Exam ST

Exam ST

Models for Stochastic Processes and Statistics

October 29, 2015

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points (each?).

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2015 and that the exam name is ST.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam ST" included in your exam packet.

   - Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

   - Candidates should employ a non-parametric test unless otherwise specified in the problem, or when there is a standard distribution that is logically or commonly associated with the random variable in question. Examples of problems with a logical or commonly associated distribution would include exponential wait times for Poisson processes and applications of the Central Limit Theorem.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2015.

END OF INSTRUCTIONS

1.

For two Poisson processes, N_1 and N_2, you are given:

- N_1 has intensity function $\lambda_1(t) = \begin{cases} 2t & \text{for} \quad 0 < t \leq 1 \\ t^3 & \text{for} \quad t > 1 \end{cases}$
- N_2 is a homogenous Poisson process.
- $\text{Var}[N_1(3)] = 4\,\text{Var}[N_2(3)]$

Calculate the intensity of N_2 at t = 3.

A. Less than 1

B. At least 1, but less than 3

C. At least 3, but less than 5

D. At least 5, but less than 7

E. At least 7

2.

For a health insurance policy, the annual number of claims follows the Poisson process with the mean, λ, equal to 5.

Calculate the probability that the third claim occurs after one year, that is $Pr(T_3>1)$.

A. Less than 0.10
B. At least 0.10, but less than 0.11
C. At least 0.11, but less than 0.12
D. At least 0.12, but less than 0.13
E. At least 0.13

3.

The number of customers who use an ATM for withdrawals follows the Poisson process with the rate equal to 100 per day. The amounts withdrawn are distributed as follows:

Amount Withdrawn	Probability
$20	0.25
$40	0.50
$50	0.10
$100	0.15

Calculate the standard deviation of the total daily withdrawals.

A. Less than $500

B. At least $500, but less than $510

C. At least $510, but less than $520

D. At least $520, but less than $530

E. At least $530

EXAMINATION CONTINUED ON NEXT PAGE

4.

You are given the following:

- X is a uniformly distributed random variable with the probability density function:

$$f_X(x) = \frac{1}{\theta} \qquad 0 \leq x \leq \theta$$

- $\hat{\theta} = cX_{max}$ where $X_{\max}$ is the largest X_i from a sample of size n.

Determine the value of c that makes $\hat{\theta}$ an unbiased estimator of θ when $n = 100$.

A. Less than 1.00

B. At least 1.00, but less than 1.02

C. At least 1.02, but less than 1.04

D. At least 1.04, but less than 1.06

E. At least 1.06

EXAMINATION CONTINUED ON NEXT PAGE

5.

You are given:

- An insurance product with a per loss limit of 200 covers losses from an exponential distribution with parameter θ.
- Based on the following table, the maximum likelihood estimate of θ is 168.

Size of Loss	Number of Claims	Sum of Losses
Less than 200	1,114	142,752
At least 200	N	200 x N

Calculate N for the table above.

A. Less than 250

B. At least 250, but less than 550

C. At least 550, but less than 850

D. At least 850, but less than 1,150

E. At least 1,150

EXAMINATION CONTINUED ON NEXT PAGE

6.

You are given two independent samples from an exponential distribution with mean, θ. The samples have three and ten observations, respectively.

The relative efficiency is the ratio of the lower bound of the variance of the estimator the mean, θ, for the first sample to the lower bound of the variance of the estimator the mean, θ, for the second sample.

Calculate the relative efficiency of the first sample to the second one using the Rao-Cramer Lower Bound.

A. Less than 20%

B. At least 20%, but less than 40%

C. At least 40%, but less than 60%

D. At least 60%, but less than 80%

E. At least 80%

EXAMINATION CONTINUED ON NEXT PAGE

7.

You are given:

- Assume losses follow a Single-parameter Pareto distribution.
- $\theta = 150$
- Losses given are 200, 300, 350, 400.

Find the Maximum Likelihood Estimate of α.

