a long time to approve necessary rate increases, resulting in insurers having to keep prices low.

Theory ii

- In competitive markets, companies may undercut each other on price in order to gain market share. This can happen especially when an insurance product is new and there is little regulation or data.
- A new entrant may enter without much experience. They may be priced lower than other more experienced or sophisticated companies. The new entrant will increase the supply and drive down the price for insurance.

Theory iii

- With more capital through a capital infusion or otherwise, insurers have more surplus and can afford to write more business. When supply increases, the price decreases.
- When insurers have more capital, they have increased capacity and need to write more premium to earn their target ROEs.

Theory iv

- An increase in interest rates in the market will increase the investment income insurers earn so they will increase their supply because they are able to make enough money even with lower underwriting income.
- If GDP shrinks, unemployment rises, or there is a general economic recession, demand drops because insureds have fewer assets to protect and diminished purchasing power. If supply stays about the same, this would result in a lower equilibrium price.
- A large, unexpected increase in inflation will increase claim costs, resulting in inadequate rates.

Part b: 0.75 point

- Capital and supply of insurance. Capital infusion will give companies more capacity to write more policies, increasing supply. Shocks that reduce capital (e.g. cat event) will reduce capacity and decrease supply.
- Capital and demand curve. If customers are sensitive to quality, demand will increase after a capital infusion since the insurers have more capital and are less likely to go insolvent.
- Profit flows and capital. When the industry is profitable, capital increases due to retained earnings and capital infusions. When the industry is not profitable, capital decreases due to operating losses and exits.
- Supply and demand curves. They are linked because their intersection determines the equilibrium price and quantity of insurance.
- Supply and demand curves. They are linked by capital supply which can increase with capital infusions or decrease with shocks (such as catastrophe events). When capital rises, supply curve shifts to the right since insurers have increase capacity to write business. Demand also shifts to the right (to a smaller extent) due to perception of increase insurance quality. Overall, price decreases. The opposite movements occur when a shock happens.
- Statistical validity of the technical approach and recognition of human factors from the soft approach. The two components are linked by analyzing supply and demand, capital capacity, etc. with the incorporation of expert opinions to model the underwriting cycle.

EXAMINER'S REPORT

Candidates were expected to know four theories of the underwriting cycle and describe how each theory would explain why markets soften. Candidates were expected to provide an example of how two components function in an econometric model of the underwriting cycle.

Part a

Theory i

Candidates were expected to know that the uncertainty inherent in projecting future losses, which may be exacerbated by data reporting and regulatory delays, can result in inadequate pricing.

Common mistakes included:

- Responses related to management action/underwriter incentive plans, or changes in technology.

Theory ii

Candidates were expected to know that companies may charge lower prices due to an optimistic view of the future or a desire to increase market share, and that competitive markets tend to move to the lowest price offered (the "winner's curse").

Theory iii

Candidates were expected to know that an increase in industry capital or capacity would lead to an increase in the supply of insurance, resulting in a lower equilibrium price.

Theory iv Candidates were expected to explain how certain general economic factors can lead to a softening of insurance prices. Common mistakes included: • Vague responses that failed to sufficiently describe how a specific factor would close lower market prices. Common mistakes included: • Describing what would cause market hardening rather than softening, or market fluctuations rather than softening specifically. • Describing what would cause lower premiums but not lower premium adequacy.
Part b
Candidates were expected to either: • Name two variables used in the econometric model and describe how they are related to or influence each other; or • Describe how the econometric model blends the soft and technical styles of modeling. Common mistakes included: • Inaccurately depicting how an econometric model functions. • Naming a specific technical modeling method and soft modeling method as the two components. • Describing a time series model.