A. Less than 0.8

B. At least 0.8, but less than 1.2

C. At least 1.2, but less than 1.6

D. At least 1.6, but less than 2.0

E. At least 2.0

EXAMINATION CONTINUED ON NEXT PAGE

8.

Let $Y_1, Y_2, \dots, Y_n$ be a random sample of size n from a distribution with the probability density function.

$$f(y|\theta) = \begin{cases} \left(\frac{2y}{\theta}\right) e^{-\left(\frac{y^2}{\theta}\right)}, & 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Determine which of the following formulas is a sufficient statistic for θ using the factorization criterion.

A. $\sum_{i=1}^{n} Y_i$

B. $\sum_{i=1}^{n} Y_i^2$

C. $\sum_{i=1}^{n} Y_i^4$

D. $\sum_{i=1}^{n} e^{(Y_i)}$

E. The distribution has no sufficient statistic for θ.

EXAMINATION CONTINUED ON NEXT PAGE

9.

Let $X_1, X_2, \ldots, X_n$ be a random sample of independent and identically distributed observations from a population with probability density function

$$f(x|\theta) = \begin{cases} \left(\frac{x^3}{6\theta^4}\right) e^{-\left(\frac{x}{\theta}\right)}, & 0 \le x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Determine the maximum likelihood estimator of θ.

A. $\bar{x}$

B. $\frac{1}{4}\bar{x}$

C. $\bar{x}^2$

D. $3\bar{x}^2$

E. $n/\sum_1^n 3/x_i$

EXAMINATION CONTINUED ON NEXT PAGE

10.

You are given:

- A random sample of size n is observed from a Pareto distribution with parameters $\theta = 20$ and unknown α.
- H_0: $\alpha = 4$.
- H_1: $\alpha = 6$.

Using the Neyman-Pearson Lemma, determine the form of the best critical region.

A. $\sum \ln(x_i) \geq c$

B. $\sum \ln(x_i) \leq c$

C. $\bar{x} \geq c$

D. $\sum \ln(x_i + 20) \geq c$

E. $\sum \ln(x_i + 20) \leq c$

11.

You are given:

- A sample of size n is observed from a normal distribution with variance 64.
- H_0: $\mu = 2$.
- H_1: $\mu = 6$.
- H_0 is rejected when $\overline{X} \geq 5.25$.

Calculate the smallest sample size, n, needed so that the probability of a Type II error is less than 0.10.

A. Less than 150

B. At least 150, but less than 200

C. At least 200, but less than 250

D. At least 250, but less than 300

E. At least 300

EXAMINATION CONTINUED ON NEXT PAGE

12.

You are given:

- 100 decks of 52 cards are combined and shuffled to put the cards in a random order.
- Each deck contains only red and black cards.
- The first 4 cards are revealed to be all red cards.
- H_0: 50% of the cards are red.
- H_A: More than 50% of the cards are red.

Calculate the smallest significance level for which H_0 can be rejected.

A. Less than 0.005

B. At least 0.005, but less than 0.010

C. At least 0.010, but less than 0.050

D. At least 0.050, but less than 0.100

E. At least 0.100

EXAMINATION CONTINUED ON NEXT PAGE

13.

You are given:

- The following table of values for samples A and B:

	Number of Observations	$\frac{1}{n}\sum x$	$\frac{1}{n}\sum(x-\bar{x})^2$
Sample A	11	30	5
Sample B	17	30	6

- H_0: $\sigma_A{}^2 = \sigma_B{}^2$
- H_A: $\sigma_A{}^2 > \sigma_B{}^2$
- Samples A and B are Normally distributed

Calculate the smallest p-value for which H_0 can be rejected.

A. Less than 0.010

B. At least 0.010, but less than 0.025

C. At least 0.025, but less than 0.050

D. At least 0.050, but less than 0.100

E. At least 0.100

EXAMINATION CONTINUED ON NEXT PAGE

14.

You observe the following distribution for a random sample of 100 claims.

Layer	Number of Claims
Less than 5,000	31
Between 5,000 and 10,000	19
Between 10,000 and 15,000	26
Greater than 15,000	24

You are testing the null hypothesis that claim severity follows a Weibull distribution with θ=10,000 and τ=1.10, against the alternative hypothesis that claim severity does not follow the distribution specified in the null hypothesis.

Using the chi-square goodness-of-fit test to evaluate the null hypothesis, what is the conclusion?

A. Do not reject the null hypothesis at the 5.0% significance level

B. Reject the null hypothesis at the 5.0% significance level, but not at the 2.5% significance level

C. Reject the null hypothesis at the 2.5% significance level, but not at the 1.0% significance level

D. Reject the null hypothesis at the 1.0% significance level, but not at the 0.5% significance level

E. Reject the null hypothesis at the 0.5% significance level

EXAMINATION CONTINUED ON NEXT PAGE

15.

You are given that a random variable follows one of two discrete probability distribution functions where each categorical value from I to XII is assigned a probability. The two probability distribution functions are displayed in the table below:

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
D_0	0.025	0.025	0.050	0.075	0.125	0.200	0.200	0.125	0.075	0.050	0.025	0.025
D_1	0.200	0.125	0.075	0.050	0.025	0.025	0.025	0.025	0.050	0.075	0.125	0.200

- H_o : D_o is the probability distribution function
- H_1 : D_1 is the probability distribution function
- The significance level is set at 10%.

Determine the combination of values that defines the critical region which provides the most powerful test.

A. I + II + III

B. I + II + X

C. I + II + XI+XII

D. III + X

E. X+XI+XII

16.

You are given the following information:

- A random sample consisting of n observations is taken from a normal distribution with mean μ and variance 1000.
- The sample mean is 3.5.
- H_0: $\mu = 0$.
- H_1: $\mu \neq 0$.
- A 90% confidence interval calculated from this sample is (-2.109, 9.109) for the sample mean using the Central Limit Theorem.

Calculate the smallest significance level at which the null hypothesis would be rejected.

A. Less than 10%

B. At least 10%, but less than 18%

C. At least 18%, but less than 26%

D. At least 26%, but less than 34%

E. At least 34%

17.

You are given a random sample of 7 observations from a continuous distribution:

1.2 2.6 3.4 4.5 5.0 5.2 8.1

The following hypothesis test is performed using the Sign Test:

- H_0: median = 6.0
- H_1: median < 6.0

Calculate the p-value of this test using the exact calculation.

A. Less than 5%

B. At least 5%, but less than 7.5%

C. At least 7.5%, but less than 10%

D. At least 10%, but less than 12.5%

E. At least 12.5%

EXAMINATION CONTINUED ON NEXT PAGE

18.

You are given the following information:

- For a general liability policy loss amounts, Y, follow the Pareto distribution with probability density function:

$$f(y) = 3\,\theta^3(y+\theta)^{-4}, \quad \theta = 1000, \quad 0 < y$$

- For reinsurance purposes we are interested in the distribution of the largest loss amount in a random sample of size 10, which is denoted by Y_{10}.

Calculate the 90$^{\text{th}}$ percentile of Y_{10}.

A. Less than 3,500

B. At least 3,500, but less than 3,550

C. At least 3,550, but less than 3,600

D. At least 3,600, but less than 3,650

E. At least 3,650

EXAMINATION CONTINUED ON NEXT PAGE

19.

An insurance company is examining the median age, m, of its policyholders. The company performs the following hypothesis test:

- $H_0: m = 40.$
- $H_1: m \neq 40.$

The following age values are observed from its portfolio of policies:

26 30 34 35.5 36 41 42 43 45 47 48 55 60

Calculate the signed-rank Wilcoxon statistic for this test.

A. Less than 13.5

B. At least 13.5, but less than 14.5

C. At least 14.5, but less than 15.5

D. At least 15.5, but less than 16.5

E. At least 16.5

EXAMINATION CONTINUED ON NEXT PAGE

20.

Let $Y_1 < Y_2 < ... < Y_{15}$ be the order statistics of a random sample from a uniform distribution on [0, 1].

Calculate the probability that $0.75 < Y_{15} < 0.85$.

A. Less than 5%
B. At least 5%, but less than 6%
C. At least 6%, but less than 7%
D. At least 7%, but less than 8%
E. At least 8%.

21.

You wish to explain Y using the following multiple regression model and 32 observations:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

A linear regression package generates the following table of summary statistics:

	Estimated Coefficient	Standard Error
Intercept	44.200	5.960
β_1	-0.295	0.118
β_2	9.110	6.860
β_3	-8.700	1.200

For the Intercept and each of the betas, you decide to reject the null hypothesis which is that the Estimated Coefficient is zero at α=10% significance.

Which variables have coefficients significantly different from zero?

A. Intercept
B. Intercept, X_1
C. Intercept, X_2
D. Intercept, X_1, X_3
E. Intercept, X_2, X_3

EXAMINATION CONTINUED ON NEXT PAGE

22.

In an experiment with three blocks, three treatments, and a total of nine observations you are given the following summary of results:

	Sum of Squares	Mean Square
Between Treatments	$\frac{146}{9}$	$\frac{73}{9}$
Between Blocks	$\frac{50}{9}$	$\frac{25}{9}$
Residual	$\frac{10}{9}$	$\frac{5}{18}$
Total	$\frac{206}{9}$	

- H_0: The treatment means are all equal.
- H_1: The treatment means are not all equal.
- A block is defined to be a group of three observations.
- The results are Normally distributed.

Calculate the smallest p-value for which H_0 can be rejected.

(A) Less than 1%

(B) At least 1%, but less than 2.5%

(C) At least 2.5%, but less than 5%

(D) At least 5%, but less than 10%

(E) At least 10%

23.

For a large number of health insurance policies, the annual number of claims per policy follows the binomial distribution with parameters $m = 10$ and q, which varies by policy. The prior probability density function of q is:

$$h(q) = 12q^2(1-q), \qquad 0 < q < 1.$$

In the first year, two claims are reported for a policy.

Calculate the standard deviation of the posterior distribution of q for this policy.

A. Less than 0.10

B. At least 0.10, but less than 0.20

C. At least 0.20, but less than 0.30

D. At least 0.30, but less than 0.40

E. At least 0.40

EXAMINATION CONTINUED ON NEXT PAGE

24.

You are given:

- For a general liability policy, the log (natural logarithm) of paid claim amounts conditionally follows the normal distribution with mean μ, and variance $\sigma^2 = 1$.
- The posterior distribution of μ follows the normal distribution with variance $= \frac{1}{n+1}$, where n denotes the number of observed claims paid.
- The following are the five observed log of paid claim amounts:

3.22	4.34	5.98	7.32	2.78

Calculate the length of the symmetric 95% Bayesian credible interval for μ.

A. Less than 1.0
B. At least 1.0, but less than 1.5
C. At least 1.5, but less than 2.0
D. At least 2.0, but less than 2.5
E. At least 2.5

EXAMINATION CONTINUED ON NEXT PAGE

25.

You are given:

- Daily claim counts follow a Poisson distribution with mean λ.
- The prior distribution of λ has probability density function:

$$f(\lambda) = \frac{1}{5}\ e^{-\lambda/5}$$

- Three claims were observed on the first day.

Calculate the variance of the posterior distribution of λ.

A. Less than 1.0

B. At least 1.0, but less than 2.0

C. At least 2.0, but less than 3.0

D. At least 3.0, but less than 4.0

E. At least 4.0

END OF EXAMINATION

Fall 2015 Exam ST Answer Key

question#	answer
1	B
2	D
3	C
4	B
5	A
6	B, E
7	C
8	B
9	B
10	E
11	B
12	D
13	E
14	C
15	C
16	D
17	B
18	C
19	E
20	D
21	D
22	A
23	B
24	C
25	C